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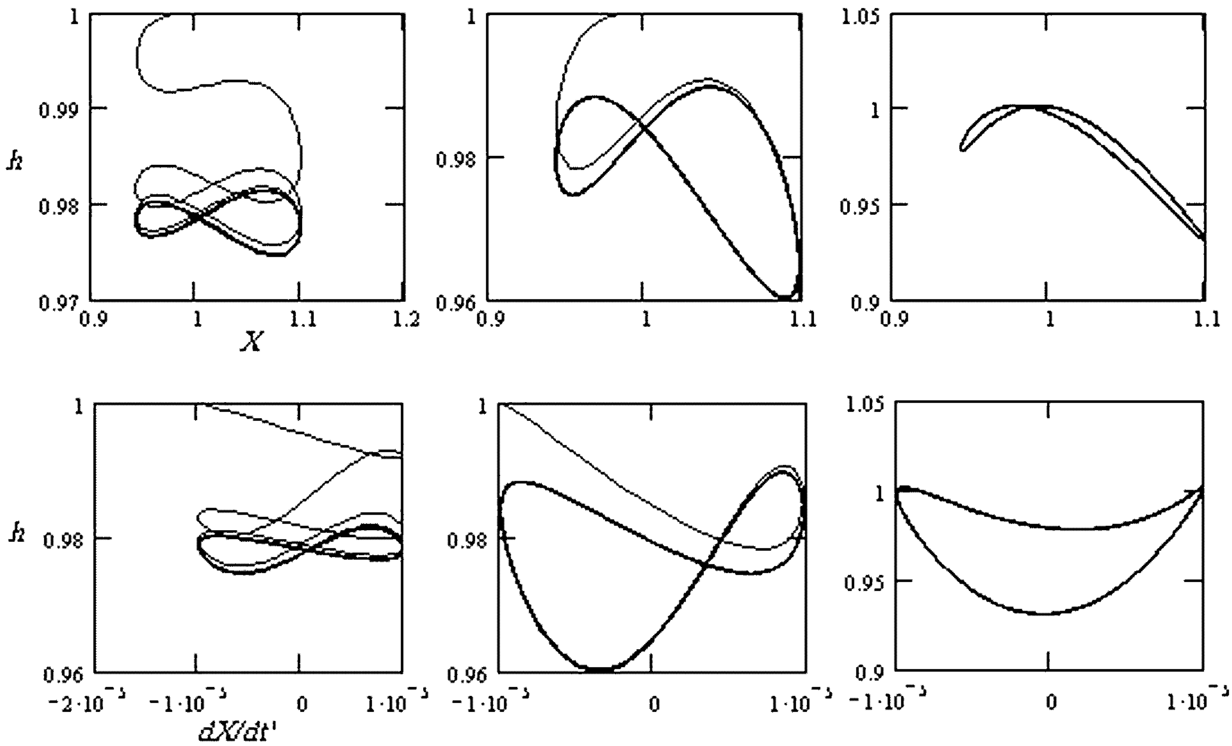

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PERFECT FLUID FRW MODELS WITH TIME VARYING CONSTANTS REVISITED

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In this paper we revise a perfect fluid FRW model with time-varying constants "but" taking into account the effects of a "c-variable" into the curvature tensor. We study the model under the following assumptions, $div(T) = 0$ and $div(T) \neq 0$, and in each case the flat and the non-flat cases are studied. Once we have outlined the new field equations, it is showed in the flat case i.e. $K = 0$, that there is a non-trivial homothetic vector field i.e. that this case is self-similar. In this way, we find that there is only one symmetry, the scaling one, which induces the same solution that the obtained one in our previous model. At the same time we find that "constants" G and c must verify, as integration condition of the field equations, the relationship $G/c^2 = const.$ in spite of that both "constants" vary. We also find that there is a narrow relationship between the equation of state and the behavior of the time functions G, c and the sign of Λ in such a way that these functions may be growing as well as decreasing functions on time t, while Λ may be a positive or negative decreasing function on time t. In the non-flat case it will be showed that there is not any symmetry. For the case $div(T) \neq 0$, it will be studied again the flat and the non-flat cases. In order to solve this case it is necessary to make some assumptions on the behavior of the time functions G, c and Λ. We also find the flat case with $div(T) = 0$, is a particular solution of the general case $div(T) \neq 0$.

1. Introduction

Since the pioneering work of Dirac ([1]), who proposed, motivated by the occurrence of large numbers in Universe, a theory with a time variable gravitational coupling constant G, cosmological models with variable G and nonvanishing and variable cosmological term have been intensively investigated in the physical literature (see for example [2]-[14]).

Recently , the cosmological implications of a variable speed of light during the early evolution of the Universe have been considered. Varying speed of light (VSL) models proposed by Moffat ([15]) and Albrecht and Magueijo ([16]), in which light was travelling faster in the early periods of the existence of the Universe, might solve the same problems as inflation. Hence they could become a valuable alternative explanation of the dynamics and evolution of our Universe and provide an explanation for the problem of the variation of the physical "constants". Einstein's field equations (EFE) for Friedmann–Robertson–Walker (FRW) spacetime in the VSL theory have been solved by Barrow ([17]), who also obtained the rate of variation of the speed of light required to solve the flatness and cosmological constant problems (see J. Magueijo ([18]) for a review of these theories).

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This model is formulated under the strong assumption that a c variable (where c stands for the speed of light) does not introduce any corrections into the curvature tensor, furthermore, such formulation does not verify the covariance and the Lorentz invariance as well as the resulting field equations do not verify the Bianchi identities either (see Bassett et al [19]).

Nevertheless, some authors (T. Harko and M. K. Mak [20] and P.P. Avelino and C.J.A.P. Martins [21]) have proposed a new generalization of General Relativity which also allows arbitrary changes in the speed of light, c, and the gravitational constant, G, but in such a way that variations in the speed of light introduce corrections to the curvature tensor in the Einstein equations in the cosmological frame. This new formulation is both covariant and Lorentz invariant and as we will show the resulting field equations (FE) verify the Bianchi identities. We will be able to obtain the energy conservation equation from the field equations as in the standard case.

The purpose of this paper is to revise a FRW perfect fluid model with time varying constants (see [22] and [23]) but taking into account the effects of a c variable into the field equations. In our previous models ([22]-[23]) we worked under the assumption that a $c-$variable does not induce corrections into the curvature tensor and hence the classical Friedman equations remain valid. We will show that such effect is minimum but exists. In section 2, once we have outlined the Einstein tensor, we check that it verifies that its covariant divergence vanishes. Consequently we impose the same condition to the right hand of the Einstein equation i.e. $div\left(\frac{8\pi G}{c^4}T^{ij}+g^{ij}\Lambda\right)=0$. In this way we set the new field equations as well as the Kretschmann scalars. As we will see, in order to integrate the resulting field equations we will need to consider some assumptions. In section 3, we will consider the condition $div\left(T^{ij}\right)=0$, and it will be studied two particular cases, the flat case, i.e. $K=0$, and the non-flat case, $K\neq 0$. Since with the proposed method we are not able to solve the so called flat problem we need to study separately both cases (the flat and the non-flat cases) introducing such conditions as an assumption.

In this approach, in the case $K=0$, we will show that the model is self-similar since it is found a non-trivial homothetic vector field. In order to integrate the FE under the assumptions $div(T)=0$ and $K=0$, we use the Lie group tactic which allows us to find a particular form of G and c for which our FE admit symmetries i.e. are integrable. As we will show under these assumptions the FE, only admit one symmetry, the scaling symmetry. This is the main difference with respect to our previous approach ([22]-[23]) where we did not consider the effects of a c-variable into the curvature tensor in such a way that the resulting FE admitted more symmetries. We also obtain as integration condition that the "constants" G and c must verify the relationship $G/c^2=const.$ in spite of that both "constants" vary. In this work we have found three solutions. The first one is very similar to the de-Sitter solution i.e. the energy density vanishes, the cosmological constant is a true constant while G and c follow an exponential law as the scale factor. The second and third solutions behave as the standard FRW where all the quantities follow power law with respect to time t, nevertheless the third solution is non-singular. In the second studied case, the non-flat case $K\neq 0$, we will show that the FE do not admit any symmetry. Nevertheless we will try to find a particular solution imposing some restrictions.

In section 4 we consider the possibility that $div(T)\neq 0$, and we will study again two particular cases, the flat case, i.e. $K=0$, and the non-flat case, $K\neq 0$. The possibility that the covariant conservation condition $div(T)=0$ be relaxed has been advanced by Rastall ([24]), who pointed out that a non-zero divergence of the energy–momentum tensor has not been ruled out experimentally at all yet. In order to integrate the resulting FE we will need to make some assumptions (scaling assumptions) about the behavior of the "constants" G,c and Λ, which allow us to find particular solutions to the FE. This scaling assumptions work well in the flat (homothetic) case, as it is expected, but they seem very restrictive in the non-flat case. It is also showed that we can recover the solution obtained in the $div(T)=0$ as a particular solution of this model in such a way that G and c must verify the relationship $G/c^2=const.$ etc... as it is expected.

In section 5 we end with a brief conclusions.

2. The model

We will use the field equations in the form:

$$R_{ij}-\frac{1}{2}g_{ij}R=\frac{8\pi G(t)}{c^4(t)}T_{ij}+\Lambda(t)g_{ij}, \tag{1}$$

where arbitrary variations in c and G will be allowed. We assume that variations in the speed of light introduce corrections to the curvature terms in the Einstein equations in the cosmological frame. In our model variations in the velocity of light are always allowed to contribute to the curvature terms. These contributions are computed from the metric tensor in the usual way. The line element is defined by (we are following the O'Neill's notation [25]):

$$ds^2=-c(t)^2dt^2+f^2(t)\times$$
$$\times\left[\frac{dr^2}{1-Kr^2}+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)\right], \tag{2}$$

and the energy momentum tensor is:

$$T_{ij}=(\rho+p)\,u_iu_j-pg_{ij}, \tag{3}$$

where p and ρ satisfie the usual equation of state, $p=\omega\rho$ in such a way that $\omega=const.$, usually ω is taken such that, $\omega\in(-1,1]$, that is to say, our universe is modeled by a perfect fluid. The $4-$velocity u^i is defined as follows $u^i=\left(c^{-1}(t),0,0,0\right)$ such that $u_iu^i=-1$.

The field equations are as follows:

$$2\frac{f''}{f}-2\frac{c'}{c}\frac{f'}{f}+\frac{f'^2}{f^2}+\frac{Kc^2}{f^2}=-\frac{8\pi G}{c^2}p+\Lambda c^2, \tag{4}$$

$$\frac{f'^2}{f^2}+\frac{Kc^2}{f^2}=\frac{8\pi G}{3c^2}\rho+\frac{1}{3}\Lambda c^2, \tag{5}$$

where as we can see, the only difference with respect to the usual one is the factor $-2\frac{c'}{c}\frac{f'}{f}$ in eq. (4). Here a prime denotes differentiation with respect to time t.

Since the divergence of $\left(R_{ij}-\frac{1}{2}g_{ij}R\right)$ vanishes then we impose that the right hand of equation (1) has zero divergence too. Therefore applying the covariant divergence to the second member of the field equation we get:

$$div\left(\frac{8\pi G}{c^4}T^{ij}+g^{ij}\Lambda\right)=0, \tag{6}$$

where we are considering that $\frac{8\pi G}{c^4}$ is a function on time t. Hence simplifying it yields:

$$\rho'+3\left(\omega+1\right)\rho H=-\left(\frac{G'}{G}-4\frac{c'}{c}\right)\rho-\frac{\Lambda' c^4}{8\pi G}, \tag{7}$$

where $H = f'/f$.

Therefore the new field equations are as follows:

$$2H' - 2\frac{c'}{c}H + 3H^2 + \frac{Kc^2}{f^2} = -\frac{8\pi G}{c^2}p + \Lambda c^2, \tag{8}$$

$$H^2 + \frac{Kc^2}{f^2} = \frac{8\pi G}{3c^2}\rho + \frac{1}{3}\Lambda c^2, \tag{9}$$

$$\rho' + 3(\omega+1)\rho H = -\left(\frac{G'}{G} - 4\frac{c'}{c}\right)\rho - \frac{\Lambda' c^4}{8\pi G}, \tag{10}$$

As we have mentioned in the introduction these field equations are not new in the literature. They have been outlined by T. Harko and M. Mak ([20]) where they study a perfect fluid model and the influence of the time variation of the constants in the matter creation. Later they study Bianchi I and V model with time varying constants (taking into account the influence of a c-variable into the curvature tensor). Other author are P.P. Avelino and C.J.A.P. Martins ([21]) and H. Shojaie and M. Farhoudi, have obtained similar equations ([26]) since these authors consider G as a true constant.

We would like to emphasize that deriving eq. (9) and substituting this result into eq. (8) it is obtained eq.(7) i.e. the covariant divergence of the right hand of our field equation in the same way as in the standard cosmological model.

Curvature is described by the tensor field R^i_{jkl}. It is well known that if one uses the singular behavior of the tensor components or its derivates as a criterion for singularities, one gets into trouble since the singular behavior of the coordinates or the tetrad basis rather than the curvature tensor. In order to avoid this problem, one should examine the scalars formed out of the curvature. The invariants $RiemS$ and $RiccS$ (the Kretschmann scalars) are very useful for the study of the singular behavior, being these as follows:

$$RiemS = R_{ijlm}R^{ijlm}, \tag{11}$$

$$RiemS := \frac{12}{c^4}\left[\left(\frac{f''}{f}\right)^2 - 2\frac{f''}{f}\frac{f'}{f}\frac{c'}{c} + \left(\frac{f'}{f}\right)^2\left(\frac{c'}{c}\right)^2 + \left(\frac{f'}{f}\right)^4 + 2\left(\frac{f'}{f}\right)^2\frac{c^2K}{f^2} + \frac{c^4K}{f^4}\right] \tag{12}$$

and

$$RiccS := R_{ij}R^{ij},$$

$$RiccS := \frac{12}{c^4}\left[\left(\frac{f''}{f}\right)^2 - 2\frac{f''}{f}\frac{f'}{f}\frac{c'}{c} + \left(\frac{f'}{f}\right)^2\left(\frac{c'}{c}\right)^2 + \left(\frac{f'}{f}\right)^4 + 2\left(\frac{f'}{f}\right)^2\frac{c^2K}{f^2} + \frac{c^4K}{f^4} + \frac{f''}{f}\left(\frac{c^2K}{f^2} + \left(\frac{f'}{f}\right)^2\right) - \frac{f'}{f}\frac{c'}{c}\frac{c^2K}{f^2} - \frac{c'}{c}\left(\frac{f'}{f}\right)^3\right]. \tag{13}$$

In order to try to solve eqs. (8)-(10) we need to make some hypotheses on the behavior of the quantities.

3. Solution I. $divT = 0$.

In this first solution we make the following assumption

$$divT = 0 \tag{14}$$

and we will consider two subclasses, the flat and the non-flat cases.

Our tactic consists in studying the field equations through the Lie method (see [27]−[28]−[29]). In particular we seek the forms of G and c for which our field equations admit symmetries i.e. are integrable.

We will consider the following assumption $divT = 0$, which transforms eq.(10) into these two new equations:

$$\rho' + 3(\omega+1)\rho H = 0, \tag{15}$$

$$\frac{G'}{G} - 4\frac{c'}{c} = -\frac{\Lambda' c^4}{8\pi G\rho}. \tag{16}$$

In order to use the Lie method, we rewrite the field equations as follows. From (8 and 9) we obtain

$$2H' - 2\frac{c'}{c}H - 2\frac{Kc^2}{f^2} = -\frac{8\pi G}{c^2}(p+\rho), \tag{17}$$

From equation (15), we can obtain

$$H = -\frac{1}{3(\omega+1)}\frac{\rho'}{\rho} \Longrightarrow H = -C_1\frac{\rho'}{\rho}, \tag{18}$$

where $C_1 = \frac{1}{3(\omega+1)}$, therefore

$$-2C_1\left(\frac{\rho''}{\rho} - \frac{\rho'^2}{\rho^2}\right) + 2C_1\frac{\rho'}{\rho}\frac{c'}{c} - 2\frac{Kc^2}{f^2} = = -\frac{8\pi G}{c^2}(\omega+1)\rho, \tag{19}$$

hence

$$\rho'' = \frac{\rho'^2}{\rho} + \rho'\frac{c'}{c} - 2\frac{Kc^2}{f^2}\rho + A\frac{G}{c^2}\rho^2, \tag{20}$$

where $A = 12\pi(\omega+1)^2 > 0, \forall\omega$. Following this tactic we try to make the smallest hypothesis number and to obtain the exact behavior of the "constants" G, c and Λ. Following other tactics we are obliged to make assumptions that could be unphysical.

3.1. Flat case. Self-similar approach.

As have been pointed out by Carr and Coley ([30]), the existence of self- similar solutions (Barenblatt and Zeldovich ([31])) is related to conservation laws and to the invariance of the problem with respect to the group of similarity transformations of quantities with independent dimensions. This can be characterized within general relativity by the existence of a homothetic vector field and for this reason one must distinguish between geometrical and physical self-similarity. Geometrical similarity is a property of the spacetime metric, whereas physical similarity is a property of the matter fields (our case). In the case of perfect fluid solutions admitting a homothetic vector, geometrical self-similarity implies physical self-similarity.

As we show in this section as well as in previous works, the assumption of self-similarity reduces the mathematical complexity of the governing differential equations. This makes such solutions easier to study mathematically. Indeed self-similarity in the broadest Lie sense refers to an invariance which allows such a reduction.

Perfect fluid space-times admitting a homothetic vector within general relativity have been studied by Eardley ([32]). In such space-times, all physical transformations occur according to their respective dimensions, in such a way that geometric and physical self-similarity coincide. It is said that these space-times admit a transitive similarity group and space-times admitting a non-trivial similarity group are called self-similar. Our model i.e. a flat FRW model with a perfect fluid stress-energy tensor has this property and as already have been pointed out by Wainwright ([33]), this model has a power law solution.

Under the action of a similarity group, each physical quantity ϕ transforms according to its dimension q under the scale transformation. For space-times with a transitive similarity group, dimensionless quantities are therefore spacetime constants. This implies that the ratio of the pressure of the energy density is constant so that the only possible equation of state is the usual one in cosmology i.e. $p = \omega\rho$, where ω is a constant. In the same way, the existence of homothetic vector implies the existence of conserved quantities.

In the first place we would like to emphasize that under the hypothesis $K = 0$, we have found that the space-time (M, g) is self-similar since we have been able to calculate a homothetic vector field, $X_H \in \mathfrak{X}(M)$, that is to say

$$L_X g = 2g, \tag{21}$$

where

$$X_H = \left(\frac{\int c(t)dt + C_1}{c(t)}\right)\partial_t + \\ + \left(1 - \left(\frac{\int c(t)dt + C_1}{c(t)}\right)H\right)(x\partial_x + y\partial_y + z\partial_z). \tag{22}$$

This kind of space-times have been studied by (Eardly ([32]), Wainwright ([33]), K. Rosquits and R. Jantzen ([34]) and for reviews see Carr and Coley ([30]) and Duggal et al ([35])).

As we have mentioned above we are only interested in the case $K = 0$ for this reason eq. (20) yields

$$\rho'' = \frac{\rho'^2}{\rho} + \rho'\frac{c'}{c} + A\frac{G}{c^2}\rho^2. \tag{23}$$

Now, we apply the standard Lie procedure to this equation. A vector field $X = \xi(t,\rho)\partial_t + \eta(t,\rho)\partial_\rho$, is a symmetry of (23) iff

$$\xi_\rho + \rho\xi_{\rho\rho} = 0, \tag{24}$$

$$\eta c - 2c'\rho^2\xi_\rho - c\rho\eta_\rho + c\rho^2\eta_{\rho\rho} - 2c\rho^2\xi_{t\rho} = 0, \tag{25}$$

$$-cc'\rho\xi_t - 3\rho^3 AG\xi_\rho + \left(c'^2\rho - cc''\rho\right)\xi + \\ +2c^2\rho\eta_{t\rho} - c^2\rho\xi_{tt} - 2c^2\eta_t = 0, \tag{26}$$

$$\rho^2 AG\eta_\rho + c^2\eta_{tt} - cc'\eta_t - 2\rho AG\eta - \\ -2\rho^2 AG\xi_t + \rho^2 A\xi\left(2G\frac{c'}{c} - G'\right) = 0, \tag{27}$$

Solving (24-27), we find that

$$\xi = at + b, \qquad \eta = -2a\rho, \tag{28}$$

subject to the following constrains, from eq. (26):

$$c'' = \frac{c'^2}{c} - \frac{ac'}{at+b}, \tag{29}$$

and from eq. (27)

$$\frac{G'}{G} = 2\frac{c'}{c}, \tag{30}$$

where a and b are numerical constants.

We would like to emphasize that this is the main difference with respect to our previous work ([22]) where we found that the model admitted more symmetries. Note that in this paper we are considering that a c-variable introduces corrections into the curvature tensor, this possibility brings us to obtain a new eq. (23) which contains an extra term $\left(\rho'\frac{c'}{c}\right)$ with respect to the employed one in our previous paper ([22]).

In this situation we have found that this model only admits two symmetries $X_1 = \partial_t$ i.e. a movement and $X_2 = t\partial_t - 2\rho\partial_\rho$, i.e. a scaling symmetry as it is expected in this kind of models (self-similar model). In such a way that $[X_1, X_2] = X_1$ i.e. they form a L_2 Lie-algebra, see ([28]).

From (30) we can see that

$$G \approx c^2 \qquad i.e. \qquad \frac{G}{c^2} = const := B, \tag{31}$$

(where we will assume that $B > 0$) that is to say, that both constants vary but in such a way that the relationship $\frac{G}{c^2}$ remains constant for any t and independently of the any value of the constants a and b. For this reason our equation (23) may be rewritten as follows

$$\rho'' = \frac{\rho'^2}{\rho} + \rho'\frac{c'}{c} + A\rho^2, \tag{32}$$

where $A = 12\pi(\omega+1)^2 B = const > 0, \forall\omega, (\omega \neq -1)$, in such a way that all the restrictions come from eq. (29).

The knowledge of one symmetry X might suggest the form of a particular solution as an invariant of the operator X i.e. as solution of

$$\frac{dt}{\xi}=\frac{d\rho}{\eta}, \tag{33}$$

this particular solution is known as an invariant solution (generalization of similarity solution) furthermore an invariant solution is in fact a particular singular solution.

In order to solve (32), we consider the following cases.

3.1.1. Case I.

Taking $a=0, b\neq 0$, we get

$$c''=\frac{c'^2}{c}, \qquad \Longrightarrow \qquad c(t)=K_2 e^{K_1 t}, \tag{34}$$

where the $(K_i)_{i=1}^2$ are integration constants. In this way we find that from eq. (30) G behaves as:

$$G=K_2^2 e^{2K_1 t}. \tag{35}$$

Therefore the equation (32) yields:

$$\rho''=\frac{\rho'^2}{\rho}+\rho' K_1+A\rho^2, \tag{36}$$

where $X_1=\partial_t$. The use of the canonical variables brings us to an Abel ode:

$$y'=-Ax^2y^3-K_1y^2-\frac{y}{x}, \tag{37}$$

where $x=\rho$ and $y=1/\rho'$, since we cannot solve it, then we try to obtain a solution through the invariants

This method brings us to the following relationship

$$\frac{dt}{\xi}=\frac{d\rho}{\eta}\Longrightarrow \rho\approx const:=\rho_0, \tag{38}$$

which seems not to be physical. This particular solution must satisfy eq. (32) which means that $\rho=0$. With this solution we go back to eq. (9)

$$3H^2=\Lambda c^2, \tag{39}$$

and from eq. (16) it is obtained the behavior of "constant" Λ

$$\left(\frac{G'}{G}-4\frac{c'}{c}\right)\rho=-\frac{\Lambda' c^4}{8\pi G}=0\Longrightarrow \Lambda=\Lambda_0=cons., \tag{40}$$

therefore

$$3H^2=\Lambda_0 K_2^2 e^{2K_1 t}\Longrightarrow \tag{41}$$

$$f=C_1\exp\left(\frac{\sqrt{\frac{\Lambda_0}{3}}}{K_1}\exp(K_1 t)\right), \tag{42}$$

with

$$RiemS:=24K_1^4, \qquad RiccS:=36K_1^4. \tag{43}$$

This symmetry has brought us to obtain a super de Sitter solution, with a energy density vanishing and with a scale factor growing like a power of exponential functions, while G and c follow an exponential functions that depends of the constant K_1. If for example we fix $K_1=1$, then there is a sudden singularity in a finite time, in this case both "constants" G and c are growing functions on time t. While if $K_1=-1$, the scale factor reaches an asymptotic behavior with G and c decreasing on time t. In any case $\Lambda=const.>0$, i.e. is a true positive constant.

3.1.2. Case II.

Taking $b=0, a\neq 0$, we get that the infinitesimal X is $X_1=t\partial_t-2\rho\partial_\rho$, which is precisely the generator of the scaling symmetries. Therefore the invariant solution will be the same than the obtained one with the dimensional method.

With these values of a and b equation (29) yields

$$c''=\frac{c'^2}{c}-\frac{c'}{t}, \qquad \Longrightarrow \qquad c(t)=K_2 t^{K_1}, \tag{44}$$

as we expected, $c(t)$ follows a power-law solution (self-similar solution), where K_1 and K_2 are numerical constants, $K_1, K_2\neq 0$. We would like to emphasize that with this behavior for c the homothetic vector field behaves as:

$$X_H=\frac{t}{K_1+1}\partial_t+\left(1-\frac{tH}{K_1+1}\right)(x\partial_x+y\partial_y+z\partial_z) \tag{45}$$

as in the standard model (see for example Eardley ([32]) and Wainwrith ([33])).

In this way G behaves as:

$$G=K_2^2 t^{2K_1}. \tag{46}$$

Therefore equation (32) yields:

$$\rho''=\frac{\rho'^2}{\rho}+\rho'\frac{K_1}{t}+A\rho^2, \tag{47}$$

we expect that this ode admits a scaling solution (i.e. that the energy density follows a power law solution) as it is expected in this kind of models.

The canonical variables bring us to obtain an Abel ode

$$y'=x\left(2+2K_1-Ax\right)y^3-(1+K_1)y^2-\frac{y}{x}, \tag{48}$$

where $x=\rho t^2$ and $y=\frac{1}{t^2(\rho' t+2\rho)}$, and which solution is unknown

The solution obtained through invariants is:

$$\frac{dt}{\xi}=\frac{d\rho}{\eta}\Longrightarrow \rho\approx t^{-2}, \qquad \rho=\frac{2(1+K_1)}{At^2}, \tag{49}$$

finding that $K_1>-1$ from physical considerations i.e. $\rho>0$ iff $K_1>-1$. This is the kind of solution expected in a self-similar model (see for example Wainwrith ([33]) and Jantzen ([34])).

From eq. (16) it is obtained the behavior of "constant" Λ

$$\Lambda' = 16\pi B\rho\frac{c'}{c^3} \Longrightarrow \Lambda = -\frac{16\pi BK_1}{K_2^2 A t^{2(1+K_1)}}, \tag{50}$$

as it is observed if $K_1 > 0$ then Λ is a negative decreasing function on time t, while if $K_1 < 0$ then Λ is a decreasing function on time t, but with the restriction $K_1 \in (-1,0)$. We would like to emphasize that the self-similar relationship

$$\Lambda \approx \frac{1}{c^2t^2} \approx \frac{1}{K_2^2 t^{2(1+K_1)}}, \tag{51}$$

is trivially verified as it is expected in this kind of solutions.

Now, we will calculate the scale factor f. In order to do that we may follow two ways, in the first one, from eq. (15) we find that

$$f = f_0 t^{2/3(\omega+1)}, \tag{52}$$

where $f_0 = \left(\frac{A_\omega A}{2(1+K_1)}\right)^{1/3(\omega+1)}$, observing again that necessarily $K_1 > -1$.

If we follow our second way it is found that from eq. (9 with $K = 0$)

$$3H^2 = \frac{16\pi B}{At^2} \qquad \Longrightarrow \qquad f = f_0 t^{\sqrt{H_0}}, \tag{53}$$

where $H_0 = \frac{16\pi B}{3A}$, but taking into account the value of $A = 12\pi(\omega+1)^2 B$, then $H_0 = \frac{4}{9(\omega+1)^2}$, and hence $\sqrt{H_0} = \frac{2}{3(\omega+1)}$, as it is expected from eq. (52). Therefore this new way does not add any more information.

In this way we found that

$$H = \sqrt{H_0}t^{-1}, \quad q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1}{\sqrt{H_0}} - 1. \tag{54}$$

therefore $q < 0$ iff $\omega \in \left(-1, -\frac{1}{3}\right)$, note that if $\omega < -1$ (phantom cosmology), then $f(t)$ is a decreasing function on time t.

If we make the assumption (scaling symmetry) on the scale factor

$$f \approx ct \qquad \Longrightarrow \qquad f \approx t^{K_1+1} \tag{55}$$

in such a way that equating this expression with eq. (52) then

$$K_1 = -\frac{1+3\omega}{3(\omega+1)} \tag{56}$$

finding that $K_1 > 0 \Longleftrightarrow \omega \in \left(-1, -\frac{1}{3}\right)$. Hence, if $K_1 > 0 \Longleftrightarrow \omega \in \left(-1, -\frac{1}{3}\right)$, G and c are growing functions on time t, while $\Lambda < 0$, i.e. is a negative decreasing function on time t. In other case, if $K_1 < 0, G$ and c are decreasing functions on time t, while $\Lambda > 0$, i.e. is a decreasing function on time t.

The Kretschmann scalars behave as:

$$RiemS \approx \frac{1}{K_2^4 t^{4(1+K_1)}}, \qquad RiccS \approx \frac{1}{K_2^4 t^{4(1+K_1)}}, \tag{57}$$

finding in this way that if $K_1 < -1$ (forbidden possibility) then both scalars tend to zero while if $K_1 > -1$, then both scalars tend to infinity i.e. there is a true singularity.

In the first place we would like to emphasize that this is the solution that we have obtained in our previous works using D.A. (see [23] and [36]). This is due to two reasons. The first one, because D.A. is a powerful tool that may be used even when the FE are not well outlined. The second one, because in the previous works we were using only three of the FE, ignoring eq. (8) and using eqs. (9,15 and 16). Therefore the solution found in those papers work well and we refer to them for all the physical considerations.

In this approach we are supposing that $q < 0$ i.e. the universe accelerates due to a equation of state $\omega \in \left(-1, -\frac{1}{3}\right)$, but as it has been pointed out by R.G.Vishwakarma (see [37]) the acceleration of the universe may be explained through different mechanisms in such a way that $q < 0$ while $\omega \in [0,1]$. In this way we would like to stress that to solve some of the cosmological problems that present the standard model, we have found that $K_1 > -1$, with $K_1 = -\frac{1+3\omega}{3(\omega+1)}$ in such a way that if $K_1 > 0$ iff $\omega \in \left(-1, -\frac{1}{3}\right)$ then $q < 0$ and G and c are growing functions on time t, while Λ is a negative decreasing function on time t. With $K_1 < 0$, $q > 0$ and G and c are decreasing functions on time t, while Λ is a positive decreasing function on time t, i.e. there is a narrow relationship between the behavior of G and c and the sign of Λ controlled by ω.

3.1.3. Case III.

This case is a generalization of the above case, simply, we will avoid the singular case. Taking $a, b \neq 0$ we get

$$c'' = \frac{c'^2}{c} - \frac{ac'}{at+b}, \qquad \Longrightarrow \qquad c(t) = K_2(at+b)^{\frac{K_1}{a}}, \tag{58}$$

obtaining in this case that G behaves as:

$$G = K_2^2(at+b)^{\frac{2K_1}{a}}, \tag{59}$$

hence equation (23) yields:

$$\rho'' = \frac{\rho'^2}{\rho} + \rho'\frac{K_1}{at+b} + A\rho^2, \tag{60}$$

The canonical variables bring us to an Abel ode

$$y' = x\left(2a^2 + 2aK_1 - Ax\right)y^3 - (a+K_1)y^2 - \frac{y}{x}, \tag{61}$$

where $x = \rho(at+b)^2$ and $y = \frac{1}{(at+b)^2(\rho'(at+b)+2a\rho)}$, that is to say, we obtain a very complicate ode which at this time we do not know how to solve. Nevertheless, as we have pointed out in ([23]) the general solution of this kind of equations are unphysical, for this reason it is sufficient consider particular solutions obtained through the invariants.

The solution obtained through invariants is:

$$\frac{dt}{\xi}=\frac{d\rho}{\eta}\Longrightarrow\rho=\frac{2a(a+K_1)}{A\left(at+b\right)^2}, \tag{62}$$

where it is observed that $K_1>-a$ (but if you choose $a=b=1$, then we have the same result as before i.e. $K_1>-1$). Therefore the scale factor behaves as:

$$f=\left(\frac{A_\omega A\left(at+b\right)^2}{2a(a+K_1)}\right)^{1/3(\omega+1)}, \tag{63}$$

$$\Longrightarrow\qquad f=f_0\left(at+b\right)^{2/3(\omega+1)} \tag{64}$$

which is very similar to the last result (see eq. (52) but in this occasion this solution is non-singular).

We end finding the behavior of Λ, as in previous cases taking into account eq. (16) it is obtained:

$$\Lambda'=16\pi B\rho\frac{c'}{c^3} \tag{65}$$

$$\Longrightarrow\qquad\Lambda=-\frac{16\pi}{AK_2^2}\frac{BaK_1}{\left(at+b\right)^{2\left(\frac{K_1}{a}+1\right)}}, \tag{66}$$

as we can see in this case if $K_1>0$ then Λ is a negative decreasing function on time t.

The Kretschmann scalars behaves as:

$$RimS\approx\frac{1}{K_2^4\left(at+b\right)^{4(1+K_1)}}, \tag{67}$$

$$RiccS\approx\frac{1}{K_2^4\left(at+b\right)^{4(1+K_1)}}, \tag{68}$$

showing a non-singular state when t runs to zero.

This solution is very similar to the previous one except that this solution is non-singular. In fact, the above solution is a particular solution of this one.

3.2. The non-flat case.

In this subsection we go next to study the particular case $K\neq0$. One of the drawbacks of the above approach is that we need to make the assumption $K=0$ i.e. our approach is unable of solving the so-called flatness problem. In order to research if it is possible to solve such problem we go next to study eq. (20) through the Lie method, seeking symmetries that allow us to obtain any solution in closed from. But as we will see eq. (20) does not admit any symmetry in such a way that in order to obtain a particular solution we will impose a concrete symmetry, but this is precisely the method that we are trying to avoid, to make assumptions or at least to make the minor number of assumptions or to make assumptions under any physical or mathematical (symmetries) well founded reasons. We only explore one case.

Therefore the equation under study is:

$$\rho''=\frac{\rho'^2}{\rho}+\rho'\frac{c'}{c}-2\frac{Kc^2}{f^2}\rho+A\frac{G}{c^2}\rho^2, \tag{69}$$

but as

$$divT=0\Longleftrightarrow\rho=A_\omega f^{3(\omega+1)},\Longrightarrow \tag{70}$$

$$f=\left(\frac{\rho}{A_\omega}\right)^{\frac{1}{3(\omega+1)}},\Longrightarrow f=(\rho)^{\frac{1}{3(\omega+1)}}, \tag{71}$$

and hence

$$\rho''=\frac{\rho'^2}{\rho}+\rho'\frac{c'}{c}-2c^2\rho^a+A\frac{G}{c^2}\rho^2, \tag{72}$$

where $a=\frac{3\omega+1}{3(\omega+1)}$, and for simplicity we have adopted the case $K=1$.

The Lie group method brings us to obtain the following system of pdes

$$\rho\xi_{\rho\rho}+\xi_\rho=0, \tag{73}$$

$$-\rho^{-1}\eta_\rho-2\frac{c'}{c}\xi_\rho+\eta_{\rho\rho}-2\xi_{t\rho}+\rho^{-2}\eta=0, \tag{74}$$

$$\left(\left(\frac{c'}{c}\right)^2-\frac{c''}{c}\right)\xi+3\rho^2\left(2c^2\rho^{a-2}-A\frac{G}{c^2}\right)\xi_\rho-$$

$$-2\rho^{-1}\eta_t-\xi_{tt}+2\eta_{t\rho}-\frac{c'}{c}\xi_t=0, \tag{75}$$

$$\eta_{tt}-\frac{c'}{c}\eta_t+\rho^2\left(A\frac{G}{c^2}-2c^2\rho^{a-2}\right)\eta_\rho+$$

$$+2\rho^2\left(2c^2\rho^{a-2}-A\frac{G}{c^2}\right)\xi_t+$$

$$+A\rho^2\frac{G}{c^2}\left(4\rho^{a-2}\frac{c^4}{AG}\frac{c'}{c}-\frac{G'}{G}+2\frac{c'}{c}\right)\xi+$$

$$+2\rho\left(ac^2\rho^{a-2}-A\frac{G}{c^2}\right)\eta=0, \tag{76}$$

which has no solution, that is to say, eq. (72) does not admit any symmetry, for this reason we will need to follow other approaches.

For example, if we “impose” any particular symmetry X, maybe we may found some restrictions for the behavior of the quantities G,c and ρ. We will explore such possibility.

3.2.1. Case I.

In this case, we choose $(\xi = 1, \eta = 0)$, *i.e.* $X = \partial_t$, in such a way that from eq.(75) it is obtained the following restriction

$$c'' = \frac{c'^2}{c} \quad \Longrightarrow \quad c(t) = K_2 e^{K_1 t}, \tag{77}$$

and from eq.(76) it is obtained the following one

$$2\left(2\rho^{a-2}\frac{c^4}{AG}+1\right)\frac{c'}{c} = \frac{G'}{G}, \tag{78}$$

hence

$$2\left(2\rho^{a-2}\frac{K_2^4 e^{4K_1 t}}{AG}+1\right)K_1 = \frac{G'}{G}, \tag{79}$$

as we can see from eq. (78), we cannot obtain the condition $G = Bc^2$ (as in the flat solution) since such condition means that

$$4\rho^{a-2}\frac{c^4}{AG}\frac{c'}{c} = \frac{G'}{G} - 2\frac{c'}{c} = 0 \Longleftrightarrow \rho = 0, \tag{80}$$

that is to say, the energy density vanishes.

In order to find a particular solution to eq. (79) we impose the condition $a = 2$ (as mathematical condition) which means that $\omega = -\frac{5}{3} \ll -1$, although such possibility is very restrictive (and maybe unphysical, the ultra phantom equation of state). In this way it is found that

$$G' = 2K_1 G + \frac{4K_1 K_2^4 e^{4K_1 t}}{A}, \tag{81}$$

$$\Longrightarrow G(t) = C_1 e^{2tK_1} + \frac{2}{A}K_2^4 e^{4(tK_1)}, \tag{82}$$

where C_1 is an integration constant, therefore eq. (72) yields

$$\rho'' = \frac{\rho'^2}{\rho} + \rho' K_1 + \left(A\frac{C_1}{K_2^2}\right)\rho^2, \tag{83}$$

since this equation has not an explicit (analytical) solution, note that we have obtained the same eq. as in section 3.1 case I eq. (36), we find again that a particular solution is $\rho = 0$, therefore from the field equation (9)

$$H^2 + \frac{K_2^2 e^{2K_1 t}}{f^2} = \frac{\Lambda_0}{3}K_2^2 e^{2K_1 t}, \tag{84}$$

and hence in this case we have found that a solution is:

$$f = \frac{\sqrt{3}}{\sqrt{\Lambda_0}} \times$$
$$\times \left[\frac{2}{3} + \frac{1}{6}\exp\left(\frac{2\sqrt{3}}{3}\frac{\sqrt{\Lambda_0}K_2}{K_1}\left(\exp(K_1 t) + C_1\right)\right) \times \right.$$
$$\left. \times \exp\left(\frac{-\sqrt{3}}{3}\frac{\sqrt{\Lambda_0}K_2}{K_1}\left(\exp(K_1 t) - C_1\right)\right)\right] \tag{85}$$

since

$$0 = \left(\frac{G'}{G} - 4\frac{c'}{c}\right)\rho = -\frac{\Lambda' c^4}{8\pi G} \tag{86}$$

$$\Longrightarrow \Lambda = \Lambda_0 = const. > 0. \tag{87}$$

This solution looks very unphysical or at least very restrictive, a vanishing energy density and a constant cosmological constant and is very similar to the obtained one in section 3.1. case I the super de-Sitter solution, with the same behavior and the same restriction for the numerical constant K_1, i.e. sudden singularities or asymptotic behavior depending of the sign of K_1. In this case, the employed method does not allow us to obtain the behavior of G, c and Λ since the equation under study does not admit any symmetry.

4. Solution II. $divT \neq 0$.

In the previous section we have made the assumption that $div(T) = 0$ i.e. the divergence of the energy–momentum tensor vanishes. Nevertheless we have obtained as general conservation equation eq. (10) i.e.

$$\rho' + 3(\omega+1)\rho H = -\left(\frac{G'}{G} - 4\frac{c'}{c}\right)\rho - \frac{\Lambda' c^4}{8\pi G}, \tag{88}$$

i.e. $div(T) = \rho' + 3(\omega+1)\rho H \neq 0$ and we have assumed a particular case (with perfect mathematical sense) $div(T) = 0$. In this section we will study the general case $div\,(T) \neq 0$.

The possibility that cosmological and physical considerations may require that the covariant conservation condition $div(T) = 0$ be relaxed has been advanced by Rastall ([24]), who pointed out that a non-zero divergence of the energy–momentum tensor has as yet not been ruled out experimentally at all. In Rastall's theory ([24]), the divergence of T is assumed to be proportional to the gradient of the scalar curvature S, $div(T) = \lambda grad(S)$, where¸ $\lambda = constant$, and in fact the modified field equations are equivalent to standard general relativity with an additional variable Λ term. We refer to the reader to the Harko and Mark work ([20]) to see a matter creation and thermodynamical approach in this context.

4.1. The flat case.

In this case we are going to consider the field equations "but" without the condition $div(T) = 0$, i.e. the field equations will be

$$2H' - 2\frac{c'}{c}H + 3H^2 = -\frac{8\pi G}{c^2}p + \Lambda c^2, \tag{89}$$

$$3H^2 = \frac{8\pi G}{c^2}\rho + \Lambda c^2, \tag{90}$$

$$\rho' + 3(\omega+1)\rho H = -\left(\frac{G'}{G} - 4\frac{c'}{c}\right)\rho - \frac{\Lambda' c^4}{8\pi G}, \quad (91)$$

but as we can see we have 2 equations with 5 unknowns, therefore it is necessary to make some assumptions, for example in the behavior of G, c and Λ as follows:

$$G = G_0 H^a, \quad c = c_0 H^b, \quad \Lambda = \Lambda_0 c^{-2} H^2, \quad (92)$$

where G_0, c_0 and Λ_0 are dimensional constants and $a, b \in \mathbb{R}$, without any restriction i.e. we do not need to assume any concrete sing or value for these numerical constants. Furthermore, we must stress that the conditions $K = 0$, together to power law assumptions bring us to a scaling solution.

Taking into account these assumptions and form eq. (90) we obtain ρ

$$\rho = \left(\frac{3-\Lambda_0}{d_0}\right) H^{2(1+b)-a}, \qquad \rho = \rho_0 H^{2(1+b)-a}, (93)$$

where $d_0 = 8\pi G_0/c_0^2$, and taking this relationship into eq. (91), we obtain the following ode in quadrature

$$\alpha\frac{H'}{H} + 3(\omega+1) H = -a\frac{H'}{H} + 4b\frac{H'}{H} - \tilde{K}\frac{H'}{H}, \quad (94)$$

and therefore

$$\left(\alpha + a - 4b + \tilde{K}\right)\frac{H'}{H^2} = -3(\omega+1), \quad (95)$$

where $\alpha = 2(1+b) - a$, $\tilde{K} = \left(2(1-b)c_0^2\Lambda_0/8\pi G_0\rho_0\right)$, and therefore

$$\frac{H'}{H^2} = -\frac{(\omega+1)(3-\Lambda_0)}{2(1-b)}, \quad (96)$$

$$\Longrightarrow \qquad H = h_0 n t^{-1}, \Longrightarrow \qquad f = f_0 t^{h_0 n}, \quad (97)$$

where $n = \frac{(\omega+1)(3-\Lambda_0)}{2(1-b)}$, and $h_0 = const > 0$, is an integration constant and we impose that $b \neq 1, \Lambda_0 \neq 3$ and $\omega \neq -1$. As we can see, there are some restrictions, for example, if $\omega \in (-1, 1]$ then b, Λ_0 must verify at the same time that $b < 1$ and $\Lambda_0 < 3$ or $b > 1$ and $\Lambda_0 > 3$. Now if $\omega < -1$ then it should exist a combination between the signs of b, Λ_0 such that $n > 0$, in other way, the radius of the Universe decreases. It is observed that the behavior of the scale factor does not depend of the constant a, i.e. of the behavior of the gravitational "constant".

Once we have obtained the behavior of f i.e. of H we go next to complete our calculations of the rest of the quantities i.e.

$$\rho = \rho_0 H^{2(1+b)-a} \approx t^{a-2(1+b)}, \quad (98)$$

with the restrictions

$$\rho_0 (h_0 n)^{2(1+b)-a} > 0, \qquad a - 2(1+b) < 0, \quad (99)$$

i.e. we are assuming that the energy density is a positive decreasing function on time t.

It is observed too that if $2b = a$, then we obtain the particular solution $\rho \approx t^{-2}$, as well as the realtionship $G/c^2 = const.$ as in the above cases, i.e. the obtained solution under the assumption $div(T) = 0$. In this way we can se that the $div(T) = 0$ case is a particular solution of the $div(T) \neq 0$ case, as one may expected, but we have not any physical or mathematical (symmetry or integrability condition) reason to assume such relationship.

We end calculating the behavior of the "constants" G, c and Λ i.e.

$$G = G_0 h_0^a n^a t^{-a}, \qquad c = c_0 (h_0 n)^b t^{-b}, \quad (100)$$

$$\Lambda = \Lambda_0 c_0^{-2} (h_0 n)^{2(b-1)} t^{2(b-1)}, \quad (101)$$

in such a way that Λ will be a decreasing function on time iff $b < 1$.

In this way we found that

$$H = h_0 n t^{-1}, \qquad and \quad (102)$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{2(1-b)}{h_0(\omega+1)(3-\Lambda_0)} - 1, \quad (103)$$

therefore $q < 0$ iff $|2(1-b)| < |h_0(\omega+1)(3-\Lambda_0)|$.

The Kretschmann scalars behave as:

$$RiemS \approx \frac{1}{c_0^4 t^{4(1-b)}}, \qquad RiccS \approx \frac{1}{c_0^4 t^{4(1-b)}}, \quad (104)$$

finding in this way that there is a true singularity.

In this case we have found a scaling solution as it is expected for this kind of models $(K = 0)$. As we can see the scale factor f does not depend of G only of c and Λ. The energy density ρ depends of G and c. Nevertheless we have not been able to find better restrictions for the introduced (ad hoc) numerical constants a and b such that they give us more information about the behavior of the time functions G, c and Λ

4.2. The non-flat case.

As in the above section we will study the case $K \neq 0$ separately. For this purpose the FE are now:

$$2H' - 2\frac{c'}{c}H + 3H^2 + \frac{Kc^2}{f^2} = -\frac{8\pi G}{c^2}p + \Lambda c^2, \quad (105)$$

$$H^2 + \frac{Kc^2}{f^2} = \frac{8\pi G}{3c^2}\rho + \frac{1}{3}\Lambda c^2, \quad (106)$$

$$\rho' + 3(\omega+1)\rho H = -\left(\frac{G'}{G} - 4\frac{c'}{c}\right)\rho - \frac{\Lambda' c^4}{8\pi G}, \quad (107)$$

but as we can see we have 2 equations with 5 unknowns, therefore it is necessary to make some assumptions, for example in the behavior of G, c and Λ as follows:

$$G = G_0 H^a, \; c = c_0 H^b, \; \Lambda = \Lambda_0 c^{-2} H^2, \quad (108)$$

where G_0, c_0 and Λ_0 are numerical constants. We must stress that in this occasion these hypotheses could be unphysical since $K \neq 0$ and we are imposing a scaling behavior typical of the flat case.

Taking into account these assumptions and form eq. (106) we obtain ρ

$$\rho = \left(\frac{3-\Lambda_0}{d_0}\right) H^{2(1+b)-a} + \frac{Kc_0H^{2b}}{f^2}, \tag{109}$$

$$\rho = \rho_0 H^{2(1+b)-a} + \frac{Kc_0H^{2b}}{f^2}, \tag{110}$$

where $d_0 = 8\pi G_0/c_0^2$, and taking this relationship into eq. (107), we obtain the following second order for f

$$f'' = -D_1\frac{(f')^{2b}}{f^{(1+2b)}} + D_2\frac{(f')^2}{f}, \tag{111}$$

where

$$D_1 = \frac{(3\omega+1)}{2}\frac{Kc_0^2}{(1-b)}, \tag{112}$$

$$D_2 = \left(1 - \frac{(\omega+1)}{2}\frac{(3-\Lambda_0)}{(1-b)}\right). \tag{113}$$

In the first place we may note that $(b \neq 1)$ and that if $\omega = -1/3 \Longrightarrow D_1 = 0$, independently of the value of constant K, in this case we are mainly interested in the $K \neq 0$ case. If $K = 0$, or $D_1 = 0$ then we obtain again the solution already obtained in the latter (last) case.

Calculation of Eq. (111). making the following change of variables it is obtained the first order ode

$$(x = f, \quad y = \frac{1}{f'}) \quad \Longrightarrow \tag{114}$$

$$y' = D_1 x^{-(1+2b)}y^{3-2b} - D_2 x^{-1} y, \tag{115}$$

and which solution is:

$$y = \left(C_1 x^{2D_2(1-b)} + \frac{2(b-1)D_1}{(2bD_2 - 2D_2 - 2b)}x^{-2b}\right)^{\frac{1}{2b-2}},$$

therefore

$$f' = \left(C_1 f^{2D_2(1-b)} + \frac{2(b-1)D_1}{(2bD_2 - 2D_2 - 2b)}f^{-2b}\right)^{\frac{1}{2(1-b)}},$$

and hence

$$t = \int^f J(u)du + C_2, \tag{116}$$

where

$$J(u) = \Big(C_1 u^{2D_2(1-b)} + \frac{2(b-1)D_1}{(2bD_2 - 2D_2 - 2b)}u^{-2b}\Big)^{\frac{1}{2(1-b)}}.$$

Since we have not obtain information about the behavior of f then we try to find a particular solution. For this purpose, we observe that eq. (111) admits the following symmetries:

$$X_1 = \partial_t, \qquad X_2 = t\partial_t + (1-b)f\partial_f, \tag{117}$$

where we would emphasize that X_2 is a scaling symmetry, maybe induced by the hypotheses about the behavior of G, c and Λ, (scaling relationships).

The invariant solution (particular solution) that induces X_2 is the following one:

$$\frac{dt}{t} = \frac{df}{(1-b)f} \qquad \Longrightarrow \qquad f = f_0 t^{(1-b)}, \tag{118}$$

with

$$f_0 = \frac{D_1(b-1)}{D_2(b-1)-b}\sqrt{\frac{D_2(b-1)-b}{D_1(b-1)}} = \tag{119}$$

$$= \frac{Kc_0^2(1+3\omega)}{(3\omega+1-\Lambda_0(\omega+1))}\sqrt{\frac{(3\omega+1-\Lambda_0(\omega+1))}{Kc_0^2(1+3\omega)}}, \tag{120}$$

and we necessarily impose that $b < 1$ and $b \neq 0$, this means that if $b \to 1$ them $f \to f_0$. As we can observe $K \neq 0$, and we must be careful with the signs since if $K = -1$ then it should be satisfied the relationship $3\omega + 1 < \Lambda_0(\omega+1)$.

In this way we found that

$$H = (1-b)t^{-1}, \qquad and \tag{121}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1}{1-b} - 1 = \frac{b}{1-b}, \tag{122}$$

therefore $q < 0$ iff $b < 0$, which implies that c is a growing function on time t.

If we make the assumption (scaling-symmetry) on the scale factor

$$f \approx ct \qquad \Longrightarrow \qquad f \approx t^{(1-b)}, \tag{123}$$

The Kretschmann scalars behave as:

$$RiemS \approx \frac{1}{C^4 t^{4(b-1)}}, \qquad RiccS \approx \frac{1}{C^4 t^{4(b-1)}}, \tag{124}$$

finding in this way that there is a true singularity.

Calculation of ρ, and the possible restrictions for the constants a and b.

$$\rho = \rho_0 (1-b)^{2b-a+2} t^{a-2(b+1)} + K\frac{c_0}{f_0^2}(1-b)^{2b} t^{-2}, \tag{125}$$

and

$$G \approx G_0 t^{-a}, \qquad c \approx c_0 t^{-b}, \qquad \Lambda \approx \Lambda_0^2 t^{2(b-1)}. \tag{126}$$

As we have seen, we have only been able to obtain a particular solution. Imposing so many hypotheses, we lose information and we do not know how to recover it in such a way that we are not able to know better the behavior of the time functions G, c and Λ.

5. Conclusions.

In this paper we have studied a perfect fluid FRW model with time-varying constants "but" taking into account the possible effects of a $c-$variable into the curvature tensor. In this way, as other authors have already pointed out, such effects are minimum in the field equations but they exists and are very restrictive. Under the made hypotheses, we have seen that the Einstein tensor has covariant divergence zero, in this way we have imposed that the right hand of the field equations has a vanishing divergence too i.e.

$$div\left(\frac{8\pi G}{c^4}T+g\Lambda\right)=0. \tag{127}$$

In this way we have obtained the set of the new FE, emphasizing the fact that we can recover eq. (127) from the field equations as in the standard case i.e. deriving one of them and simplifying with the other one.

In other to solve the resulting FE we have considered the following cases. In the first case we have imposed the condition $div(T)=0$, as a particular case of eq. (127) and we have studied the flat and non-flat cases. We have needed to make such distinction because with the employed method we are not able of solving the so called flatness problem, for this reason we have needed to study separately then.

The flat case under this hypothesis is the already studied one in our previous works (see [22]) and under this new considerations i.e. taking into account the effects of a c-variable into the curvature tensor, we have shown that the scaling solutions the only one while in ([22]), without this new assumption, we obtained more solutions, i.e. we obtained other solutions apart from the scaling solution. We would like to emphasize that it has been obtained as integration condition that the "constants" G and c must verify the relationship $G/c^2=cosnt.$ in spite of the fact that both "constants" vary. This result is in agreement with our scaling solution obtained in our previous work (see [22]) but in those works we needed to make such relationship as assumption.

We have also shown that this model is self-similar since we have been able of obtaining a non-trivial homothetic vector field. This result is agreement with the obtained scaling solution as it is well known.

With the obtained solution, if we want to solve the acceleration of the universe i.e. $q<0$, then the time function G and c are growing function on time t, while Λ is a negative decreasing function on time t, in this case the equation of state belong to the interval: $\omega\in(-1,-1/3)$. In other cases G and c are decreasing function while Λ is a positive decreasing function and in this case the equation of state belongs to the interval: $\omega\in[-1/3,1]$.

The non-flat case does not admit any symmetry and the particular studied case looks very unphysical.

The second class of studied models verifies the condition $div(T)\neq 0$, i.e. without imposing any restriction to eq. (127) and we have studied again the flat and non-flat cases. To solve these cases we have needed to make scaling assumptions on the behavior of the time functions G, c and Λ. These assumptions work well in the flat case (self-similar case) but in the non-flat case seem very restrictive. In the flat case, it is found for the made assumptions, that there is a relationship between the numerical constants that determine the behavior of G and c that brings us to obtain again the particular case $div(T)=0$, i.e. such case could be seen as a particular solution of the more general case $div(T)\neq 0$.

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References

[1] P. A. M. Dirac, Proc. R. Soc. London A165, 199 (1938).

[2] A-M. M. Abdel-Rahman, Nuovo Cimento B102, 225 (1988).

[3] A-M. M. Abdel-Rahman, Phys. Rev. D45, 3497 (1992).

[4] A. Beesham, Gen. Rel. Grav. 26, 159 (1993).

[5] M. S. Berman, Phys. Rev. D43, 1075 (1991).

[6] J. C. Carvalho, J. A. S. Lima and I. Waga, Phys. Rev. D46, 2404 (1992).

[7] W. Chen and Y. S. Wu, Phys. Rev. D41, 695 (1990).

[8] Y. K. Lau, Aust. J. Phys. 38, 547 (1985).

[9] Y. K. Lau and S. J. Prokhovnik, Aust. J. Phys. 39, 339 (1986).

[10] J. A. S. Lima and M. Trodden, Phys. Rev. D53, 4280 (1996).

[11] T. Singh, A. Beesham and W. S. Mbokazi, Gen. Rel. Grav. 30, 573 (1988).

[12] R. F. Sistero, Gen. Rel. Grav. 23, 1265 (1991).

[13] J. A. S. Lima and J. M. F. Maia, Phys. Rev. D49, 5597 (1994).

[14] I. Waga, Astrophys. J. 414, 436 (1993).

[15] J.W. Moffat, Int. J. Mod. Phys. D 2 351 (1993)

[16] A. Albrecht and J. Magueijo. Phys. Rev. D **59**, 043516 (1999)

[17] J.D. Barrow. Phys. Rev. D **59**, 043515 (1999)

[18] J. Magueijo. Rept.Prog.Phys. **66**, 2025, (2003)

[19] B.A. Bassett et al. Phys.Rev. D**62**, 103518, (2000)

[20] T. Harko and M.K.Mak. Class. Quantum Grav. **16**, R31 (1999).

[21] P.P. Avelino and C.J.A.P. Martins. Phys.Lett. B459, 2741-2752, (1999).

[22] J.A. Belinchón and I. Chakrabarty. Int. Jour. Moder. Phys. D**12**, 1113 (2003). gr-qc/0404046.

[23] J.A. Belinchón. "An excuse for revising a theory of time-varying constants", gr-qc/0404028.

[24] P. Rastall. Phys. Rev. D 6 3357 (1972).

[25] B. O'Neill. "*Semi-Riemannian Geometry with Applications to Relativity*". Academic Press (1983)

[26] H. Shojaie and M. Farhoudi. "A varying-c cosmology". gr-qc/0406027.

[27] H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers and E. Hertl, "*Exact Solutions of Einstein's Field Equations*", 2nd Edition, Cambridge University Press, (2003)

[28] N.H. Ibragimov. "*Elementary Lie Group Analysis and Ordinary Differential Equations*", Jonh Wiley & Sons (1999)

[29] G.W Bluman and S.C. Anco. "*Symmetry and Integral Methods for Differential Equations*". Springer-Verlang (2002)

[30] B. J. Carr and A. A. Coley, Class. Quantum Grav. 16, R31 (1999).

[31] G. I. Barenblatt and Y. B. Zeldovich, Ann. Rev. Fluid Mech. 4, 285 (1972).

[32] D. M. Eardley, Commun. Math. Phys. 37, 287 (1974).

[33] J. Wainwright, "*Self-Similar Solutions of Einstein's Equations*". Published in Galaxies, Axisymmetric Systems & Relativity. ed M.A.H MacCallum CUP (1985).

J. Wainwright, Gen. Rel. Grav. 16, 657 (1984).

[34] K. Rosquits and R. Jantzen, Class. Quantum Grav., 2 (1985) L129.

K. Rosquits and R. Jantzen, "*Transitively Self-Similarity Space-Times*". Proc. Marcel Grossmann Meeting on General Relativity. Ed. Ruffini. Elsevier S.P. (1986). pg 1033.

[35] K.L. Duggal and R. Sharma. "*Symmetries of spacetimes and Riemmanian Manifolds*". Kluwer Academic Publisher. (1999)

[36] J.A. Belinchón. Int.J.Theor.Phys. 39, 1669-1686, (2000).

J.A. Belinchón and A. Alfonso-Faus, Int.J.Mod.Phys. D10, 299-310, (2001).

[37] R.G. Vishwakarma. astro-phy/0506217. to appear in the Mon. Not. Roy. Astron. Soc.

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THE NATURE OF RELATIVISTIC LENGTH SHRINKAGE

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It is shown here, that relativistic shrinkage of the length of moving body is realizing strongly synchronously with the change of its velocity and is continuously self-sustaining without the effect of any forces. This shrinkage is caused by inertial isobaric self-contraction of body matter and by propagation of changes of the strengths of inertia forces field together with the front of body intrinsic time. A mechanism of kinetic energy filling (accumulation) of a body is considered and propagation of the phase waves of perturbation of gravitational field at supraluminal velocity is substantiated here.

1. Introduction

Physical processes during their separate stages can be accompanied and can be not accompanied by transfer of matter or its excited state in space. In the first case, they are characterized by group velocity V of transfer of particles and quasi-particles (photons, phonons, excitons etc.). This velocity cannot exceed the velocity of light in free space (equal to one if distances are measured in light units of length). In the second case, they are characterized by phase velocity U of propagation of change of collective space-time state of matter. The change of this state of matter realizes, as suggested here, together with the change of graviinertial stressed state in a space, filled with matter. Therefore, not only propagation of change of collective space-time state of matter but also propagation of induction of strength of inertia forces field in intrinsic frame of reference of space coordinates and time (FR) of hypothetical incompressible (perfectly rigid) body will happen momentarily ($u \equiv \infty$). Phase velocity of propagation of induction of spatial distribution of the strength of inertia forces field in an incompressible body, which moves relatively to the hypothetically physically homogeneous physical vacuum (PV) at a constant velocity V, in the fundamental FR of physical vacuum (PVFR) will not be infinitely high, but equal to: $U = c^2/V = V^{-1}$ ($c = 1$). This fact is connected with translational motion of body, in which a wave front of induction of graviinertial stressed state propagates.

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2. Derivation of dependence of relativistic length shrinkage on velocity of the body

Let the incompressible body moves uniformly in pseudo-Euclidean Minkowski space-time of the PVFR at absolute velocity V_0 (before the induction of the strength of inertia forces field in it). Also let the initial distance along the movement route between two arbitrary points (i and j) of the body in the PVFR absolute space (in which frequency of universal background is isotropic) is equal to X_{ij0}. Then after body transition into new steady state of its uniform motion at absolute velocity $V_j = V_i = V$, the distance between these two points will be equal to:

$$X_{ij} = X_{ij0} + (V_0 \delta T_{ij0} + \delta X_j) - (V \delta T_{ij} + \delta X_i) =$$
$$= (\Gamma_0^2 X_{ij0} + \delta X_j - \delta X_i)\Gamma^{-2} = \delta T_{ij} \Gamma^{-2} V^{-1}, \qquad (1)$$

where:

$$\delta T_{ij0} = X_{ij0}(U_0 - V_0)^{-1} = \Gamma_0^2 V_0 X_{ij0}, \qquad (2)$$

and:

$$\delta T_{ij} = [X_{ij0} + (V_0 \delta T_{ij0} + \delta X_j) - \delta X_i]U^{-1} =$$
$$V(\Gamma_0^2 X_{ij0} + \delta X_j - \delta X_i) = X_{ij}(U - V)^{-1} = \Gamma^2 V X_{ij}, \qquad (3)$$

— the durations of time delay of accordingly induction and removal of the strength of inertial forces field in point j, regarding point i (equal to desynchronizations, observed in the PVFR, of all the other events, which are synchronous in these points in the inertial FR (IFR) of moving body);

— δX_i and δX_j are the traversed paths in absolute space of accordingly points i and j from the moments of induction T_{i0} and T_{j0} till the moments of removal $T_i = T_{i0} + \delta T_i$ and $T_j = T_{j0} + \delta T_j$ of strengths of inertia forces field in these points; $\Gamma_0 = (1 - V_0^2)^{-1/2}$ and $\Gamma = (1 - V^2)^{-1/2}$ are characteristics of the accordingly initial and newly-formed IFR;

— $U_0 \equiv V_0^{-1}$ and $U \equiv V^{-1}$ are velocities of propagation of fronts of processes of induction and removal of strengths of inertia forces field in the PVFR accordingly.

Let's assume that X_{ij} is a function just of V and depends neither on the value of V_0 nor on the law of body motion before it takes on the value of inertia motion velocity V. Then, according to (3), δT_{ij} does not depend on V_0 and on this law of body motion either. Basing on this fact and on the condition $V_0 = 0$, let's choose a uniformly accelerated motion (as the simplest law of irregular motion) of the point i of the body before it takes the value of velocity V:

$$V_i = \alpha_i \delta T_{ij}, \tag{4}$$

where $a_i = dV_i/dT$ is the acceleration of the motion of point i. Then multiplying the left and the right parts of the equation (4) by dT and considering the immobility of the point j during the time δT_{ij} $(dX_j = 0)$, we will obtain the following differential equation:

$$dX_{ij} = -dX_i = -\delta T_{ij} dV = -\Gamma^2 V X_{ij} dV, \tag{5}$$

solving which we will find: $X_{ij} = X_{ij0}\Gamma_0/\Gamma$,

$$\delta T_{ij} = \Gamma_0 \Gamma V X_{ij0} = \delta T_{ij0} \Gamma V / \Gamma_0 V_0, \tag{6}$$

When $V_0 = 0$: $X_{ij} = x_{ij}/\Gamma$, and $\delta T_{ij} = \Gamma V x_{ij}$, where: $x_{ij} = X_{ij}(0)$ is the distance between the points j and i, measured in the IFR of moving body and equal to the distance between them in absolute space in hypothetical state of body absolute rest relatively to the PV.

So, if an incompressible body proceeds from the state of rest relatively to the PV into a new-steady state of inertial motion, then relativistic shrinkage of body length along the direction of motion really takes place in PVFR. The value of this shrinkage is determined by the analytical dependence, discovered by Fitzgerald and Lorentz independently from each other, and does not depend on the law of change of strengths:

$$\begin{aligned} -G_j(x,V) &= (dP_A/dt)/H_A = \\ &= -(\partial \ln v_c(x,V)/\partial x)_V = \\ &= -d(P_j/m_j)/dT = -\Gamma_j^3 \alpha_j \end{aligned}$$

of removable gravitational (graviinertial) field, which originates in the intrinsic FR of the body. And consequently, the value of this shrinkage does not depend on the law of motion of body points during the proceeding of body from the state of rest or inertial motion into the state of inertial motion at another velocity. Here: P_A and H_A is accordingly linear momentum and invariable energy (conservative Hamiltonian) of free-falling (motionless in the PVFR) particle A, which are definite in intrinsic FR of the accelerating body; $v_c(x,V) = cG_i(x_i,V) \cdot G(x,V)^{-1}$ is improper values of the velocity of light in free space in body intrinsic FR (unequal in different points of physically inhomogeneous intrinsic space of body in proper quantum time t of point i); P_j and m_j - accordingly linear momentum in the PVFR and eigenvalue of mass of the point object (particle) j of the body. At this, conditions:

$$\begin{aligned} \delta X_j - \delta X_i &= \Gamma^2 X_{ij} - \Gamma_0^2 X_{ij0} = (\Gamma - \Gamma_0) x_{ij}, \\ \delta T_j - \delta T_i &= \delta T_{ij} - \delta T_{ij0} = (\Gamma V - \Gamma_0 V_0) x_{ij}, \end{aligned} \tag{7}$$

which follow from (1-3), always guarantee simultaneity of removal of strengths of inertial forces field in body intrinsic FR in all body points. And consequently, they also guarantee a momentary transition in this FR (without any transient process) of an incompressible body into equilibrium state of inertial motion. And the fulfillment of these conditions is possible only at the following distribution of strength of inertia forces field along the moving body:

$$G_j(V)^{-1} = G_i(V)^{-1} + x_{ij}. \tag{8}$$

Where, as we supposed, $G_i(V)$ can vary with time according to the arbitrary law. And it specifies at that any law of body motion. At that spatial distribution of strength of graviinertial field (inertia forces field) unconditional realization of the identity will take place:

$$U = (\partial X/\partial x)_t \cdot (\partial T/\partial x)_t^{-1} \equiv V^{-1}.$$

3. Equations of irregular motion of body points

According to (8), the motion of any point of the body in the process of its transition from inertial motion at absolute velocity V_0 to inertial motion at absolute velocity V can be described by the same parametric equations as motion of the point i:

$$\partial X_i = X_i - X_{i0} = \int_{V_0}^{V} \frac{v dv}{G_i(v) \cdot (1 - v^2)^{3/2}}, \tag{9}$$

$$\partial T_i = T_i - T_{i0} = \int_{V_0}^{V} \frac{dv}{G_i(v) \cdot (1 - v^2)^{3/2}}, \tag{10}$$

or in another form by the equation:

$$\begin{aligned} [X_i(V) - X_c(V)]^2 - [T_i(V) - T_c(V)]^2 &= \\ = [x_i - x_c(V)]^2 &= G_i V^{-2}. \end{aligned} \tag{11}$$

Where:

$$\begin{aligned} X_i(V) - X_c(V) &= \Gamma(V)[x_i - x_c(V)] = \\ &= \Gamma(V)/G_i(V), \end{aligned} \tag{12}$$

$$T_i(V) - T_c(V) = V[X_i(V) - X_c(V)] =$$

$$= V\Gamma(V)/G_i(V), \tag{13}$$

and:

$$x_c(V) = x_i - G_i(V)^{-1} \tag{14}$$

is a coordinate of an asymptotic limit (singular plane) of intrinsic space of moving body, which can be considered as an observer horizon of the body FR

$$(G_c = \infty);$$

$$X_c(V) = X_c(V_0) + \int_{x_c(V_0)}^{x_c(V)} \Gamma(V)dx_c =$$

$$= X_c(V_0) + \int_{V_0}^{V} \frac{dG_i}{dv} \cdot \frac{\Gamma(v)}{G_i(v)^2}dv, \tag{15}$$

is a coordinate in absolute space of hypothetically initial position of the observer horizon of body at the beginning of its motion ($V = 0$) on condition that distribution of strengths of inertia forces field along the body from the beginning of its motion is the same as at the given identical velocity of all its points

$$G_i(0) = G_i(V) = const(V);$$

$$T_c(V) = T_c(V_0) + \int_{x_c(V_0)}^{x_c(V)} V\Gamma(V)dx_c =$$

$$= T_c(V_0) + \int_{V_0}^{V} \frac{dG_i}{dv} \cdot \frac{\Gamma(v)v}{G_i(v)^2}dv, \tag{16}$$

is a hypothetical moment of absolute time, when the body motion in the absolute PV space would begin, if distribution of strengths of inertia forces field along the body were stationary. At that, $T_c(0) = T_i(0)$. And at coincidence of the points of origin in absolute space and in intrinsic space of the body in its hypothetical state of rest relatively to the PV ($X_i(0) = x_i(0)$) and also $X_c(0) = x_c(0)$. When dependence of strengths of inertia forces field on absolute velocities V of body points is weak, equation (11) corresponds to quasi-hyperbolic motion of these points. If the distribution of strengths of inertia forces field along the moving body is stationary ($G_i \equiv const(V)$), then all the body points perform in the PVFR not quasi-hyperbolic but definitely a hyperbolic motion. And the moving body (even if it is compressible) will be resting in an appropriate to it Möller rigid accelerating FR [1],[2]. A proportional mutual synchronization of quantum clock, located in different points of physically inhomogeneous intrinsic space of this FR, is possible only in this FR. But in general case, events in different points are considered just to coincide with each other, if they happen at the same instantaneous values of absolute velocities V of these points. Events, not having direct cause-and-effect relations with each other, are meant here under coincident events. But in the presence of common cause, mutual correlation of coincident events can take place. These events correspond to definite collective space-time state (microphase state) of particles of body matter. And they are simultaneous according to quantum clock, only in the case of homogeneity of intrinsic time t, which is possible only in the case of stationarity of spatial distribution of the velocity of light in free space in co-moving FR:

$$v_{cj} = v_{ci}G_iG_j^{-1} = v_{ci}(1 + x_{ij}G_i) = const(t).$$

And it is possible only in Moller FR [1],[2].

From the condition of the absence of increment of action S: $dS = LdT = PdX - HdT = 0$, corresponding to invariance of collective space-time state of matter, we'll have:

$$dX/dT = H/P = V^{-1} \equiv U,$$

where L - Lagrangian of body matter. In view of this, the front of induction of strength of inertia forces field in incompressible body (the same as the front of propagation of change of collective space-time state of matter) is identical to the front of coincident events.

According to Lorentz transformations, the front of coincident events of any body, which moves relatively to an observer at velocity v , will be moving in observer FR, as well as in the PVFR, not at infinitely high, but at finite phase velocity $u = v_c^2/v$.

4. Propagation of the changes of graviinertial stressed state and elastic deformation in compressible body

For an elastically compressible (deformable) body, the distance $\breve{x}_{ij}$ between the points i and j in its uniform and stable metrical intrinsic space (where the motion of its points in the process of elastic deformation of the body matter is observable) can be connected with the distance between them x_{ij} in inseparable from the body its nonuniform and metrically instable physical intrinsic space by the following dependence: $\breve{x}_{ij} = \alpha(V)x_{ij}$. Where: $\alpha(V)$ is a coefficient of the elastic shrinkage of body size along the direction of its motion, that depends on the velocity of the body when the strength of inertia forces field is instable ($G \neq const(V)$).

In contrast to a hypothetical incompressible body, in metrical intrinsic space of a compressible body, only microobjects (elementary particles) have purely relativistic size shrinkage along the direction of motion. It is connected with the elastic deformation of matter macroobjects that also can be observed in the body intrinsic FR. As well as in incompressible body, this shrinkage is caused by elementary particles adaptation (and by adaptation of the matter as the whole, owing to

Van der Waals forces, which have electromagnetic nature) to changed conditions of elementary particles interaction. This adaptation guarantees in the first place the isotropy of interaction frequency and becomes apparent in co-moving FR at the absence of anisotropy of the radiation spectrum of radiation sources, motionless relatively to the body. Lorentz considered processes, which are connected with this adaptation, first in detail [3] on the example of electrical and electromagnetic phenomena. The possibility of such adaptation follows from the wave nature of elementary particles and of matter as the whole. That's why relativistic size shrinkage realizes at elementary particles level and is connected with longitudinal self-contraction (which is initiated by motion) of wave-like formations, which correspond to elementary particles [4].

Relativistic size shrinkage along the direction of the body motion will guarantee isotropy of the velocity of light in free space only in physical intrinsic space of the compressible body. In metrical intrinsic space (considering the observability of body deformation in it) the velocity of light in free space will be anisotropic. Moreover, in contrast to physical intrinsic space, in metrical intrinsic space definition of an interval between two world points (which is invariant to transformations of coordinates only in physical spaces), as well as the definition of energy and momentum of any objects has no physical sense. That is why the analysis of dynamics of body compression and of the motion of objects is impossible in principle in this space [5]. Dynamics of the objects can be analyzed only in physical intrinsic space of a compressed body using continuous renormalization of all the measurements and spatial characteristics, determined in it, considering their change in metrical intrinsic space of the body.

In elastic compressible body, as well as in hypothetical incompressible body, the front of induction of strengths of graviinertial field in it can be identified with a wave front of change of collective space-time state of body matter. That is why, also in an intrinsic FR of elastically compressible body the propagation not only of quantum of action, but also of change of space-time distribution of the strength of graviinertial field (the main characteristic of collective space-time state of matter), takes place momentarily in principle. This is caused by the wave nature of elementary particles of matter, which becomes apparent also in collective space-time state of matter.

Let the linear momentum and therefore velocity of an elastic compressible body increase in a result of a shock action (impact). Then in the PVFR (as well as in FR of any body that moves at another velocity) a phase soliton (phase packet) of modulation of strength of graviinertial field will run along the moving body at supraluminal phase velocity. This soliton changes the value of relativistic shrinkage of molecules of body matter and does not cause their elastic deformation. After that, a soliton (wave packet) of elastic deformation and excitation of matter molecules will run along the body at a velocity of sound. Body filling with additional kinetic energy, as it were transferred by graviinertial phase soliton at supraluminal velocity, as a matter of fact is not connected with ripple-through energy transfer in it. This filling is an inert process and realizes only due to accumulation of difference of Doppler energies of exchange virtual elementary particles and quasi-particles, which propagate in the process of interaction of elementary particles, atoms and molecules in and against the direction of soliton propagation. These are virtual pi-mesons that during the process of strong interaction maintain collective dynamic equilibrium between protons and neutrons in an atom. And these also are virtual photons that during the process of electromagnetic interaction maintain collective dynamic equilibrium between protons and electrons in an atom, as well as between electrically and magnetically polarized atoms and molecules. At this, the work is executed by the forces, which disturb the mechanical equilibrium of the matter. These forces are equal to inertia forces (they are, like gravitational forces, actually only pseudoforces [4]), but they are directed oppositely. Neither entropy nor enthalpy of the matter in the body intrinsic FR vary during the process of the body filling with kinetic energy. That is why neither free photons nor phonons or any other quasi-particles are generated and transferred in the body. And therefore, ripple-through energy transfer and energy dissipation are absent. In the contrast to total energy, Helmholtz energy of body matter increases in the PVFR not only due to increase of linear momentum and of relativistic shrinkage of molecular volume, but also due to decrease of Planck relativistic temperature. The energy of soliton of elastic deformation (which runs after the phase soliton) will be dissipated and transformed into heat after multiple reflections from the body boundaries.

In this case, before coming of a soliton of elastic deformation of matter, motion of points of compressible body can be described by the same equations (8-16) as motion of points of incompressible body. Now let the body linear momentum increase due to a long-lived force. And that's why the front of removal of the graviinertial stressed state runs along the body after the induction of a definite elastic deformation of its matter. Then motion of the body points before the front running can be described by equations, different from the ones (8-16). The distance between the points j and i $\breve{x}_{ij}$ in elastically deformated state of the body matter will be used in these equations instead of the distance x_{ij}. Therefore, also in this case, removal of graviinertial stressed state will realize the same way as in an incompressible body, but considering the change of the distances x_{ij} to distances $\breve{x}_{ij}$ that take place at a point in time when the front of removal of the stressed state is passing through the point j.

In accordance with this, relativistic shrinkage of the length of moving body is not directly connected with effect of any internal or external forces (external forces have only indirect influence on the change of its value via the change of intensity of body motion - the value of ratio of body velocity to improper value of velocity of light) and before coming of soliton of elastic deformation its matter does not exhibit resistance to contraction. Absence of forces of resistance to relativistic self-contraction of the body displays the inertiality of the process of relativistic shrinkage of its length. This is like the inertiality of freon expansion in refrigerating chamber, when, because of invariability of freon internal energy, no work executes in the process of its expansion. Also this is like the inertiality of body free fall in the field of gravity pseudoforces. In the process of such body fall gravitational pseudoforce and d'Alamberian inertial pseudoforce (which both don't execute work) don't equilibrate, but only compensate each other [4]. Therefore body free fall is not equilibrium, but inertial motion. And, consequently, in the process of inertial motion, as well as in the process of inertial self-contraction of the body, no work executes, despite the presence of internal pressure in this body. And, of course, this is connected with the absence of matter resistance to its relativistic self-contraction. So, in the case of unrightful neglect of nonequilibrium of the process of relativistic length shrinkage (and, therefore, in the case of using of thermodynamic equalities, instead of inequalities) we can conventionally assume that in the process of relativistic shrinkage of body length fictious work (pseudowork) on relativistic self-contraction of matter "executes":

$$dA_\Gamma = (\partial A/\partial\Gamma)_{\tilde{\mathbf{v}}} \cdot d\Gamma = -p(\partial \mathbf{v}/\partial\Gamma)_{\tilde{\mathbf{v}}} \cdot d\Gamma = p\tilde{\mathbf{v}}\Gamma^{-2}d\Gamma.$$

It executes due to decrease of relativistic value of heat content (enthalpy) of body matter:

$$-dL_{\mathbf{H}} = (-dL_{\mathbf{H}}/\partial\Gamma)_{\tilde{\mathbf{H}}} \cdot d\Gamma =$$
$$= -\tilde{\mathbf{H}}\Gamma^{-2}d\Gamma = -\tilde{\mathbf{U}}\Gamma^{-2}d\Gamma - dA_\Gamma,$$

by certain fictious internal pseudoforces, which conventionally "equilibrate" the forces of internal pressure in the matter. Here: $\mathbf{v} = \tilde{\mathbf{v}}/\Gamma$ and $\tilde{\mathbf{v}}$ - accordingly relativistic value and eigenvalue of matter molar volume; $L_{\mathbf{H}} = -\tilde{\mathbf{H}}/\Gamma$ - Lagrangian of enthalpy $\tilde{\mathbf{H}} = \tilde{\mathbf{U}} + p\tilde{\mathbf{v}}$, $\tilde{\mathbf{U}}$ - eigenvalue of internal energy of the one mole of matter, and p -pressure in matter. Because of this, the shrinkage of body dimensions along the direction of its motion can be considered as their decrease because of relativistic "cooling down" of moving body. All the more, this decrease of body dimensions is strongly proportional not only to decrease of lagrangian of enthalpy (relativistic value of enthalpy), but also to decrease of relativistic Planck temperature when the eigenvalue of temperature is constant. At that, of course, the thermal energy of oscillatory motion of matter molecules transforms not only into internal potential energy, but also into kinetic energy of the ordered motion of these molecules. As a result of decrease of thermal energy (including due to relativistic length shrinkage) lagrangian of enthalpy reaches its minimum in physically homogeneous space (in the case of absence of gravity). The matter can be withdrawn from this stable equilibrium thermodynamic state only in the way of forced deceleration of the body.

In accordance with this, gravity phenomenon is also connected with tendency of matter to reach in physically inhomogeneous space (in gravitational field) its thermodynamic state with minimum of relativistic value of enthalpy. And, consequently, gravitational forces are not external but internal (for the matter) pseudoforces, caused by the tendency of matter to reach its more stable thermodynamic state. The fact that not gravitational pseudoforces but external forces, that equilibrate them and prevent body free fall, execute work in the process of body motion in gravitational field also denotes this. These forces execute positive work when body moves up (against gravity pseudoforces) and negative work in the case of deceleration or body free fall.

The neglect of nonequilibrium of relativistic thermodynamic state of matter in the process of the change of its linear momentum, and consequently, the neglect of nonequilibrium of inertial process of relativistic length shrinkage, leads to ambiguity in determining the relativistic temperature of moving body matter. The wrong determining of relativistic quantity of heat [6] is the consequence of considering pseudowork, which is "executed" at the expense of change of pressure in matter when the linear momentum of moving body is not being conserved, as really executed work. In fact the using in thermodynamics of relativistic Ott pseudotemperature [1],[6],[7], as well as relativistic Planck temperature (which is coordinated with the relativistic Doppler value of the frequency of radiation in the directions normal to the vector of velocity), is connected with this. Lack of understanding of the fact that relativistic self-contraction of moving body is the consequence of the change of space-time continuum, and thus the change of relativistic thermodynamic state of its matter, and that this self-contraction realizes inertially (without overcoming of any forces, resisting to it) is also the cause of conscientious long-continued fallacies, connected with dynamical interpretation of relativistic length shrinkage [8],[9].

5. Effects caused by equivalence of removable (graviinertial) and irremovable gravitational fields

Momentary propagation of induction of graviinertial stressed state (physical inhomogeneity of the body intrinsic space, that can be identified to a gravitational field) in the body is well matched in Einstein-Podolski-Rosen paradox [10],[11] with a momentary intercoordination of changes in the quantum-mechanical characteristics of previously correlated photons or elementary particles after mutual self-distancing of the ones at arbitrary long spacing. Equivalence of removable (graviinertial) and irremovable gravitational fields, connected with identity of mechanisms of effect of motion and gravity on the state of matter via the change of frequency of interaction of its elementary particles, denotes the possibility of momentary propagation of disturbance of proper gravitational field (in intrinsic FR) also in free space. Also this denotes rigid fixation in space of gravitational field to macro- or microobject, which creates it, (at the impossibility of time delay of displacement of spatial distribution of gravitational field strength relatively to displacement of this object). That is why the carriers of gravitational field in free space are photons, elementary particles and any moving macroobjects, consisting of them, but not hypothetical gravitons (their existence is impossible in principle as it was shown in [4]). Any moving macroobject (body) can be characterized by de Broglie frequency. And it lets us consider moving body also as a "graviton wave," which transfers energy.

During the rotation of a negatively charged body around a positively charged body, electromagnetic radiation is generated. But similar specific radiation is not generated during the rotation of planets around the Sun. Otherwise the planets finally would fall on the Sun due to continuous energy losses. That is why only the matter, the accretion of which is realizing from one star to another in a compact system of double star, can be considered as "graviton radiation."

Phase gravitational waves (caused, for example, by the rotation of astronomical body around the point, which is not coincide with its centre of mass) that propagate in free space at supraluminal velocity give rise to disturbance in the motion of another astronomical bodies, not executing any work. At this, only transition of internal energy of the matter of astronomical bodies into kinetic energy realizes, as it does during a free fall of macroobjects in terrestrial gravitational field. In other case periodical transformation of matter internal energy into potential energy of its deformation takes place, as it, for example, is being observed in the form of day-today variations of sea level (high and low tide) under the influence of gravitational waves, caused by the motion of the Moon relatively to Earth surface. These phase waves are progressing waves of metrical and physical microinhomogeneities of space in the form of space-time modulations of permittivity and permeability of PV, which uniquely determine metrical and physical properties of the space, filled with PV. Space-time modulations of permittivity and permeability of PV also appear in the process of propagation of radiation because of the presence of negative feedbacks, which guarantee self-restriction of improper values of electric and magnetic strengths of electromagnetic wave. And these negative feedbacks also cause the formation of wave packets of quanta of electromagnetic energy (soliton-like photons) nearby elementary particles because of the presence of very significant metrical and physical microinhomogeneities of space in their neighbourhood. These unstable inhomogeneities are (in fact) space-time modulations of nonlinery interconnected characteristics of PV, and they form in PV self-sustaining spiral-wave structural elements - autowaves of elementary particles.

6. Conclusions

The relativistic shrinkage of the length of moving body realizes strongly synchronously with the change of intensity of its motion (ratio of body velocity to velocity of light) and is self-sustaining when this intensity is constant without the participation of any forces. Therefore this shrinkage can be considered as purely kinematics effect. This effect is caused by the change of relativistic values of matter thermodynamic parameters and is not connected with elastic deformation of this matter. At any law of body motion the value of relativistic shrinkage of its dimensions along the direction of its motion is uniquely determined by the intensuty of body motion. Relativistic shrinkage of body length appears and varies during the process of inertial isobaric self-contraction or self-expansion of its matter. This shrinkage is amed at the reaching the minimum of relativistic value of matter enthalpy and at the guaranteeing of isotropy of rates of physical processes in isotropic mediums of moving bodies. This process of isobaric relativistic deformation of matter always passes ahead of the process of change of matter elastic compression. And all this is caused by propagation (together with front of body intrinsic time) of not only change of collective space-time state of its matter, but also changes of the strengths of inertia forces field.

Phase waves of perturbation of removable (graviinertial) and irremovable gravitational fields don't transfer energy themselves. They only create necessary conditions for non-through local energy transfer in the process of the interaction of matter elementary particles. This energy transfer realizes due to accumulation of difference of Doppler values of energies of exchange virtual elementary particles and quasi-particles, which propagate in and against the direction of the body motion. That's why phase gravitational waves, which transfer the change of collective space-time state of PV and matter (that is only the excited state of PV), can propagate at supraluminal velocity.

References

[1] C. Möller, "The Theory of Relativity," Clarendon Press, Oxford, 1972.

[2] P. Danylchenko, "Phenomenological justification of relativistic shrinkage of the length of moving body," in: *The Gauge-evolutional theory of the Creation*, Vinnytsia, Ukraine, **1**, 5 (1994)(in Russian); "The nature of relativistic length shrinkage," in: *Gauge-Evolutional Interpretation of Special and General Relativities*, O. Vlasuk, Vinnytsia, 3 (2004).

[3] H. Lorentz, "The theory of electrons," Teubner, Leipzig, 1916.

[4] P. Danylchenko, "Foundations of the Gauge-evolutional theory of the Creation (space, time, gravitation and the Universe expansion)," Vinnytsia, Ukraine, 1994; see also "Gauge-evolutional interpretation of special and general relativities" (in Russian) on: http://pavlo-danylchenko.narod.ru/myarticles-rus.html (English version will be available soon).

[5] P. Danylchenko, "Nonrigid frames of reference contracting in Minkowski space-time," in: *The Gauge-evolutional theory of the Creation*, Vinnytsia, Ukraine, **1**, 52 (1994) (in Russian).

[6] I.P. Bazarov, "Thermodynamics," Pergamon Press, Oxford, 1964.

[7] H.Z. Ott, *Phys.*, **175**, 70 (1963).

[8] E.L. Feinberg, "Is it possible to consider relativistic change of scales of length and time as the result of effect of some forces?" in: Einsteinovski sbornik (Einstein Studies) 1975 -1976, Nauka, Moscow, 43 (1978) (in Russian).

[9] E.L. Feinberg, "Special theory of relativity - how good-faith misconceptions come about," *Uspekhi Fizicheskih Nauk*, **40**, 4, 455 (1997) (in Russian); see also on: http://ufn.ioc.ac.ru/ufn97/ufn97_4/Russian/r974f.pdf.

[10] A. Einstein, B. Podolski, N. Rosen, *Philos. Rev.*, **47**, 7 (1935).

[11] J.-P. Vigier, "paper on Einstein-Podolski-Rosen paradox," in Einstein Centenarium, ed. H.-J. Treder, Akademie Verlag, Berlin, 1982.

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CAUSAL BULK VISCOUS LRS BIANCHI I MODELS WITH VARIABLE GRAVITATIONAL AND COSMOLOGICAL "CONSTANT"

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In this paper we have investigated an LRS Bianchi I anisotropic cosmological model of the universe by taking time varying G and Λ in the presence of bulk viscous fluid source described by full causal non-equilibrium thermodynamics. We obtain a cosmological constant as a decreasing function of time and for $m, n > 0$, the value of cosmological "constant" for this model is found to be small and positive which is supported by the results from recent supernovae observations.

1. Introduction

The conventional theory of evolution of the universe includes a number of dissipative processes. Dissipative thermodynamics processes in cosmology originating from a bulk viscosity are believed to play an important role in the dynamics and evolution of the universe. Misner[1] suggested that large-scale isotropy of the universe observed at the present epoch is due to action of neutrino viscosity which was not negligible when the universe was less than a second old. A number of processes responsible for producing bulk viscosity in the very early universe are such as the interaction between radiation and matter[2], gravitational string production [3, 4], viscosity due to quark and gluon plasma, dark matter or particle creation[5, 6]. It is important that each dissipative process is subject to as detailed analysis as possible. However, it is also important to develop a robust model of dissipative cosmological processes in general, so that one can analyze the overall dynamics of dissipative without getting lost in the details of particular complex processes. In requirements of such a model Maartens[7] pointed out that the model should (i) be causal and stable, and (ii) provide a constant relativistic thermodynamics in the 'conventional' post-inflationary regime.

In order to study these phenomena, the theory of dissipative was first developed by Eckart[8] and subsequently an essential equivalent formulation was given by Landau and Lifshitz[9]. But Eckart theory has several drawbacks including violation of causality and instabilities of equilibrium states. Readers interested in the general theory of causal thermodynamics are urged to consult the excellent survey report of Maartens[7] and Zimdahl[10] and references cited therein. A relativistic second-order theory was found by Israel[11] and developed by Israel and Stewart[12]. The advantages of the causal theory are as follows[13]: (1) For stable fluid configurations, the dissipative signals propagate causally. (2) Unlike Eckart-type's theory, there is no generic short- wavelength secular instability in causal theory. (3) Even for rotating fluids, the perturbations have a well-posed initial value problem. Therefore, the best currently available theory for analyzing dissipative processes in the universe is the full (i.e. non-truncated) Israel-Stewart causal thermodynamics, which we consider in this work.

In recent years, models with relic cosmological constant Λ have drawn considerable attention among researchers for various aspects such as the age problem, classical tests, observational constraints on Λ, structure formation and gravitational lenses have been discussed in the literature (see Refs.[14] – [16]). Lindey[17] has suggested that cosmological "constant" may be

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considered as a function of temperature and related to the spontaneous symmetry breaking process. Therefore, Λ should be a function of time in a homogeneous universe as temperature varies with time. Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant are given by Ratra and Peebles [18], Dolgov[19] − [21], Sahni and Starobinsky[22], Peebles[23], Padmanabhan[24] and Vishwakarma[25]. Recent cosmological observations suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. There are several aspects of the cosmological constant both from cosmological and field theoretical perspectives. Presently, determination of Λ has become one of the main issues of modern cosmology as it provides the gravity vacuum state and make possible to understand the mechanism which led the early universe to the large scale structures and to predict the fate of the whole universe. The cosmological "constant" can be measured by observing quasars whose light gets distorted by gravity of galaxies that lies between the quasars and Earth. Krauss and Turner[26] have mentioned that as Λ term dominates the energy density of the universe, cosmologists are correct in their attempt to evoke it once again for better understanding of both the universe and fundamental physics.

In the last few decades there have been numerous modifications of general relativity in which gravitational "constant" (G) varies with time[27]. Considering the principle of absolute quark confinement, Der Sarkissian[28] has suggested that gravitational and cosmological "constant" may be considered as functions of time parameter in Einstein's theory of relativity. A number of authors[29] − [40] have considered time-varying G and Λ within the framework of general relativity.

Motivated by the fact that bulk viscosity, gravitational and cosmological "constants" , are more relevant during early stages of the universe, in this paper, we have considered the evolution of a LRS Bianchi I model with bulk viscous fluid in full causal non-equilibrium thermodynamics, in presence of time-varying gravitational and cosmological "constants".

2. The Basic Equations

A locally rotationally symmetric (LRS) Bianchi I space-time may be represented by the line element

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2), \tag{1}$$

where metric potentials A and B are depending on cosmic time t only.
The Einstein's field equations with variable G and Λ are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda(t)g_{ij} = -8\pi G(t)T_{ij}, \tag{2}$$

where R_{ij} is the Ricci tensor; $R = g^{ij}R_{ij}$ is the Ricci scalar; and T_{ij} is the energy-momentum tensor of cosmic fluid in the presence of bulk viscosity given by

$$T_{ij} = (\rho + p + \Pi)u_iu_j - pg_{ij}. \tag{3}$$

Here ρ, p and Π are the energy density, equilibrium pressure and bulk viscous pressure respectively and u^i is the flow vector satisfying the relations $u^iu_i = 1$.
The Einstein's field equations (2) for the line element (1) lead to the following set of equations

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi G(t)\rho + \Lambda(t), \tag{4}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = -8\pi G(t)[p + \Pi] + \Lambda(t), \tag{5}$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -8\pi G(t)[p + \Pi] + \Lambda(t). \tag{6}$$

A combination of equations (4) - (6) yield the continuity equation

$$8\pi\dot{G}\rho + 8\pi G\left[\dot{\rho} + (\rho + p + \Pi)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right)\right] + \dot{\Lambda} = 0. \tag{7}$$

The usual energy-momentum conservation equation $T^{ij}_{;j} = 0$ suggests

$$\dot{\rho} + (\rho + p + \Pi)\left[\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right] = 0. \tag{8}$$

From equations (7) and (8), we get

$$\dot{\Lambda} = -8\pi\dot{G}\rho. \tag{9}$$

For the full causal non-equilibrium thermodynamics, the causal evolution equation for bulk viscosity is given by[7]

$$\tau\dot{\Pi} + \Pi = -\xi\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) - \\ -\frac{\epsilon}{2}\,\tau\,\Pi\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right), \tag{10}$$

where $T \geq 0$ is the temperature, ξ the bulk viscous coefficient and $\tau \geq 0$ the relaxation coefficient for the transient bulk viscous effect (relaxation time i.e. the time which system takes in going back to equilibrium once the divergence of the four velocity has been switched off). For $\tau = 0$, equation (10) gives evolution equation for the non-causal theory. For $\epsilon = 0$, we get causal evolution equation for truncated theory, which implies a drastic condition on the temperature, while ϵ takes value unity in full causal theory.

3. The Model

Equations (5) and (6) yield

$$\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{\ddot{A}}{A}-\frac{\dot{A}\dot{B}}{AB}=0. \tag{11}$$

In order to find exact solutions of the field equations, we require more physically plausible relations amongst the variables. Considering a power law relation between A and B as $A\sim B^n$, eq.(11) suggests

$$B=B_0\left(\frac{t}{t_0}\right)^{\frac{1}{n+2}}, \tag{12}$$

$$A=A_0\left(\frac{t}{t_0}\right)^{\frac{n}{n+2}}, \tag{13}$$

where A_0 and B_0 are the values of A and B at present time $t=t_0$ and n is a positive constant.
By use of (12) and (13), equations (4) - (6), reduce to

$$\frac{2n}{(n+2)^2}\frac{1}{t^2}=8\pi G\rho+\Lambda, \tag{14}$$

$$\frac{2n+1}{(n+2)^2}\frac{1}{t^2}=8\pi G(p+\Pi)-\Lambda. \tag{15}$$

Further following[21] − [25], we assume $G=t^m$. Hence equations (9) and (14) suggest

$$\dot{\Lambda}-\frac{m}{t}\Lambda=-\frac{2mn}{(n+2)^2}\frac{1}{t^3}, \tag{16}$$

which is a linear equation and it has solution

$$\Lambda=\frac{2mn}{(n+2)^2(m+2)}\frac{1}{t^2}. \tag{17}$$

From equations (14) and (17) one can easily obtain expression for energy density in terms of cosmic time t as

$$\rho=\frac{n}{2\pi(m+2)(n+2)^2}\frac{1}{t^{m+2}}. \tag{18}$$

Considering the usual barotropic equation of state relating the perfect fluid pressure p to the energy density as

$$p=(\gamma-1)\rho, \tag{19}$$

where $\gamma(1\le\gamma\le 2)$ is a constant, equations (13) and (14) lead to

$$\Pi=\frac{n(m+1)}{2\pi(m+2)(n+2)^2}\frac{1}{t^{2+m}}. \tag{20}$$

Now, we consider[7, 10][41] − [44] following phenomenological widely accepted relations

$$\xi-\xi_0\rho^{\alpha} \quad and \quad \tau=\frac{\xi}{\rho} \tag{21}$$

for the bulk viscosity coefficient ξ and mass density ρ and also for bulk viscosity coefficient and the relaxation time τ, respectively, where $\xi_0\ge 0$ and α are constants. If $\alpha=1$, eq.(21) may correspond to a radiative fluid, whereas $\alpha=\frac{3}{2}$ may correspond to a string-dominated universe. However, more realistic models are based upon α in the region $0\le\alpha\le\frac{1}{2}$.
Using (21), equation (10) on integration yields

$$T=T_0\ exp\left[\frac{2}{\epsilon}g(t)\right]\ exp\left[\frac{2}{\epsilon}f(t)\right]\frac{\Pi^{\frac{2}{\epsilon}}R^3\tau}{\xi}, \tag{22}$$

where $f(t)$ and $g(t)$ are anti-derivatives of $\frac{1}{\tau}$ and $\frac{3\rho H}{\Pi}$ respectively.
With the help of equations (18), (20) and (21), we obtain the expressions for $f(t)$ and $g(t)$ as

$$f(t)=\frac{\tau_0}{t^{(m+2)(\alpha-1)-1}}, \tag{23}$$

$$g(t)=\frac{1}{m(m+1)t^m}, \tag{24}$$

where

$$\tau_o=\frac{1}{1-(m+2)(\alpha-1)}\times$$
$$\times\left[\frac{n}{2\pi(m+2)(n+2)^2}\right]^{\alpha-1}. \tag{25}$$

We observe from eq. (17) that the cosmological constant in this model is decreasing function of time and it approach a small value as time increases (i.e., the present epoch). For $m,n>0$ the value of cosmological "constant" for this model is found to be small and positive which is supported by the results from recent supernovae observations recently obtained by the High - z Supernova Team and Supernova Cosmological Project (Garnavich *et al.*;[45] Perlmutter *et al.*;[46] Riess *et al.*;[47] Schmidt *et al.* [48]).
Using the observational values $\dot{G}/G=10^{-11}yr^{-1}$ and $H_p=7.5\times10^{-11}yr^{-1}$, and the relation of the present age of the universe with Hubble parameter ($H_p\,t_p\sim\frac{2}{3}$), relation $G=t^m$ suggest

$$m=\frac{2}{22.5}.$$

This clearly shows that gravitational parameter G turns out to be an increasing function of time.
From eq (18), it is observed that the energy density ρ is decreasing with evolution of the universe. For all $m>-2$, $n>0$, the energy conditions are satisfied.
It is also observed from eq (20) that bulk viscous pressure Π decreases with time. When $t\to 0$, $\Pi\to\infty$. When $t\to\infty$, $\Pi\to 0$.
In this model, expressions for expansion and shear are

$$\theta=\frac{1}{t}\quad and \quad \sigma^2=\frac{2(n-1)^2}{3(n+2)^2t^2}. \tag{26}$$

$$\frac{\sigma^2}{\rho} = \frac{4\pi(n-1)^2(m+2)}{3n} t^m. \tag{27}$$

Equation (28) clearly shows the effect of time varying G on the relative anisotropic. For, $-2 < m < 0$, relative anisotropy is decreasing. Further, it can be observed that $\sigma^2 \propto \theta^2$, which indicate that the model does not approach isotropy for large value of t.

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References

[1] C.W. Misner, *Phys. Rev. Lett.*, **19**, 533 (1966).

[2] S. Weinberg, *Astrophy. J.*, **168**, 175 (1971).

[3] N. Turok, *Phys. Rev. Lett.*, **60**, 549 (1988).

[4] J.D. Barrow, *Nucl. Phys.* B, **310**, 243 (1988).

[5] M.K. Mak, T. Harko, *J. Math. Phys.*, **39**, 5458 (1998); *Aust. J. Phys.* **52**, 659 (1998).

[6] L.P. Chimento, A.S. Jakubi, *Phys. Lett.* A, **212**, 320 (1996).

[7] R. Maartens, *Class. Quant. Grav.*, **12**, 1455 (1995).

[8] C. Eckart, *Phys. Rev.*, **58**, 919 (1940).

[9] L.D. Landau, E.M. Lifshitz, *Fluid Mechanics* (Butterworth Heinemann, 1987).

[10] W. Zimdahl, *Phys. Rev.* D, **53**, 5483 (1996).

[11] W. Israel, *Ann. Phys.*, **100**, 310 (1996).

[12] W. Israel, J.M. Stewart, *Phys. Lett.* A, **58**, 213 (1976).

[13] A.M. Anile, D. Pavon, V. Romano, "*The case of hyperbolic theories of dissipation in relativistic fluids*", [gr-qc/9810014].

[14] J.A. Frieman, I. Waga, *Phys. Rev.* D, **57**, 4642 (1998).

[15] R. Carlberg *et al.*, *Astrophy. J.*, **462**, 32 (1996).

[16] M. Özer, M.O. Taha, *Nucl. Phys.* B, **287**, 776 (1987); K. Freese, F.C. Adams, J.A. Frieman, E. Motta, *ibid.* B, **287**, 797 (1987); J.C. Carvalho, J.A.S. Lima, I. Waga, *Phys. Rev.* D, **46**, 2404 (1992); V. Silviera, I. Waga, *ibid.* D, **50**, 4890 (1994).

[17] A.D. Lindey, *ZETP Lett.*, **19**, 183 (1974).

[18] B. Ratra, P.J.E. Peebles, *Phys. Rev.* D, **37**, 3406 (1988).

[19] A.D. Dolgov, *Phys. Rev.* D, **55**, 5881 (19970).

[20] A.D. Dolgov, in *The Very Early Universe*, eds. G. W. Gibbons, S.W. Hawking, S.T.C. Siklos, (Cambridge Univerity Press, 1983).

[21] A.D. Dolgov, M.V. Sazhin, Ya.B. Zeldovich, *Basics of Modern Cosmology* (Editions Frontiers, 1990).

[22] V. Sahni, A. Starobinsky, *Int. J. Mod. Phys.* D, **9**, 373 (2000); (gr-qc/9904398).

[23] P.J.E. Peebles, B. Ratra, *Rev. Mod. Phys.*, **75**, 559 (2003); (astro-ph/0207347).

[24] T. Padmanabhan, *Phys. Rep.*, **380**, 235 (2003); (hep-th/0212290).

[25] R.G. Vishwakarma, *Class. Quant. Grav.*, **17**, 3833 (2000); *Class. Quant. Grav.*, **18**, 1159 (2001); *Gen. Rel. Grav.*, **33**, 1973 (2001); *Class. Quant. Grav.*, **19**,776 (2002).

[26] L.M. Krauss, M.S. Turner, *Gen. Rel. Grav.*, **21**, 1137 (1995).

[27] P.S. Wesson, *Gravity, particles and Astrophysics*, (Reidel, Dordrecht, 1980).

[28] M. Der Sarkissian, *Nuovo Cimento* B, **88**, 29 (1995).

[29] O. Bertolami, *Nuovo Cimento* B, **93**, 36 (1986); *Fortschr. Phys.*, **34**, 829 (1986).

[30] M.S. Berman, *Gen. Rel. Grav.*, **23**, 465 (1991).

[31] A.M.M. Abdel-Rahman, *Gen. Rel. Grav.*, **22**, 655 (1990).

[32] A. Beesham, *Gen. Rel. Grav.*, **26**, 159 (1994); *Phys. Rev.* D, **48**, 3539 (1999).

[33] D. Kallingas, P.S. Wesson, C.W.F. Everitt, *Gen. Rel. Grav.* **24**, 351 (1992).

[34] Abdussattar and R.G. Vishwakarma, *Class. Quant. Grav.*, **14**, 945 (1997).

[35] I. Chakrabarty, A. Pradhan, *Grav.& Cosm.*, **25**, 55 (2001).

[36] A. Pradhan, A.K. Vishwakarma, *SUJST* **XII**, Sec. B, 42 (2000);
A. Pradhan, A. Kumar, *Int. J. Mod. Phys.* D, **10**, 291 (2001);
A. Pradhan, V.K.Yadav, *Int. J. Mod. Phys.* D, **11**, 893 (2002).

[37] A.I. Arbab, *Gen. Rel. Grav.* **29**, 61 (1997).

[38] T. Singh, A. Beesham, W.S. Mbokazi, *Gen. Rel. Grav.* **30**, 573 (1998).

[39] T. Harko, M.K. Mak, *Gen. Rel. Grav.*, **31**, 849 (1999).

[40] J.A. Belinchón, I. Chakrabarty, *Int. J. Mod. Phys.* D, **12**, 861 (2003).

[41] N. O. Santos, R. S. Dias, A. Banerjee, *J. Math. Phys.*, **26**, 878 (1985).

[42] O. Grön, *Astrophys. Space Sci.*, **58**, 919 (1990).

[43] D. Pavon, *Class. quant. Grav.*, **7**, 487 (1990).

[44] D. Pavon, J. Bafaluy, D. Jou, *Class. Quant. Grav.*, **8**, 347 (1991).

[45] P.M. Garnavich *et al.*, *Astrophys. J.*, **493**, L53 (1998a), Hi-z Supernova Team Collaboration (astro-ph/9710123); *Astrophys. J.*, **509**, 74 (1998b), Hi-z Supernova Team Collaboration (astro-ph/9806396).

[46] S. Perlmutter *et al.*, *Astrophys. J.* **483**, 565 (1997), Supernova Cosmology Project Collaboration (astro-ph/9608192); *Nature*, **391**, 51 (1998); Supernova Cosmology Project Collaboration (astro-ph/9712212); *Astrophys. J.* **517**, 565 (1999).

[47] A.G. Riess *et al.*, *Astron. J.*, **116**, 1009 (1998), Hi-z Supernova Team Collaboration (astro-ph/9805201).

[48] B.P. Schmidt, *Astrophys. J.*, **507**, 46 (1998), Hi-z Supernova Team Collaboration (astro-ph/9805200).

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THERMAL VARIATION IN THE EXTERNED ONE-ZONE RR LYRAE MODEL. II NON LINEAR RESULTS

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The extended one-zone stellar pulsation model is proposed as a tool to investigate the factors affecting thermal variation of pulsating stars. Linear analyse of the resulting equations is described. The results are in very good agreement with the detailed calculations.

Key words: RR Lyrae – variable stars

1. Introduction

Stellingwerf's nonlinear one-zone model (Stellingwerf, 1972) was extended (Pricopi, 2005) by considering a slow and uniform rotation that lead to a very small oblateness of the star and that the matter in the core-surrounding shell consist of a mixture of ideal gas and radiation. In this paper, we use this model to investigate the factors affecting the linear variation of the thermal structure of RR Lyrae-like pulsating stars.

The actual mechanisms responsible for the pulsations of most kind of variable stars are envelope ionization mechanisms. The second helium ionization zone seems to be the main agent responsible for the pulsations in most common types of variable stars. Hydrogen ionization may even be the main cause of the instability in the red variables (Cox, 1980).

If the opacity law is of the form $\kappa \sim \rho^n T^{-s}$ and if s is large and negative, there may be a dumming up of radiation upon compression, and hence driving, even if Γ_3 has a value close to its normal value of 5/3. This fact has been found to be important in some cases by Stellingwerf (1978, 1979) in his calculations of the pulsational stability of models for δ Scuti-like stars and for β Cephei stars. Stellingwerf has referred to this effect as the "bump mechanism."

"Bump mechanism" is responsible for the pulsations of our extended one-zone model. The results are qualitative and, however, this model is not intended to be a substitute for finely zoned nonlinear calculations. In Section 2 we resort the well-known equations of stellar structure (e.g. Kippenhahn and Weigert, 1991; Lungu, 1982) and the equations of the our model are written down. The linear results are presented in Section 3. From the condition of pulsational instability we obtain a condition for s in terms of physical parameters. The linear variation of the opacity, temperature and effective temperature is presented for strictly periodic pulsations.

2. Basic equations

The equations of stellar structure are (Kippenhahn and Weigert, 1991; Lungu, 1982):

1. The motion equation:

$$\frac{\partial^2 r}{\partial t^2} = -\frac{Gm}{r^2} - \frac{3}{5}\frac{Gma^2\lambda}{r^4}(1 - 3\cos^2\theta) + \omega^2 r\sin^2\theta - 4\pi r^2\frac{\partial P}{\partial m}. \quad (1)$$

2. The continuity equation:

$$dm = 4\pi r^2(1-\lambda)\rho dr. \quad (2)$$

3. The energy equation:

$$\frac{\partial l}{\partial m} = -c_V\frac{\partial T}{\partial t} + \frac{\delta}{\alpha}\frac{P}{\rho^2}\frac{\partial \rho}{\partial t}. \quad (3)$$

4. And the radiative energy transport equation in the diffusion approximation:

$$l = \left[4\pi r^2(1-\lambda)\right]^2 \frac{4\sigma}{3\kappa}\frac{\partial T^4}{\partial m}, \quad (4)$$

where the notations are: ω = (small) angular velocity, λ = oblateness, a = semimajor axis of the ellipsoid, θ = polar angle. The others notations are usual. The pressure $P = P_{gas} + P_{rad}$. Following Kippenhahn and Weigert (1991), let $\beta = P_{gas}/P$. It follows $\alpha = 1/\beta$, $\delta = (4-3\beta)/\beta$, $\nabla_{ad} = [1 + (1-\beta)(4+\beta)/\beta^2]/[5/2 + 4(1-\beta)(4+\beta)/\beta^2]$ and $\Gamma_1 = 1/(\alpha - \delta\nabla_{ad})$. If $\beta = 1$

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(pure gas) we have $\Gamma_1 = 5/3$ and if $\beta = 0$ (pure radiation) we have $\Gamma_1 = 4/3$.

Like Stelligwerf (1972) we introduce the following relations referring to the core-surroundig shell:

$$\frac{\partial P}{\partial m} = -\frac{P}{m_s}, \quad \frac{\partial l}{\partial m} = \frac{L - L_i}{m_s}, \quad \frac{\partial T^4}{\partial m} = -\frac{T^4}{m_s}, \tag{5}$$

where P, L, T stands for the pressure, radiative energy flux, temperature in the shell, respectively, L_i = luminosity at the base of the shell and m_s = shell mass. Let M = stellar mass, R = stellar radius and R_c = rigid core radius.The equations (1), (3) and (4) become respectively:

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2} - \frac{3}{5}\frac{GMa^2\lambda}{R^4}(1 - 3\cos^2\theta) + $$
$$+\omega^2 R\sin^2\theta + 4\pi R^2\frac{P}{m_s}; \tag{6}$$

$$\frac{L - L_i}{m_s} = -c_V\frac{\partial T}{\partial t} + \frac{\delta}{\alpha}\frac{P}{\rho^2}\frac{\partial \rho}{\partial t}; \tag{7}$$

$$L = -\frac{64\pi^2(1-\lambda)^2\sigma R^4}{3\kappa}\frac{T^4}{m_s}. \tag{8}$$

From definition, $\lambda = (a-b)/(a+b)$, b = semiminor axis of ellipsoid, and we can write

$$a = \frac{R}{\sqrt{1 + \frac{4\lambda\cos^2\theta}{(1+\lambda)^2}}}. \tag{9}$$

Taking into account (9), the eq. (6) become

$$\frac{d^2R}{dt^2} = -\frac{GM}{R^2}\chi + \omega^2 R\sin^2\theta + 4\pi R^2\frac{P}{m_s}, \tag{10}$$

where

$$\chi = 1 + \frac{3}{5}\frac{\lambda(1 - 3\cos^2\theta)}{1 + \frac{4\lambda\cos^2\theta}{(1+\lambda)^2}}. \tag{11}$$

For a static star ($\omega = 0$), we have $\lambda = 0$ and, from (11) $\chi = 1$.

The hydrostatic equilibrium state implies

$$4\pi R_0^2\frac{P_0}{m_s} = \frac{GM}{R_0^2}\chi - \omega^2 R_0\sin^2\theta, \tag{12}$$

where the "0" subscript correspond to the equilibrium model.

Following Stellingwerf (1972), we denote

$$X = \frac{R}{R_0}. \tag{13}$$

The geometry is introduced via the function $m = m(X)$ such that (Rudd and Rosenberg, 1970)

$$\frac{\rho}{\rho_0} = X^{-m}, \tag{14}$$

where

$$m(X) = \frac{\ln\left(\frac{X^3 - \eta^3}{1 - \eta^3}\right)}{\ln X}, \tag{15}$$

with $\eta = R_c/R_0$. The equilibrium value of m is $m_0 = 3/(1-\eta^3)$.

The non-adiabatic effects are contained in the function h defined by

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^{\Gamma_1} h, \tag{16}$$

where $\Gamma_1 = (\partial \ln P/\partial \ln \rho)_{ad}$. With these definitions, eq. (10) become

$$\frac{d^2X}{dt^2} = \xi(hX^{-q} - X^{-2}) - \zeta(hX^{-q} - X), \tag{17}$$

where $\xi = GM\chi/R_0^3$, $\zeta = \omega^2\sin^2\theta$ and $q = m\Gamma_1 - 2$.

As regards the properties of the stellar matter, we consider the following formulae for the equation of state and opacity law, respectively:

$$\rho = \rho_k P^\alpha T^{-\delta}; \tag{18}$$

$$\kappa = \kappa_k \rho^n T^{-s}, \tag{19}$$

where ρ_k and κ_k are constants.

Using (14) and (16) we write

$$\frac{P}{P_0} = X^{-m\Gamma_1}h. \tag{20}$$

From (14), (18) and (20) we obain

$$\frac{T}{T_0} = X^{-m(\Gamma_3 - 1)}h^{\frac{\alpha}{\delta}}, \tag{21}$$

where $\Gamma_3 - 1 = (\gamma - 1)/\delta$, $\gamma = c_P/c_V = \alpha\Gamma_1$. For the opacity we write using (14), (19) and (21):

$$\frac{\kappa}{\kappa_0} = X^{-m[n - s(\Gamma_3 - 1)]}h^{-s\frac{\alpha}{\delta}}. \tag{22}$$

From (8), (21) and (22) we obtain for luminosity:

$$\frac{L}{L_0} = X^{4 + m[n - (4+s)(\Gamma_3 - 1)]}h^{(4+s)\frac{\alpha}{\delta}} \tag{23}$$

and the variation of the luminosity at the base of the shell is supposed to be

$$\frac{L_i}{L_0} = X^{-u}, \tag{24}$$

where u is a parameter that ranges from 0 to 20 (Stellingwerf and Donohoe, 1987). Using the relation $L = 4\pi R^2\sigma T_{eff}^4$ we obtain for effective temperature

$$\frac{T_{eff}}{T_{eff0}} = X^{\frac{1}{2} + \frac{m}{4}[n - (4+s)(\Gamma_3 - 1)]}h^{\frac{1}{4}(4+s)\frac{\alpha}{\delta}}. \tag{25}$$

Using these relations, eq. (7) becomes

$$\frac{dh}{dt} = -\varepsilon\frac{\delta}{\alpha}X^{m(\Gamma_3-1)}h^{1-\frac{\alpha}{\delta}}\times$$

$$\times\left(X^{4+m[n-(4+s)(\Gamma_3-1)]}h^{(4+s)\frac{\alpha}{\delta}} - X^{-u}\right) -$$

$$-3\frac{\delta}{\alpha}(\Gamma_3-1)X^2h\left(X^3-\eta^3\right)^{-1}\times$$

$$\times\left(X^{m[\Gamma_3-\Gamma_1]}h^{1-\frac{\alpha}{\delta}}-1\right)\frac{dX}{dt}, \qquad (26)$$

where $\varepsilon = L_0/m_s c_V T_0 = L_0/E_s$ (E_s is the internal energy of the shell).

Equations (17) and (25) constitute our final set of relations for the unknowns X and h.

3. Linear results

We assume small amplitude motion and put $x = X-1$, $h' = h-1$. The linear form of eq. (17) is

$$\frac{d^2x}{dt^2} = [\xi(2-q)+\zeta(1+q)]x + (\xi-\zeta)h'. \qquad (27)$$

The linear form of eq. (25) is

$$\frac{dh'}{dt} = -\frac{\delta}{\alpha}\varepsilon[(b+u)x+(4+s)h'], \qquad (28)$$

where $b = 4+m_0[n-(4+s)(\Gamma_3-1)]$. As usual, we assume a time variation of $e^{i\sigma t}$ for all quantitties. From eq. (27) and (28) we obtain:

$$h' = \frac{\sigma^2-[\xi(2-q)+\zeta(1+q)]}{(\xi-\zeta)}x; \qquad (29)$$

$$h' = -\frac{\frac{\delta}{\alpha}\varepsilon(b+u)}{i\sigma+\varepsilon(4+s)}x. \qquad (30)$$

These equations may be combined to yield

$$(i\sigma)^3 + A(i\sigma)^2 + B(i\sigma) + C = 0, \qquad (31)$$

where $A = \varepsilon(4+s)$, $B = [\xi(2-q)+\zeta(1+q)]$ and $C = \varepsilon(4+s)[\xi(2-q)+\zeta(1+q)] + (\delta/\alpha)\varepsilon(b+u)(\zeta-\xi)$.

The condition for vibrational instability (Stellingwerf, 1972)

$$b+u > 0; \qquad (32)$$

results in a condition for s:

$$s < \frac{1}{\Gamma_3-1}\left(n+\frac{4+u}{m_0}\right) - 4. \qquad (33)$$

Exterior to an ionization zone we have $n=1$, $\gamma = 5/3$ and $\delta = 1$ (if $\beta = 1$). Then, putting $u=0$ we obtain $s < -0.5$ for $m_0 = 3$ (homogeneous star) and $s < -1.9$ for $m_0 = 10$ (RR Lyrae star). The "bump mechanism" will thus be more effective for main sequence (low m_0) stars than for giants. Also, the "bump mechanism" is more effective if the luminosity variation at the base of the shell is increasing: putting $u = 10$, we obtain $s < 3.5$ for $m_0 = 3$ and $s < -0.4$ for $m_0 = 10$.

Let $N = A^2-3B$, $Q = 2A^3-9AB+27C$, $S = -4N^3+Q^2$, $Z = [(-Q+\sqrt{S})/2]^{1/3}$. By solving eq. (31) we obtain:

$$Re(i\sigma) = -\frac{1}{6}\left(\frac{N}{Z}+Z+2A\right); \qquad (34)$$

$$Im(i\sigma) = \pm\frac{i\sqrt{3}}{6}\left(-\frac{N}{Z}+Z\right).$$

From (34) we can obtain the value of s for the case of strictly periodic pulsations putting $Re(i\sigma) = 0$. Also, in that case is easy to show that $Im(i\sigma) = \pm\sqrt{B}$, where $B > 0$ (i.e. condition of dynamical stability).

The opacity variation will be controled by the energy equation. We may define $\kappa' = \kappa/\kappa_0 - 1$ and combine eqs.(22) and (30) to obtain (for strictly periodic pulsations)

$$\kappa' =$$

$$= \frac{i\sigma\left(4-b-4m_0(\Gamma_3-1)\right)+\varepsilon(4s-4m_0n+su)}{i\sigma+A}x \qquad (35)$$

or, after some manipulation,

$$\kappa' = \frac{[\varepsilon AE+BD]-i\sqrt{B}(b+u)\varepsilon s}{A^2+B}x, \qquad (36)$$

where $D = 4-b-4m_0(\Gamma_3-1) = -m_0[n-s(\Gamma_3-1)]$ and $E = 4s-4m_0n+su$. From the relation (36) we see that the opacity can increase upon compression leading to driving in this region.

The linear variation of the temperature and effective temperature is easily to obtain putting $t = T/T_0 - 1$ and $t_{eff} = T_{eff}/T_{eff0} - 1$ and using (21), (25) and (30):

$$t = \frac{i\sigma m_0(\Gamma_3-1)+\varepsilon(4+m_0n+u)}{i\sigma+A}x; \qquad (37)$$

$$t_{eff} = \frac{1}{4}\frac{i\sigma(2+b)+\varepsilon(4+s)(2-u)}{i\sigma+A}x$$

or, after some manipulation (for strictly periodic pulsations)

$$t = \frac{[\varepsilon AF+Bm_0(\Gamma_3-1)]-i\sqrt{B}(b+u)\varepsilon}{A^2+B}x; \qquad (38)$$

$$t_{eff} =$$

$$= \frac{1}{4}\frac{[(A^2+B)(2-u)+B(b+u)]+iA\sqrt{B}(b+u)}{A^2+B}x,$$

where $F = 4 + u + m_0 n$. From (38) we see that $t_{\max}$ lags before $x_{\min}$ by $0°\text{div}\,90°$ i.e. the temperature is increasing upon compression (at least close to the maximum compression of the shell). Also, $t_{eff\,\max}$ lags before $x_{\max}$ by $0°\text{div}\,180°$. An detailed discussion of eqs. (38) will be made elsewhere.

Finally, we conclude that this simple model can be used to shed much light upon (linear) thermal variation of the RR Lyrae type pulsating stars.

References

[1] Baker, N., 1966, *Stellar Evolution*, Eds. R. F. Stein and A. W. G. Cameron, Plenum Press, New York, p. 333

[2] Castor,J.I., 1968, *Ap.J.* **154**, 793

[3] Castor,J.I., 1971, *Ap.J.* **166**, 109

[4] Cox J.P., Giuli, 1968, *Principles of Stellar Structure*, Gordon and Breach, New York

[5] Cox, J.P., King, D.S., Stellingwerf, D.F., 1972, *Ap.J.* **171**, 93

[6] Goldreich and Weber, 1980, *Ap.J.* **238**, 991

[7] Iben,I.,Jr., 1971, *Ap.J.* **166**, 131

[8] Kippenhahn R.,Weigert A., 1991, *Stellar Structure and Evolution,* Springer-Verlag, Berlin

[9] Ledoux,P., Walraven,Th., 1958, *Hbd. Phys.* **50**, 353

[10] Lungu,N., 1982, *Stellar Pulsations*, Scientific and Encyclopedic Editure, Bucharest

[11] Okamoto, I., Unno,W., 1967, *Publ. Astr.Soc. Japan*, **19**, 154

[12] Pricopi D., 2005, *S.A.J.* (submited)

[13] Rudd, T.J., Rosenberg, R.M., 1970, *Astr.Astrophys.* **6**, 193

[14] Stellingwerf, R.F., 1972, *Astron&Astrphys.***21**, 91

[15] Stellingwerf, R.F., Gautshy, A., Dickens, R,J., 1987, *Ap.J.* **313**, 75

[16] Stellingwerf, R.F., Donohoe, M., 1987, *Ap.J.* **314**, 252

[17] Sweet, P.A., 1950, *M.N.R.A.S.* **110**, 548

[18] Usher, P.D., Whitney, C.A., 1968, *Ap.J.* **154**, 203

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ON THE CONSTANCY OF NATURAL CONSTANTS

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Three kinds of "constants" are applied in natural sciences, namely the products of definition (e.g. Avogardo number), time independent constants (e.g. light velocity) and time evolving constants (the Universe critical density). In many cases dimensionless constants have been introduced into natural sciences. Individual kinds of constants are discussed and exemplified. A profound relationship between the ENU (Expansive Nondecelerative Universe) and dimensionless constants of the fundamental physical interactions is presented. The contribution provides an alternative to the Dirac presumption on a time decrease of the gravitational constant G. Using simple relations the contribution precises the values of the vector bosons x and y.

1. Introduction

An issue of the time dependence of natural constants is of fundamental importance in natural sciences [1]. At present three kinds of constants are applied in natural sciences. The first group consists of the constants introduced "artificially" by scientists. As an example, Avogadro constant ($N_A = 6.0221367(36)$ x 10^{23} mol^{-1}) defining a number of units in one mol of a substance may be given. For the majority of constants their values are given in common literature sources without, however, a deep discussion of a real constancy (i.e. time independence). Some constants were subjected to such analysis and it was postulated that their values are evolving in the course of time. In connection with a extremely low value of the ratio of α_g/α_e Dirac suggested [2] that the value of some fundamental constants can change in time, e.g. gravitational constant G should decrease in time (of course, it relates to time of cosmological dimension). In this paper, the issue of time dependence of some constants is analysed.

2. Dimensionless constants and their time evolution

There is no problem in defining dimensionless physical constants, namely that of strong interaction α_s

$$\alpha_s = 1 \tag{1}$$

and electromagnetic interaction α_e

$$\alpha_e = \frac{e^2}{4\,\pi\,\varepsilon_o\,\hbar\,c} = 7.29 \times 10^{-3}. \tag{2}$$

For the constant of weak interaction α_w various modes of expression are used, the simplest one being [1]

$$\alpha_w = \frac{g_F\,m_P^2\,c}{\hbar^3} \approx 10^{-6}, \tag{3}$$

where g_F is the Fermi constant ($g_F = 1.41 \times 10^{-62}$ J m^3) and m_p is the proton mass. Ambiguousness appears in the gravitational constant α_g, for which the most familiar expression and value is

$$\alpha_g = \frac{G\,m_P^2}{\hbar\,c} \approx 10^{-39}. \tag{4}$$

The constant α_g depends on the mass option. As a matter of tradition, the proton mass m_P is used, there is, however, no justified reason for it. On the other hand, there should be a certain mass m_x, the introduction into α_g of which will be rationalized and justifiable. Gravitational forces are far-reaching, theoretically boundless, forces. Due to the operation of the hierarchical rotational gravitational systems (HRGS), the actual gravitational effects are considered as being infinitive. Gravitation can manifest itself only where the density of the gravitational energy of HRGS is higher than the critical density ε_{crit}. In the ENU model [3-5] it holds

$$\varepsilon_g = -\frac{c^4 R}{8\,\pi\,G} = -\frac{3\,m\,c^2}{4\pi\,a\,r^2}, \tag{5}$$

where ε_g is the density of gravitational energy created by a body with the mass m at the distance r, R is the scalar curvature, a is the gauge factor that at present

$$a = 1.229 \times 10^{26} \text{ m}. \tag{6}$$

In the ENU model ε_{crit} is expressed as

$$\varepsilon_{crit} = \frac{3\,c^4}{8\,\pi G\,a^2}. \tag{7}$$

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In the cases when

$$|\varepsilon_{\mathrm{g}}| = \varepsilon_{\mathrm{crit}} \tag{8}$$

using equations (5) and (7), the following relation is obtained

$$r = r_{\mathrm{ef}(m)} = \left(a\, R_{\mathrm{g}(m)}\right)^{1/2} \tag{9}$$

in which $r_{\mathrm{ef}(m)}$ represents the effective gravitational range of a body with the mass m, $R_{g(m)}$ is its gravitational radius. Since for the Compton wave λ of the particle with the mass m

$$\lambda = \frac{\hbar}{m\,c} \tag{10}$$

a particle having the mass m_x for which $\lambda = r_{\mathrm{ef}(m)}$ must exist. It thus represents the lighest particle able to gravitationally influence its environment. Relating (9) and (10) leads to

$$m_x = \left(\frac{\hbar^2}{2\,G\,a}\right)^{1/3}. \tag{11}$$

At present

$$m_x \approx 10^{-28}\ \mathrm{kg}. \tag{12}$$

This value can be introduced into α_{g} without additional presumptions and then

$$\alpha_{\mathrm{g}} = \frac{G m_x^2}{\hbar\, c} = \frac{m_x^2}{m_{\mathrm{Pc}}^2} \approx 10^{-40}, \tag{13}$$

where m_{Pc} is the Planck mass

$$m_{\mathrm{Pc}} = 2.176711 \times 10^{-8}\ \mathrm{kg} \tag{14}$$

In the ENU model it was proved that

$$a = c\,t = \frac{2\,G\,m_{\mathrm{U}}}{c^2}, \tag{15}$$

where t is the cosmological time, m_{U} is the mass of the Universe. It follows from (11), (13) and (15)

$$\alpha_{\mathrm{g}} \sim (t)^{-2/3}. \tag{16}$$

It is worth pointing out at the importance of relation (16). It brings an evidence that α_{g} is not a true constant but a time decreasing quantity. This fact leads to some interesting consequences.

The Compton dimension of particle m_x is about 10^{-15} m and corresponding $t_x \sim 10^{-23}$ s. Consequently, for the cosmologic time t expressed in t_x units using (11), (13) and (15), relation (17) can be obtained

$$\frac{t}{t_x} = \frac{1}{\alpha_{\mathrm{g}}}. \tag{17}$$

This fact was known to Dirac who, however, t_x put equal to the atomic time. It was, however, only a coincidence since at present the Compton wavelength of the particle m_x approaches the dimension of atomic nucleus. Since Dirac introduced the constant mass of the proton when expressing α_{g} and at the same time relied on the validity of relation (17), he formulated a false conclusion on time decrease of the gravitational constant G. In reality, G is a true constant and α_{g} is a time decreasing quantity (see relations 11, 13 and 16).

In the ENU model the Universe is mass-space-time (in the language of special relativity) closed. In the framework of Friedmann model for such systems it is supposed that

$$m_U \approx \frac{m_{\mathrm{P}}}{\alpha_{\mathrm{g}}^2}. \tag{18}$$

The above relation cannot be unambiguously proved, using (11), (13), (15) it can be, however, derived that

$$m_{\mathrm{U}} \approx \frac{m_x}{\alpha_{\mathrm{g}}^2}. \tag{19}$$

In addition, it holds in Friedmann model that

$$k\,T \approx \left(\frac{\hbar^3\, c^5}{G\,t^2}\right)^{1/4}, \tag{20}$$

where T is the temperature of the relict radiation. For closed systems it is supposed that

$$k\,T_{\min} \approx \alpha_{\mathrm{g}}^{1/4} m_{\mathrm{p}}\, c^2. \tag{21}$$

Relation (21) cannot be derived, however, it follows from (11), (13) and (20) that

$$k\,T \approx \alpha_{\mathrm{g}}^{1/4} m_x\, c^2. \tag{22}$$

If in a known empirical formula [1]

$$H \approx \frac{\alpha_{\mathrm{g}}\, m_{\mathrm{P}}\, c^2}{\hbar}, \tag{23}$$

where H is the Hubble constant, the mass m_{P} is substituted for m_x, relation (24) emerges which can be derived directly using (11), (13) and (15)

$$H \approx \frac{\alpha_{\mathrm{g}}\, m_x\, c^2}{\hbar}. \tag{24}$$

It should be worth mentioning that relations (19), (22) and (24) are in fact closely connected to dependence (17) which can be directly derived from them. It is obvious that the introduction of m_x into the "constant" α_{g} enables to formulate known empirical formulae as self-sustaining, rationalizable and derivable relations that are or should be valid in our Universe.

Due to the time decreasing of α_{g} (16) there had to be the time in the past when

$$\alpha_{\mathrm{g}} = \alpha_s. \tag{25}$$

It follows of (1), (13) and (25) that at that time

$$m_{\mathrm{Pc}} = m_x = 10^{19}\ \mathrm{GeV}. \tag{26}$$

It really happened at the beginning of the Universe expansion ($t \sim 10^{-43}$ s).

At about $t \sim 10^{-40}$ s following the beginning of the expansion the values of $\alpha_{\rm e}$ and $\alpha_{\rm g}$ were equal and, consequently, treatment of (2), (13) and (25) leads to relation

$$m_x = m_{\rm Pc}\, \alpha_{\rm e}^{1/2} \approx 10^{18} \text{ GeV}. \tag{27}$$

At about $t \sim 10^{-34}$ s following the begining of the expansion, it held

$$\alpha_{\rm g} = \alpha_{\rm w} \tag{28}$$

and treating (3), (13) and (28) we get

$$m_x = m_{\rm Pc}\, \alpha_{\rm w}^{1/2} \cong 10^{16} \text{ GeV}. \tag{29}$$

We supposed the values m_x represent the mass of vector bosons x and y.

Dimensionless constant of gravitational interaction is not probably the only "constant" changing in time. Based on (at least) three experimental observations, namely: far-distance supernovas spectra; changes in the ratio of radioactive elements distribution in a nuclear reactor of Oklo; measurements of the anisotropy of cosmic radiation background it follows that there is a actual possibility of very slow increase in dimensionless "constant" of electromagnetic interaction.

Such a time evolution would have a very strong impact to the future of our Universe. Stemming from a consistency of the ENU model and de Aquino's experiments with gravitational H and G systems [6,7] a possibility of an increase of vacuum permeability in time follows. Taking the velocity of light as a time independent constant, we must expect a vacuum permittivity time decreasing. This is in line with a supposed increase of fine-structure constant in time. When accepting a presumption of the constancy of strong interaction, in far future the magnitude of electromagnetic interaction shall approach the magnitude of strong interaction. In a moment of their identity a disintegration of nuclear matter will happen. Only black holes, gravitones, photons, leptons and protons will exist in the Universe.

3. Conclusions

1. The present contribution rationalize the "constant" $\alpha_{\rm g}$ and makes possible to prove on a exact base some relations describing our Universe that up to now are formulated as empirical formulae.

2. The contribution corrects Dirac presumptions on a time decrease of constant G.

3. Using simple relations the contribution determines the mass of vector bosons x and y.

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References

[1] I. L. Rozentahl, *Adv. Math. Phys. Astr. 31* (1986) 241

[2] P. A. M. Dirac, *Nature 139* (1937) 323

[3] V. Skalský, M. Súkeník, *Astrophys. Space Sci., 178* (1991) 169

[4] M. Súkeník, J. Šima, *Spacetime and Substance 2* (2001) 79

[5] J. Šima, M. Súkeník, *Entropy 4* (2002) 152

[6] F. de Aquino, *J. New Energy 5* (2000) 67

[7] F. de Aquino, xxx.lanl.gov/abs/physics/0205089

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LINEAR WAWES IN AN IMPERFECT ANISOTROPIC MHD FLUID IN RELATIVISTIC FORMALISM

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Imperfect fluids are characterised by dissipative effects like bulk viscosity, shear and thermal conduction. They are present right from the terestrial plasma to the depth of time in the early universe. Isotropic fluid with finite dissipation was studied by S. Wienberg for its astrophysical implications. On the other extreme, propagation of linear waves have been under constant investigation for relativistic anisotropic magnetohydrodynamic plasmas. However anisotropic fluid with dissipative effects have not been given due consideration especially for plasmas inside a magnetic field. Here we intend to carry out studies for plasma with finite bulk viscosity. Conditions for growth and decay of linear waves are obtained in special cases of isotropy and Chew-Goldberger-Low (CGL) state while giving indicative results for general scenario.

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I. INTRODUCTION

Relativistic plasma embedded in a strong magnetic field exhibits itself in a variety of interesting astrophysical events from pulsar winds to blackhole magnetospheres [1, 2, 3]. Waves and instabilities for a single fluid with isotropic pressure have been investigated in the relativistic magnetohydrodynamic framework [4, 5]. However in some situations the plasma does not remain isotropic, as in Cosmic Pinches where pressures transverse to the ambient magnetic field dominates while the reverse is true for Pulsar winds. The equation of state of plasma have also been subject of study when the strong magnetic field suppresses the equilibration of pressures parallel and perpendicular to itself [6]. Plane magnetosonic waves have been studied for a single relativistic fluid with anisotropic pressures [7, 8]. However, there are systems in which radiation can provide the mechanism for viscosity and heat conduction. In these systems the bulk shear can be effective. (Imperfect) fluid systems with isotropic pressure and dissipative effects have been put to investigations by Wienberg [2, 9]. At the other level of interest, Winfried Zimdahl [10] applied the theory of dissipative processes in relativistic fluids to a flat homogeneous and isotropic universe with bulk viscosity to exhibit the possibility of inflationary solution. The bulk viscous pressure is then interpreted as an effective description for particle production processes. Roy Maartens and Vicenc Mendez [11] made a non-linear generalization to fluids with bulk viscosity which indicate to give a thermodynamically consistant inflationary solutions for the early universe. Karsten Jedamzik, Visnja Katalinic and Angela Olinto [12] examined the effect of dissipation on evolution of magnetic fields in an expanding fluid composed of matter and radiation. The cosmological implications of the decaying magnetic field is further explored.

The present paper intends to consider the effect of bulk viscosity on the plane wave propagation in relativistic magnetized plasma which has developed an anisotropy in pressures. The paper is organized as follows. In Sec. II we present the basic equations used in the present paper emphasizing the new energy-momentum tensor. In Sec. III the general dispersion relations are obtained and their derivation method is described briefly. In Sec. IV, to the first order of dissipation, the dispersion relation is discussed under various limits.

II. BASIC EQUATIONS

We consider a plasma embedded in a magnetic field. Plasma fluid velocity and temperature both are allowed to be relativistic. Hereafter, we will use units where velocity of light $c = 1$. Let U^α be the plasma 4-velocity and $F^{\alpha\beta}$ be the electromagnetic field tensor. The plasma rest-frame electric and magnetic fields are defined [7] as

$$E^\alpha \equiv F^{\alpha\beta} U_\beta, \tag{1}$$

$$B^\alpha \equiv \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} U_\beta F_{\gamma\delta}, \tag{2}$$

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respectively. The electromagnetic field tensor can then be written in the following form:

$$F^{\alpha\beta} = (E^\alpha U^\beta - E^\beta U^\alpha) + \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}(B_\gamma U_\delta - B_\delta U_\gamma), \quad (3)$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is a completely antisymmetric tensor and $\epsilon^{0123} = 1$.

For a hydrodynamical system whose pressure, density and velocity vary appreciably over distances of the order of mean free path, thermal equibrium is not strictly maintained and the fluid kinetic energy is dissipated as heat. The presence of weak space-time gradients in an imperfect fluid has the effect of modifying the energy-momentum tensor [9]. For a fluid system interacting with radiation and wherein bulk viscosity is the dominant dissipative component, the stress energy-momentum tensor is

$$T^{\alpha\beta} = W_1 U^\alpha U^\beta - W_2 \eta^{\alpha\beta} -$$
$$-W_3 n^\alpha n^\beta - \zeta U^\gamma_{,\gamma}(U^\alpha U^\beta - \eta^{\alpha\beta}), \quad (4)$$

where

$$W_1 = \varepsilon + p_\perp + \frac{B^2}{4\pi}, \quad (5)$$

$$W_2 = p_\perp + \frac{B^2}{8\pi}, \quad (6)$$

$$W_3 = p_\perp - p_{||} + \frac{B^2}{4\pi}. \quad (7)$$

Here $B^\alpha = Bn^\alpha$ and the coefficient of bulk viscosity ζ is positive definite. The plasma 4-velocity is normalized as $U^\alpha U_\alpha = 1$. The magnetic field unit vector n^α obeys $n^\alpha n_\alpha = -1$. The energy density ε is in general a function of mass density ρ and the magnetic field strenght B, and can be put in the following functional form [7]:

$$\varepsilon = \rho e(\rho, B) \quad (8)$$

The pressures are

$$p_{||} = \rho^2 \frac{\partial e}{\partial \rho} = \frac{\partial \varepsilon}{\partial ln\rho} - \varepsilon, \quad (9)$$

$$p_\perp = p_{||} + \rho B \frac{\partial e}{\partial B} = \frac{\partial \varepsilon}{\partial ln\rho} + \frac{\partial \varepsilon}{\partial lnB} - \varepsilon. \quad (10)$$

These equations are in their most general form and are representative of relativistic anisotropic state.

Invoking the usual frozen-in condition $E^\alpha = 0$ for the relativistic magnetohydrodynamic fluid, the electromagnetic dual tensor becomes

$$G^{\alpha\beta} = B(n^\alpha U^\beta - n^\beta U^\alpha), \quad (11)$$

and the Maxwell equations yields

$$G^{\alpha\beta}_{,\beta} = 0. \quad (12)$$

The equations of motion of the fluid are contained in the particle and energy-momentum conservation laws [2]:

$$J^\alpha_{,\alpha} = 0, \quad (13)$$

$$T^{\alpha\beta}_{,\beta} = 0, \quad (14)$$

where $J^\alpha (= \rho U^\alpha)$ is the 4-current density.

III. DISPERSION RELATION

The perturbed state is characterized by $\rho \to \rho + \delta\rho, B \to B + \delta B, U^\alpha \to U^\alpha + \delta U^\alpha, n^\alpha \to n^\alpha + \delta n^\alpha$, and plane wave solutions for the perturbations require that $\delta\rho, \delta B, \delta U^\alpha, \delta n^\alpha \propto \exp(ik_\alpha x^\alpha)$. It should be noted that the condition $U^\alpha U_\alpha = 1$ and $n^\alpha n_\alpha = -1$ yields the following constraints, $U^\alpha \delta U_\alpha = n^\alpha \delta n_\alpha = 0$.

It is assumed that the coefficient of bulk viscosity ζ does not vary appreciably with variations in above system parameters (ρ, B, U^α).

Substituting in Eqs.(12)-(14) $\partial/\partial x^\alpha \to ik_\alpha$, one obtains the following form:

$$k_\alpha \delta J^\alpha = 0, \quad (15)$$

$$k_\alpha \delta G^{\alpha\beta} = 0, \quad (16)$$

$$k_\alpha \delta T^{\alpha\beta} = 0. \quad (17)$$

The frequency and parallel component of the wave vector of the plane waves are defined in the rest frame of the fluid as:

$$\omega = k_\alpha U^\alpha, \quad k_{||} = -k_\alpha n^\alpha \quad (18)$$

respectively, and

$$k_\alpha k^\alpha = s^2 = \omega^2 - k^2 = \omega^2 - k_{||}^2 - k_\perp^2. \quad (19)$$

The coefficients W_i in the expression for the energy-momentum tensor will be function of number density ρ and magnetic field strength B. Hence,

$$\delta W_i = \frac{\partial W_i}{\partial \rho}\delta\rho + \frac{\partial W_i}{\partial B}\delta B.$$

The four-vector defined as $l^\alpha \equiv \epsilon^{\alpha\beta\mu\nu} k_\beta U_\mu n_\nu$ [7] will be orthogonal to the independent four vectors k^α, U^α and n^α i.e., $l^\alpha U_\alpha = l^\alpha n_\alpha = l^\alpha k_\alpha = 0$. Multiplication of Eqs.(15)-(17) by l_α , one obtains

$$\begin{pmatrix} k_{||} & \omega \\ \omega W_1 & k_{||} W_3 \end{pmatrix} \begin{pmatrix} l_\alpha \delta U^\alpha \\ l_\alpha \delta n^\alpha \end{pmatrix} = 0. \quad (20)$$

This yields the corresponding dispersion relation for the Alfven wave:

$$\begin{aligned} \omega^2 &= \frac{W_3}{W_1} k_{||}^2, \\ &= \left(\frac{p_\perp - p_{||} + \frac{B^2}{4\pi}}{\varepsilon + p_\perp + \frac{B^2}{4\pi}} \right) k_{||}^2. \end{aligned} \quad (21)$$

It requires that $l_\alpha \delta U^\alpha$ and $l_\alpha \delta n^\alpha$ be non zero which then reads that the fluid oscillations be tied to the magnetic oscillations.

Multiplying Eqs.(15)-(17) by the other three independent vectors k^α, U^α and n^α gives the following matrix

$$\begin{pmatrix} \omega & 0 & 0 & 0 & 0 & 1 \\ 0 & -k_{\|} & 1 & -\omega & 0 & 0 \\ 0 & \omega & 0 & 0 & -k_{\|} & 1 \\ A_1 & B_1 & 0 & k_{\|}W_3 & 0 & W_1 \\ A_2 & B_2 & 2k_{\|}W_3 & 0 & 0 & (-i\zeta k^2 + 2\omega W_1) \\ A_3 & B_3 & W_3 & 0 & \omega W_1 & -i\zeta k_{\|} \end{pmatrix} \times \begin{pmatrix} \delta ln\rho \\ \delta lnB \\ k_\alpha \delta n^\alpha \\ U_\alpha \delta n^\alpha \\ n_\alpha \delta U^\alpha \\ k_\alpha \delta U^\alpha \end{pmatrix} = 0, \tag{22}$$

where

$$A_1 = \omega \frac{\partial}{\partial ln\rho}(W_1 - W_2),$$

$$A_3 = k_{\|} \frac{\partial}{\partial ln\rho}(W_2 - W_3),$$

$$A_2 = \omega A_1 + k_{\|}A_3 + k_\perp^2 \frac{\partial}{\partial ln\rho} W_2,$$

$$B_1 = \omega \frac{\partial}{\partial lnB}(W_1 - W_2),$$

$$B_3 = k_{\|} \frac{\partial}{\partial lnB}(W_2 - W_3),$$

$$B_2 = \omega B_1 + k_{\|}B_3 + k_\perp^2 \frac{\partial}{\partial lnB} W_2.$$

The corresponding dispersion relation for the slow and fast magnetosonic modes are given by

$$\left[\omega^2 \frac{\partial(W_1 - W_2)}{\partial ln\rho} - k_{\|}^2 \frac{\partial(W_2 - W_3)}{\partial ln\rho}\right] \times \times \left[k_{\|}^2 W_3 - \omega^2 W_1 + k_\perp^2 \frac{\partial W_2}{\partial lnB}\right] + k_\perp^2 \frac{\partial W_2}{\partial ln\rho}\left[k_{\|}^2\left(W_3 + \frac{\partial(W_2 - W_3)}{\partial lnB}\right) + + \omega^2\left(W_1 - \frac{\partial(W_1 - W_2)}{\partial lnB}\right)\right] - i\zeta\omega\left[k_{\|}^2\left(k_{\|}^2 W_3 - \omega^2 W_1\right) + + k_\perp^2\left(\omega^2 \frac{\partial(W_1 - W_2)}{\partial lnB} + k_{\|}^2 \frac{\partial(W_3)}{\partial lnB}\right)\right] = 0. \tag{23}$$

In the limit of bulk viscosity $\zeta \to 0$ (perfect RAM fluid), this equation reduces to the dispersion relation obtained by Gedalin [7].

IV. DISCUSSION

Unmagnetized Plasma :

In the absence of magnetic field, the energy density $\varepsilon = \varepsilon(\rho)$ and isotropy prevails i.e., $W_3 = 0$; and to the first order of dissipative term [9], we obtain

$$\omega = \omega_0 + iLk^2, \tag{24}$$

where the decay characteristic length

$$L = \frac{\zeta}{2(\varepsilon + p)}, \tag{25}$$

and the frequency of the sound waves with the bulk shear being neglected

$$\omega_0 = k\left\{\frac{\frac{\partial W_2}{\partial ln\rho}}{\frac{\partial}{\partial ln\rho}(W_1 - W_2)}\right\}^{\frac{1}{2}}, = k\left(\frac{\partial p}{\partial \varepsilon}\right)^{\frac{1}{2}}. \tag{26}$$

The amplitude of the sonic waves decays at the rate $\Gamma = Lk^2$.

Magnetized Plasma :

(i) In the special case of propagation of magnetosonic modes in the direction of the magnetic field, the frequency is found to be

$$\omega = \omega_0 + iL_{\|}k^2, \tag{27}$$

with the decay characteristic length

$$L_{\|} = \frac{\zeta}{2(\varepsilon + p_{\|})}, \tag{28}$$

and the viscosity neglected frequency

$$\omega_0 = k\left\{\frac{\frac{\partial(W_2 - W_3)}{\partial ln\rho}}{\frac{\partial}{\partial ln\rho}(W_1 - W_2)}\right\}^{\frac{1}{2}}, = k\left\{\frac{\frac{\partial p_{\|}}{\partial ln\rho}}{(\varepsilon + p_{\|})}\right\}^{\frac{1}{2}}. \tag{29}$$

It is to be noted that $L_{\|}$ is always positive. This implies that the wave decays for all possible values of the system parameters.

(ii) For the magnetosonic waves propagating perpendicular to the ambient magnetic field, the frequency is

$$\omega = \omega_0 - iL_\perp k^2, \tag{30}$$

where the spatial scale of decay

$$L_\perp = \frac{\zeta(\frac{\partial \varepsilon}{\partial lnB} + \frac{B^2}{4\pi})}{2(\varepsilon + p_\perp + \frac{B^2}{4\pi})\frac{\partial \varepsilon}{\partial ln\rho}}, \tag{31}$$

and the frequency in the absence of dissipation

$$\omega_0 = k\left\{\frac{\frac{\partial(W_1 - W_2)}{\partial ln\rho}\frac{\partial W_2}{\partial lnB} + \frac{\partial W_2}{\partial ln\rho}[W_1 - \frac{\partial(W_1 - W_2)}{\partial lnB}]}{W_1 \frac{\partial(W_1 - W_2)}{\partial ln\rho}}\right\}^{\frac{1}{2}}, = k\left\{\frac{\frac{\partial p_\perp}{\partial lnB} + \frac{B^2}{4\pi} + \frac{\frac{\partial p_\perp}{\partial ln\rho}}{\frac{\partial \varepsilon}{\partial ln\rho}}(\varepsilon + p_\perp - \frac{\partial \varepsilon}{\partial lnB})}{\varepsilon + p_\perp + \frac{B^2}{4\pi}}\right\}^{\frac{1}{2}}. \tag{32}$$

In this case we observe that the magnetosonic modes can grow or decay accordingly as $L_\perp > 0$ or $L_\perp < 0$. In the latter case the system tends to return to its equilibrium state while in the former instability grows. A state characterized by $L_\perp = 0$ represents propagation of harmonics in the direction transverse to the magnetic field unlike the unmagnetized-isotropic state wherein the harmonics can propagate only upto its few characteristic lengths.

Shrauner [13] proposed a generalized polytrope model for the MHD set of equations for considering the collisional, collisionless and the transitional regimes of plasma state. According to Shrauner, the anisotropic pressures can be written as

$$p_{||} = C_{||}\rho^{\nu}B^{-\alpha}, \quad p_\perp = C_\perp \rho^{\delta}B^{\beta}, \tag{33}$$

where $C_{||}$ and $C_\perp$ are positive constants. The polytropic indices ν, α, δ and β are positive constants and these polytropic relations can be reduced in certain special cases to well known equations of state for the pressure. From equations (8), (9), (10) and (33) we get the generalized energy density

$$\varepsilon = a_1\rho + a_2 p_{||} + a_3 p_\perp, \tag{34}$$

where a_1, a_2 and a_3 are positive constants. The condition for decay of the transverse modes i.e., $L_\perp < 0$ translate into

$$a_2\alpha p_{||} > a_3\beta p_\perp + \frac{B^2}{4\pi}. \tag{35}$$

For $a_1 = 1, a_2 = 1/2$ and $a_3 = 1$ we recover the energy density for a plasma state characterized by nonrelativistic temperatures; $\varepsilon = \rho + \frac{1}{2}p_{||} + p_\perp$. Further, if $\nu = 3, \alpha = 2, \delta = 1$ and $\beta = 1$, then the system represents the well known CGL [14] state with $p_{||} = C_{||}\rho^3/B^2$ and $p_\perp = C_\perp \rho B$. It is evident that the condition for decay transverse modes in the CGL plasma reads as

$$p_{||} > p_\perp + \frac{B^2}{4\pi}. \tag{36}$$

This is the usual criterion for firehose instability in MHD.

We would like to point out that a simple gas of structureless point particles will have negligible bulk viscosity in the extreme-relativistic or nonrelativistic limits. However, this need not be the case for a fluid composed of a mixture of highly relativistic and nonrelativistic particles. It is well known that the exchange of energy between translational and rotational degrees of freedom gives ordinary diatomic gases an appreciable bulk viscosity. Although the present analysis assumes that coefficient of bulk viscosity ζ does not vary appreciably, it does depend on the dynamical variables of the system in the general form $\zeta = 4bT^4\tau[\frac{1}{3} - (\frac{\partial p}{\partial \varepsilon})_\rho]^2$. Here T is plasma temperature, b is of the order of Stefan-Boltzmann constant and τ is the free mean time of radiation quanta [9]. In view of this, the present work is an indicative analysis of plasma as an imperfect fluid and can be explored further.

References

[1] E. Asseo and D. Beaufils, Astrophys. Space Sci. **89**, 133(1983).

[2] S. Weinberg, *Gravitation and Cosmology* (John Wiley, New York, 1972).

[3] M.C. Begelman, R.D. Blandford, and M.D. Rees, Rev. Mod. Phys. **56**, 255 (1984)

[4] F. Hoffman and E. Teller, Phys.Rev. **80**, 692 (1950)

[5] A. Lichnerowitz, *Relativistic Hydrodynamics and Magnetohydrodynamics* (Benjamin, New York, 1967).

[6] M.Gedalin, Phys. Fluids B **3**(8) (1991).

[7] M.Gedalin, Phys. Rev. E **47**, 4354 (1993).

[8] E.G.Tsikarishvili and A.D.Rogava, Phys. Rev. E **50(4)**, 3050(1994).

[9] S.Weinberg, Ap.J. **168**, 175 (1971).

[10] Winfried Zimdahl, Phys. Rev. D **53**, 10 (1996).

[11] Roy Maartens and Vicenc Mendez, astro-ph/9611205 25 Nov 1996.

[12] Karsten Jedamzik, Visnja Katalinic and Angela Olinto, astro-ph/9606080 27 Aug 1997.

[13] Shrauner, B.A.,Plasma Phys. **15**, 375 (1973).

[14] G.Chew, M.Goldberger, and F.Low, Proc.R.Soc.London Ser.A **236**, 112 (1956).

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THERMAL VARIATION IN THE EXTERNED ONE-ZONE RR LYRAE MODEL. II NON LINEAR RESULTS

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We use the extended one-zone stellar pulsation model to investigate the factors affecting thermal variation of pulsating stars. Non-linear analyses of the resulting equations are described. The results are in very good agreement with the detailed calculations.

Key words: RR Lyrae – variable stars

1. Introduction

The extended one-zone model (Stellingwerf, 1972; Pricopi, 2005 hereafter Paper I) was used to investigate the linear effects on the thermal variation of different factors such as: interior luminosity variation, radiation, thickness of the shell, rotation and effective temperature of equilibrium model. In this paper, we use this model to investigate the nonlinear effects of factors above on the adiabaticity, temperature, effective temperature and opacity variation of RR Lyrae-like pulsating stars.

"Bump mechanism" (see Stellingwerf, 1978,1979) is responsible for the pulsations of our extended one-zone model (and, as expected, we found that the opacity are increasing upon compression). However, this model is not intended to be a substitute for finely zoned nonlinear calculations. The results are qualitative and we are content to seek out and explain only the simplest features in terms of basic physical processes. In Section 2 we resort the equations of our one-zone model (see Paper I). Section 3 deals with the physical input of the model. In Section 4 nonlinear results are presented. The conclusions of this paper are summarized in Section 5.

2. Basic equations

We resort the equations of our extended one-zone model (see Paper I):

1. The motion equation:

$$\frac{d^2X}{dt^2} = \xi(hX^{-q} - X^{-2}) - \zeta(hX^{-q} - X). \qquad (1)$$

2. The energy equation:

$$\frac{dh}{dt} = -\varepsilon\frac{\delta}{\alpha}X^{m(\Gamma_3-1)}h^{1-\frac{\alpha}{\delta}}\times \times\left(X^{4+m[n-(4+s)(\Gamma_3-1)]}h^{(4+s)\frac{\alpha}{\delta}} - X^{-u}\right) - -3\frac{\delta}{\alpha}(\Gamma_3-1)X^2h\left(X^3-\eta^3\right)^{-1}\times \times\left(X^{m[\Gamma_3-\Gamma_1]}h^{1-\frac{\alpha}{\delta}} - 1\right)\frac{dX}{dt}. \qquad (2)$$

These equations forms a non-linear third-order set of ordinary differential equations with time as independent variable. After integration, we can obtain: the temperature variation $T/T_0 = X^{-m(\Gamma_3-1)}h^{\frac{\alpha}{\delta}}$; the opacity variation $\kappa/\kappa_0 = X^{-m[n-s(\Gamma_3-1)]}h^{-s\frac{\alpha}{\delta}}$ and, using the relation $L = 4\pi R^2\sigma T_{eff}^4$, we obtain the effective temperature variation

$$T_{eff}/T_{eff0} = X^{\frac{1}{2}+\frac{m}{4}[n-(4+s)(\Gamma_3-1)]}h^{\frac{1}{4}(4+s)\frac{\alpha}{\delta}}.$$

The variation of the luminosity at the base of the shell is supposed to be $L_i/L_0 = X^{-u}$ where u is a parameter that ranges from 0 to 20 (Stellingwerf and Donohoe, 1987).

From pressure variation $P/P_0 = (\rho/\rho_0)^{\Gamma_1}h$ (Stellingwerf, 1972) and temperature variation in the form $T/T_0 = (\rho/\rho_0)^{-1/\delta}(P/P_0)^{\alpha/\delta}$ (see Paper I), we obtain the gradients

$$\frac{d\ln P}{d\ln\rho} = \Gamma_1 + \frac{d\ln h}{d\ln\rho}; \qquad (3)$$
$$\frac{d\ln T}{d\ln\rho} = (\Gamma_3-1) + \frac{\alpha}{\delta}\frac{d\ln h}{d\ln\rho}$$

and

$$\nabla = \frac{d\ln T}{d\ln P} = \nabla_{ad} + \frac{\left(\frac{\alpha}{\delta}-\nabla_{ad}\right)\frac{d\ln h}{d\ln\rho}}{\Gamma_1 + \frac{d\ln h}{d\ln\rho}}, \qquad (4)$$

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where we used the thermodynamic relation $(\alpha\Gamma_1 - 1)/\delta = \Gamma_3 - 1$, $\nabla_{ad} = (\Gamma_2 - 1)/\Gamma_2 = (\Gamma_3 - 1)/\Gamma_1$. It is easy to show that $\alpha/\delta > \nabla_{ad}$. Using $\rho/\rho_0 = (1-\eta^3)/(X^3-\eta^3)$, $\eta = R_c/R_0$ (R_c=core radius), we obtain

$$\frac{\partial \ln h}{\partial \ln \rho} = -\frac{1}{h}\frac{X^3-\eta^3}{3X^2}\frac{dh/dt}{dX/dt}. \tag{5}$$

From (4) and (5) we may say the follows:

I. If $dX/dt > 0$ (expansion) then: 1) if $dh/dt > 0$ (h is increasing) then $d\ln h/d\ln\rho < 0$ and, from (25), $\nabla < \nabla_{ad}$ i.e. the expansion is sub-adiabaticaly; 2) if $dh/dt < 0$ (h is decreasing) then $d\ln h/d\ln\rho > 0$ and $\nabla > \nabla_{ad}$ i.e. the expansion is supra-adiabaticaly;

II. If $dX/dt < 0$ (compression) then: 1) if $dh/dt > 0$ (h is increasing) then $d\ln h/d\ln\rho > 0$ and $\nabla > \nabla_{ad}$ i.e. the expansion is supra-adiabaticaly; 2) if $dh/dt < 0$ (h is decreasing) then $d\ln h/d\ln\rho < 0$ and $\nabla < \nabla_{ad}$ i.e. the expansion is sub-adiabaticaly;

III. If $dh/dt = 0$ and $dX/dt \neq 0$ the shell is moving adiabaticaly: $\nabla = \nabla_{ad}$;

IV. If $dX/dt = 0$ (maximum compression/expansion) and $dh/dt \neq 0$ (as for the series of models which are investigated below), we have $\nabla = \alpha/\delta$. From (2) it is easily to show that a necessary contition for $dh/dt = 0$ (in the case of $dX/dt = 0$) is $h < 1$ at maximum extension and $h > 1$ at maximum compression.

3. Physical input

For our model we take $M = 0.5M_\odot$, $R_0 = 3.41 \times 10^{11} cm$ (tipical values for RR Lyrae-like stars), $n = 1$, $\varepsilon = 10^{-4}$. The shell thickness is chosen to comprises the outer 10 - 15% of the stellar radius. Also, s is determined by the periodicity condition ("bump mechanism"). Angular velocity $\omega = (2\pi/2.6)\times 10^{-6}$ (period of the rotation $\approx$ 30 days) and oblateness $\lambda = 5\times 10^{-7}$ (such that the radial pulsations remains an adequate working hypothesis). The pressure $P = P_{gas} + P_{rad}$. Following Kippenhahn and Weigert (1991), let $\beta = P_{gas}/P$. It follows $\alpha = 1/\beta$, $\delta = (4-3\beta)/\beta$, $\nabla_{ad} = [1 + (1-\beta)(4+\beta)/\beta^2]/[5/2 + 4(1-\beta)(4+\beta)/\beta^2]$ and $\Gamma_1 = 1/(\alpha - \delta\nabla_{ad})$. If $\beta = 1$ (pure gas) we have $\Gamma_1 = 5/3$ and if $\beta = 0$ (pure radiation) we have $\Gamma_1 = 4/3$.

4. Nonlinear results

To facilitate nonlinear calculations, we use the time normalization $t' = t/10^4$ and integrate eqs. (1) and (2) (using Mathcad soft) for $t' \in [0, 150]$. The integrations were started at $X = 1$, $h = 1$ and $dX/dt' = 10^{-5}$ (leading to a maximal radial velocity of few tens m/s for this model). We investigate the influence of interior luminosity variation, radiation, thickness of the shell, rotation and luminosity per internal energy of the shell ratio on the h function, temperature variation, effective temperature variation and opacity variation. We mention that, from Figures 1-9, it would be obtained more information about the themal variation of our model than we pointed out.

We begin by examining the effect of interior luminosity variation taking $u = 3$, 5 and 10. In addition, we take $\beta = 1$, $\eta = 0.87$, $\varepsilon = 10^{-4}$ and $\omega = (2\pi/2.6)\times 10^{-6}$. The resulting curves are shown in Figures 1-4.

From the discussion of ∇ and with help of Figure 1, it is easy to find when the shell is moving adiabaticaly ($\nabla = \nabla_{ad}$), super- ($\nabla > \nabla_{ad}$) and sub-adiabaticaly ($\nabla < \nabla_{ad}$). So far, we may say that, for this models, the shell is expanding/compressing adiabaticaly close to maximal velocity (as expected) and close to maximal expansion, short time after it start to compress. Also, from Figure 1 we may say that an increase of interior luminosity variation results in an increase of non-adiabaticity. From Figures 2-4 show us that the temperature, effective temperature and the opacity are increasing upon compression. This *increase* in κ upon compression (normally, κ *decreases* upon compression) enhances the dummng up of the radiation and contributes to the driving in this region. Also, we observe that, approximatively, $\kappa_{\max}$ is at maximum compression and $\kappa_{\min}$ is at maximum expansion of the shell. The increase of u results in a increase of amplitude of T_{eff}-curve. The amplitude of κ -curve is decreasing (as expected) and the T-curve remains almost unchanged for all three values of u.

To investigate the effect of radiation, we take $\beta = 0.99995$, $\beta = 0.9999$ and $\beta = 0.99985$. In addition, we take $u = 10$, $\eta = 0.87$, $\varepsilon = 10^{-4}$ and $\omega = (2\pi/2.6)\times 10^{-6}$. The resulting h-curve and κ-curve are shown in Figures 5,6.

From Figure 5 and taking into account (2) we can dicuss the adiabaticity as before. But, it is interesting to observe that the presence of radiation make the shell to move adiabaticaly not close to maximal velocity, but close to maximal compression/expansion.While radiation pressure is increasing, the variation of κ is decreasing (more radiation in the shell means smaller κ) and $\kappa_{\max}$ lags behind $X_{\min}$.

To investigate the effect of thickness of the shell, we take $\eta = 0.7$, $\eta = 0.8$ and $\eta = 0.9$. In addition, we take $u = 4$, $\beta = 1$, $\varepsilon = 10^{-4}$ and $\omega = (2\pi/2.6)\times 10^{-6}$. The resulting κ-curve is shown in Figure 7.

While the shell thickness is decreasing, the variation of κ is increasing (κ must increase suficiently upon compression to compensate the thiness of the shell and assure periodic pulsations), the variation of T and h is increasing, but, the variation of T_{eff} is decreasing.

Now, let us investigate the effect of rotation. We take $\omega = 0.1\times(2\pi/2.6)\times 10^{-6}$, $\omega = 5\times(2\pi/2.6)\times 10^{-6}$ and $\omega = 10\times(2\pi/2.6)\times 10^{-6}$. In addition, we take $u = 4$, $\beta = 1$, $\varepsilon = 10^{-4}$ and $\eta = 0.87$. The resulting

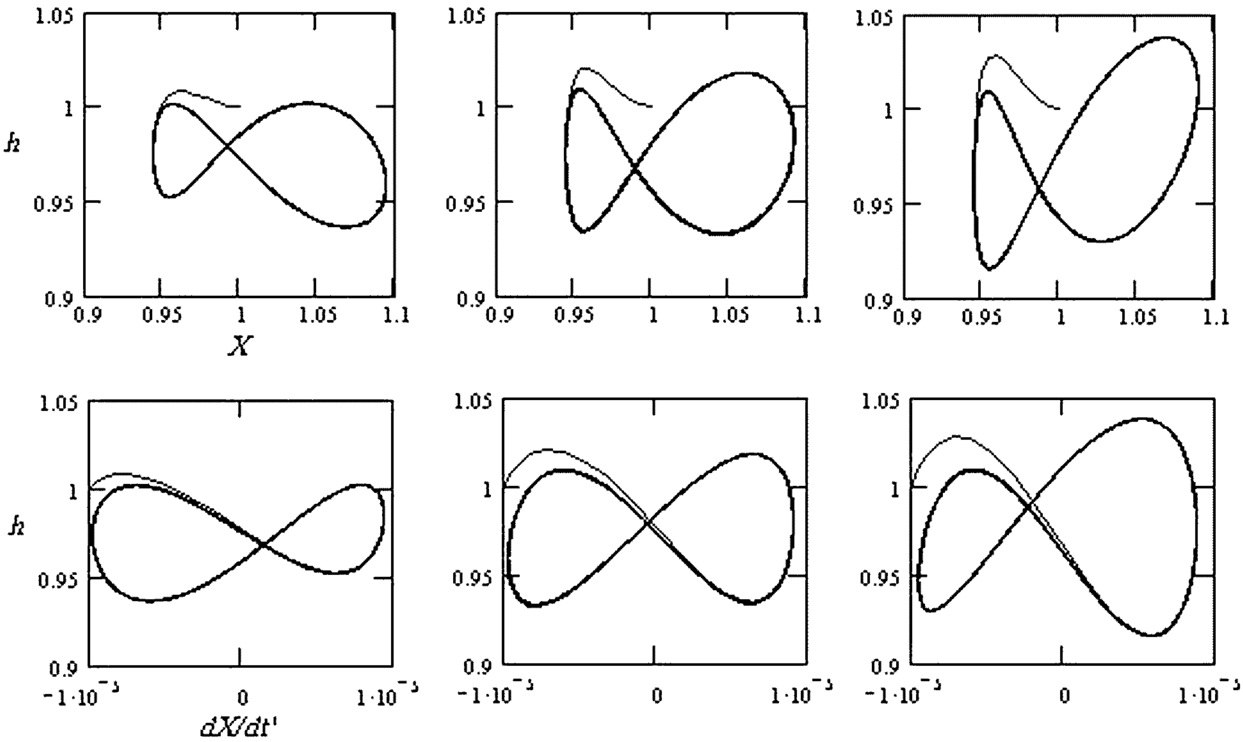

Figure 1: The influence of interior luminosity variation on the h function

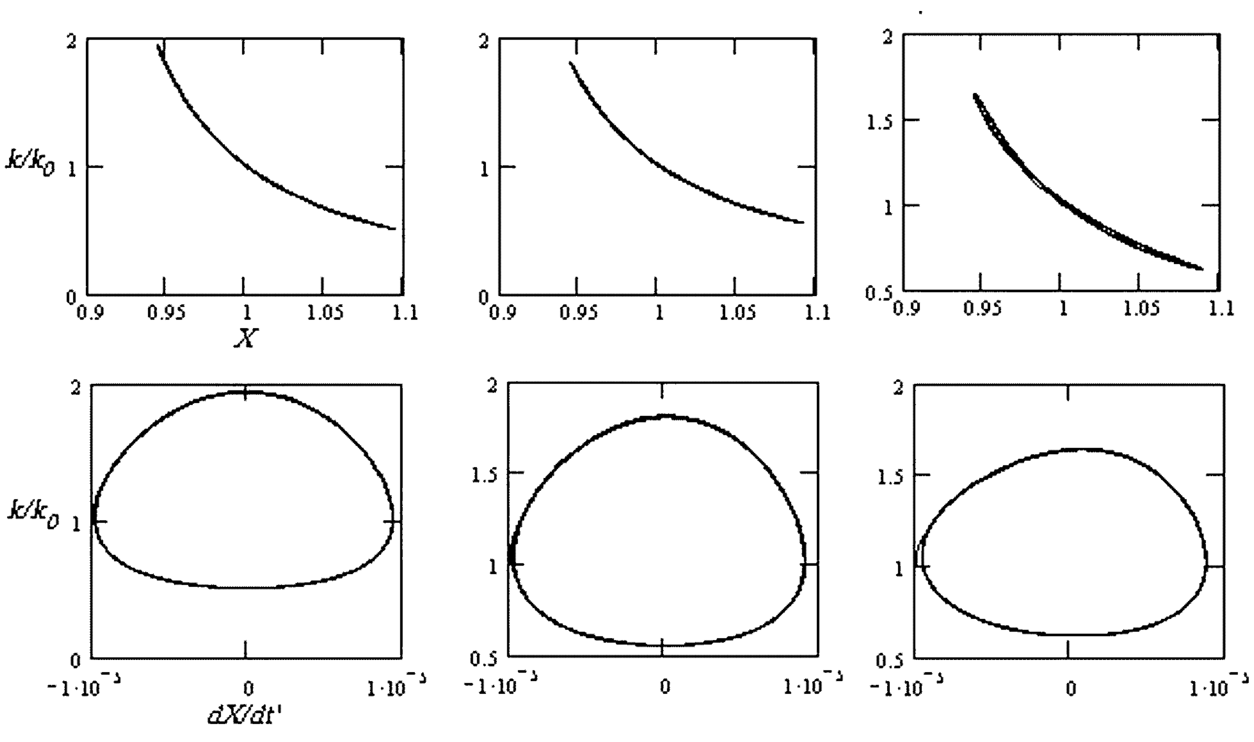

Figure 2: The influence of interior luminosity variation on the temperature variation

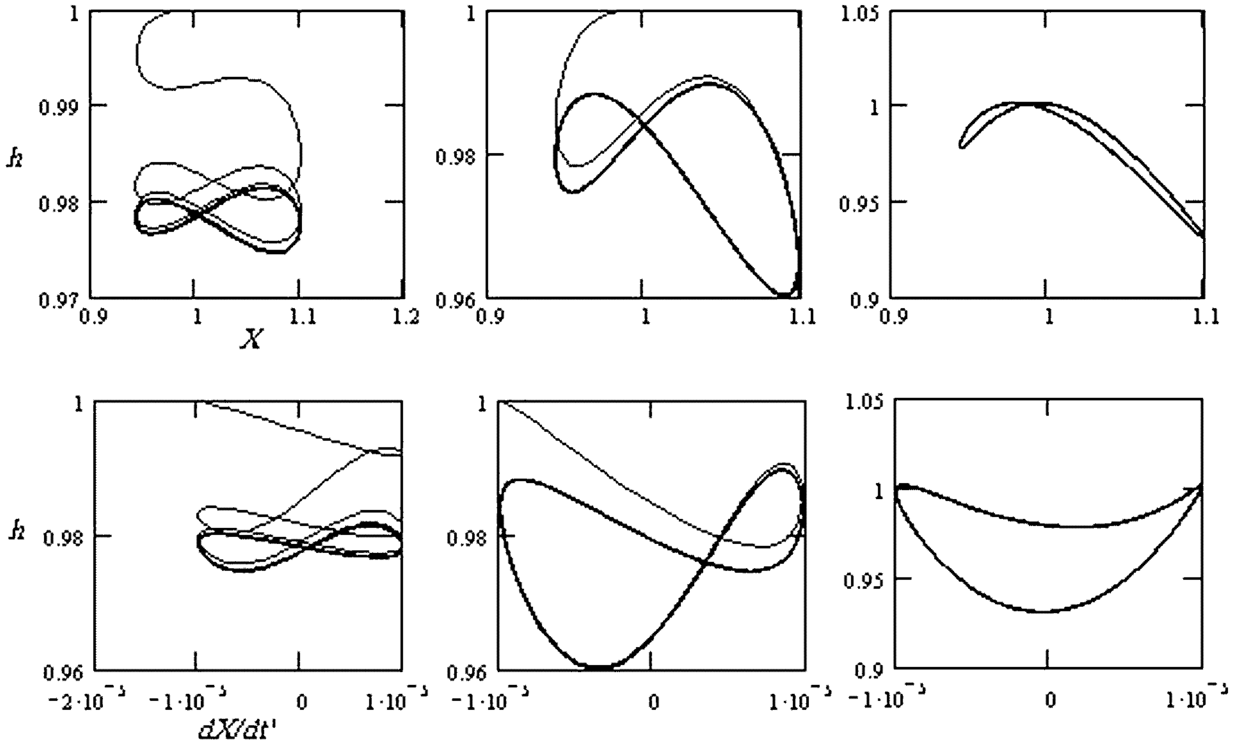

Figure 3: The influence of interior luminosity variation on the effective temperature variation

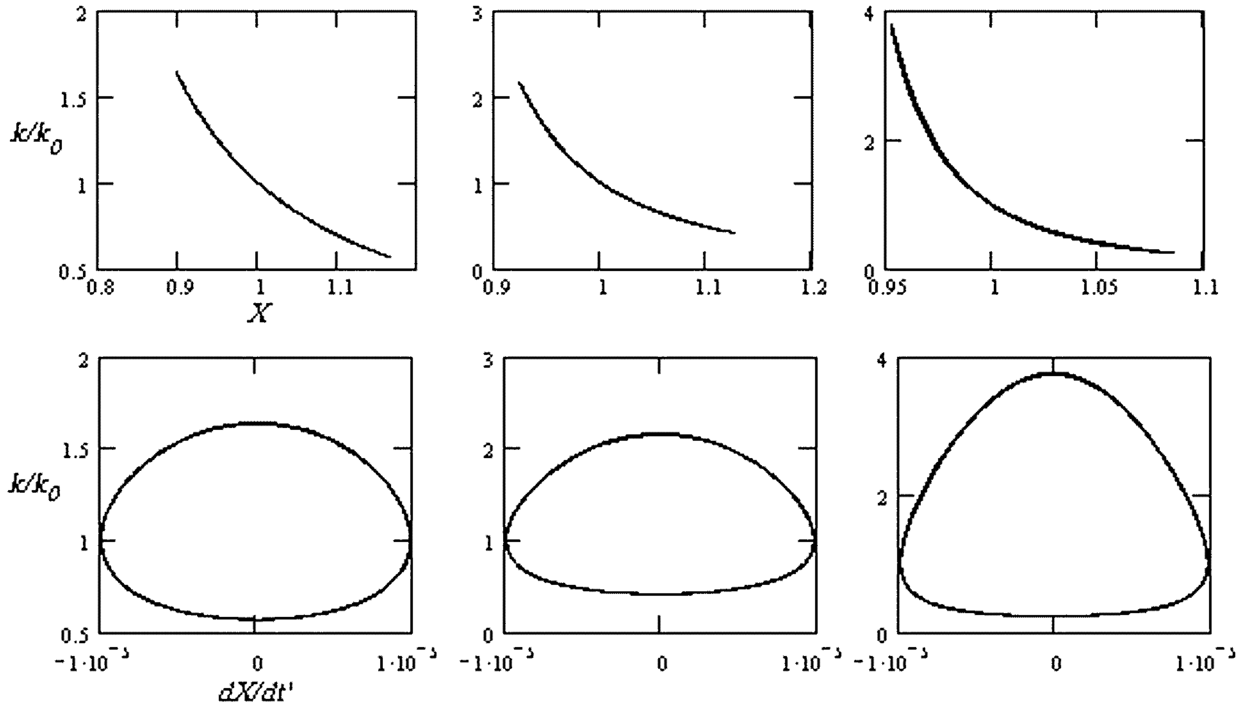

Figure 4: The influence of interior luminosity variation on the opacity variation

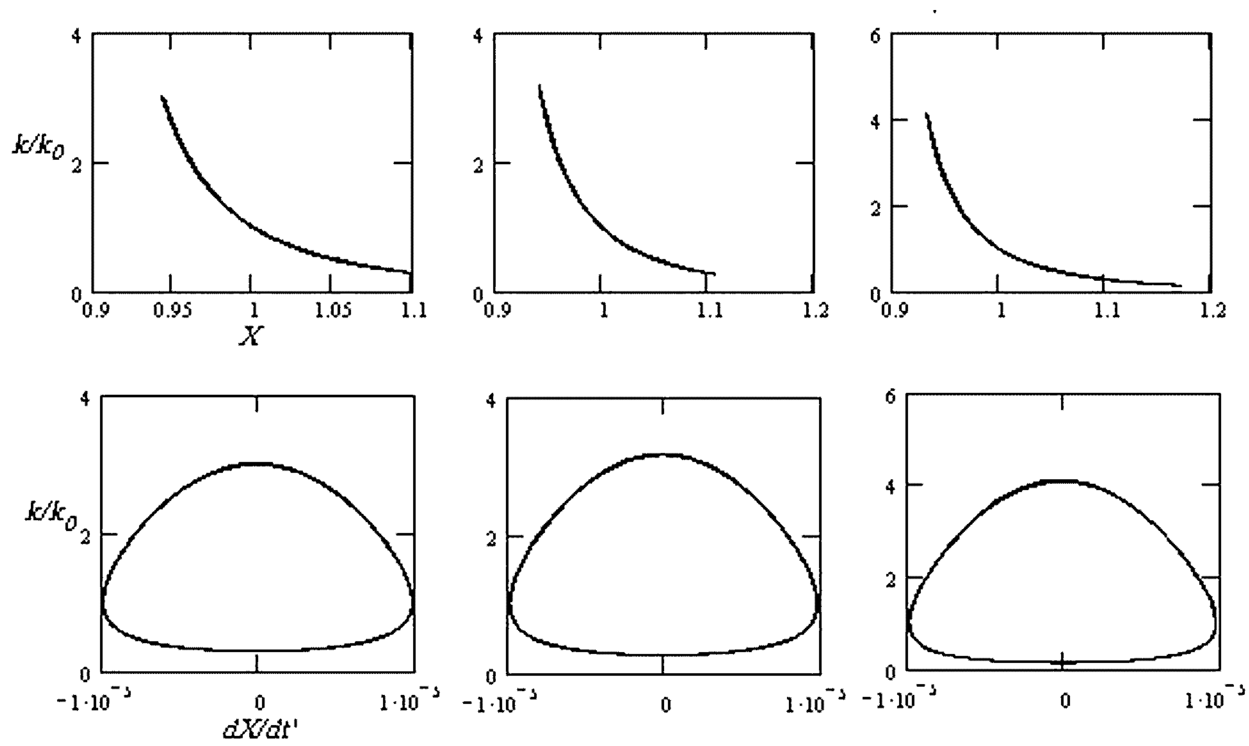

Figure 5: The influence of radiation on the h function

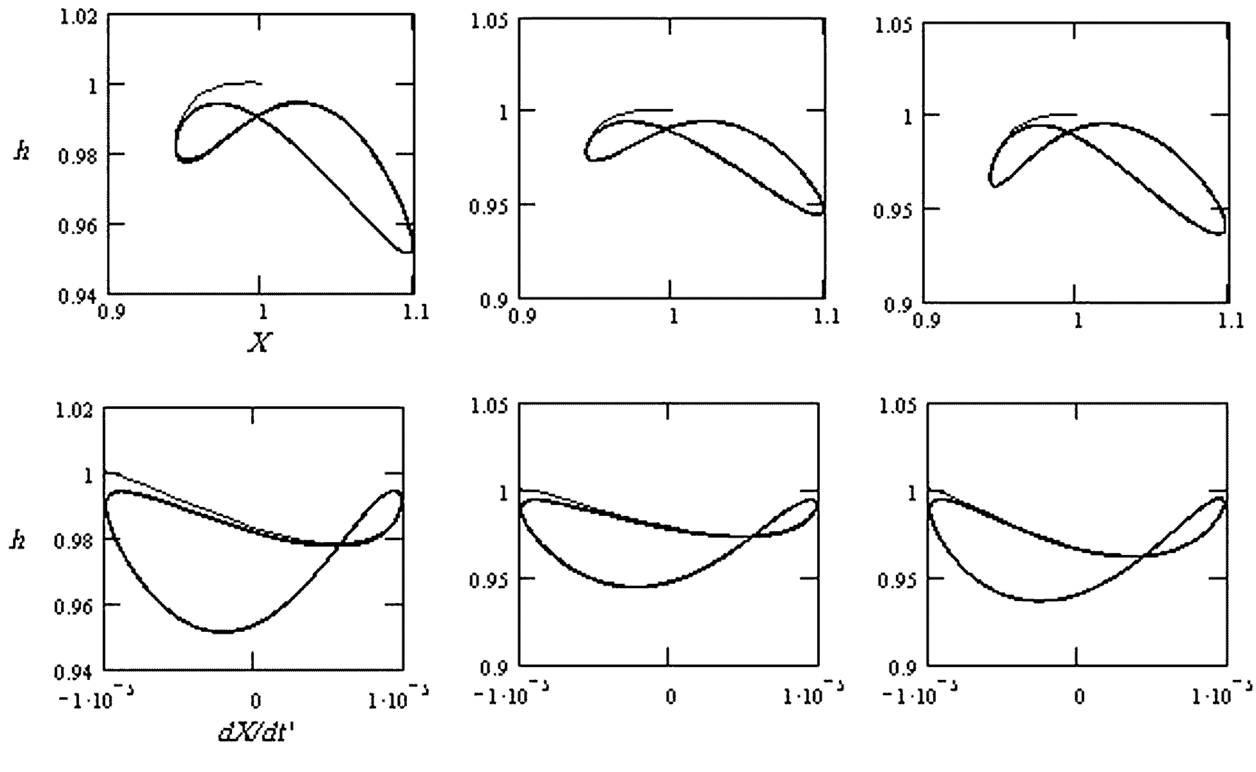

Figure 6: The influence of radiation on the opacity variation

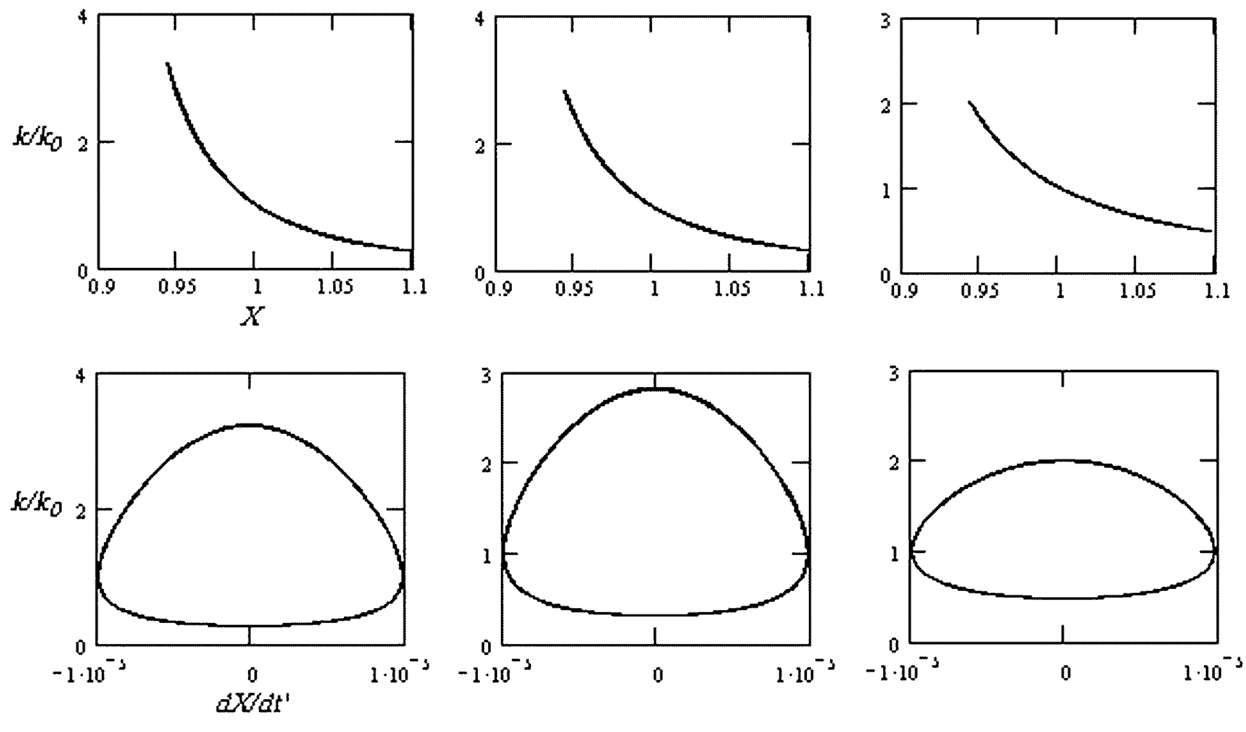

Figure 7: The influence of thickness of the shell on the opacity variation

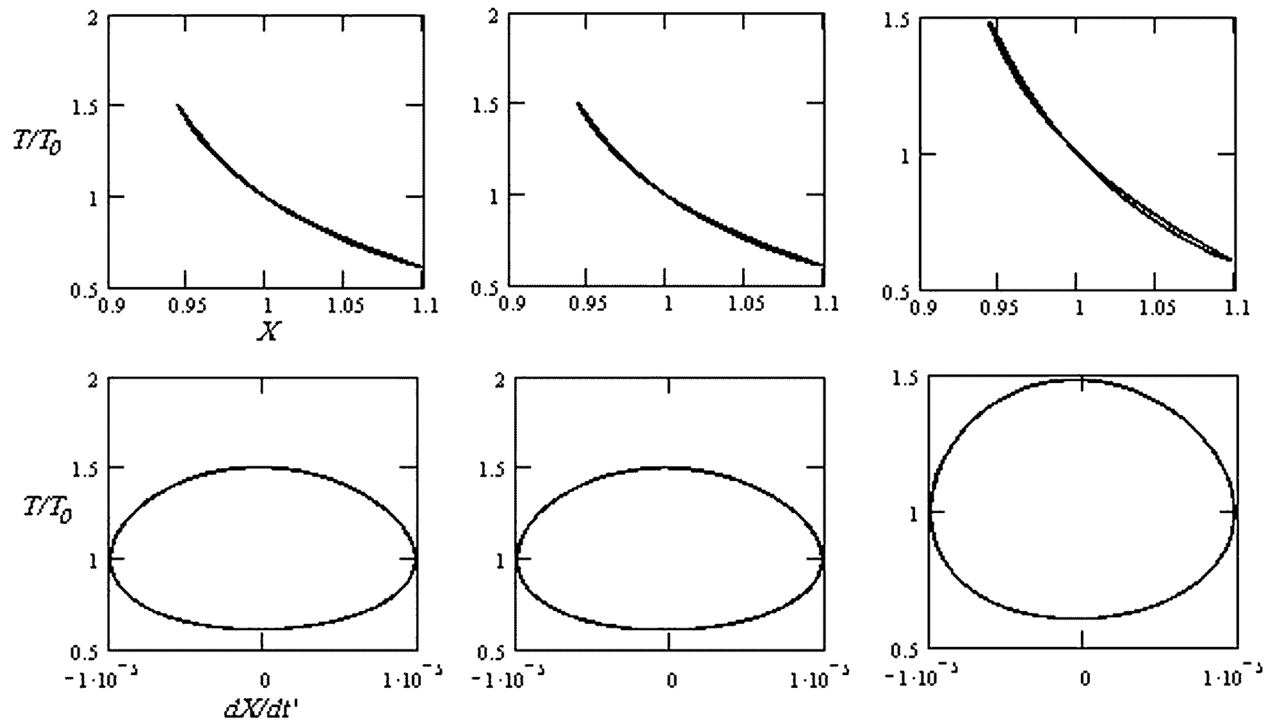

Figure 8: The influence of rotation on the opacity variation

κ-curve are shown in Figure 8. The rotation have a small influence on the thermal

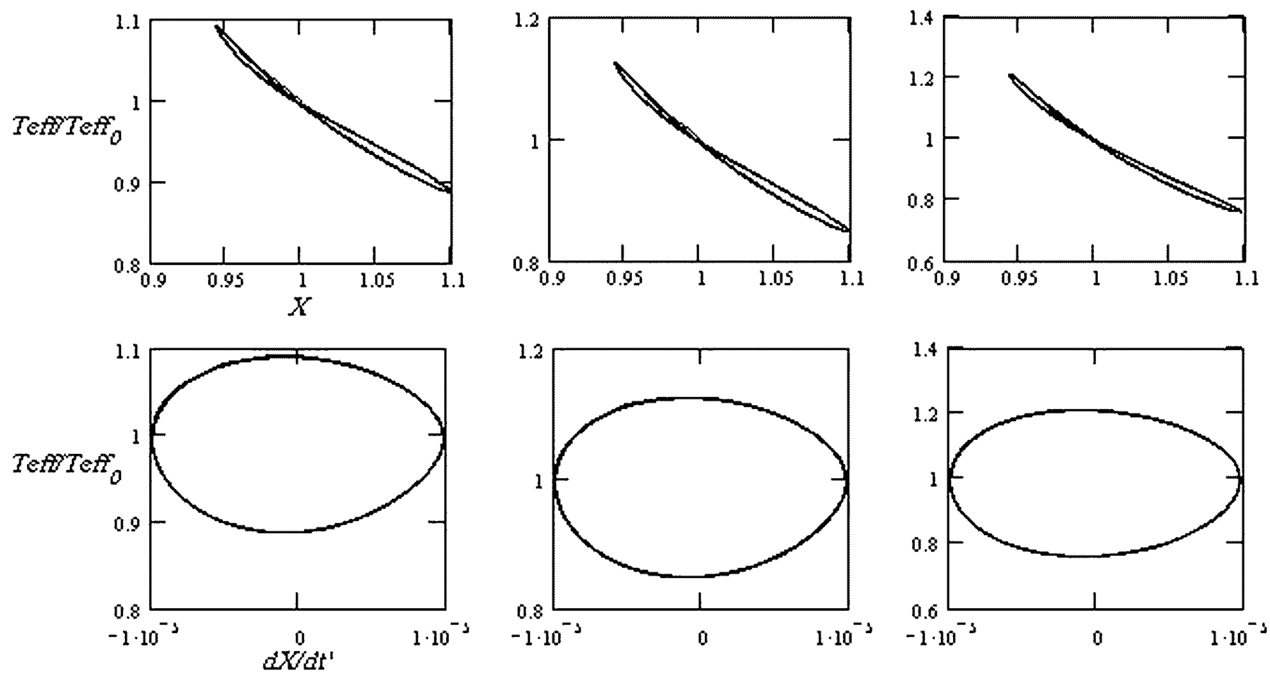

Figure 9: The influence of ε on the h function

variation of the pulsating star. The most important ones are those on the h function and opacity. This may be understood having in mind the stabilizing effect of the rotation (the rotation tends to stabilize the star).

To investigate the influence of ε, we take $\varepsilon = 0.1 \times 10^{-4}$, $\varepsilon = 0.5 \times 10^{-4}$ and $\varepsilon = 5 \times 10^{-4}$. In addition, we take $u = 4$, $\beta = 1$, $\eta = 0.87$ and $\omega = (2\pi/2.6) \times 10^{-6}$. The resulting h-curve are shown in Figure 9.

We see that the most important influence of ε is on the variation of h function. This become clear if we observe that we can take ε as a measure of effective temperature of the equilibrium model. As the ε is growing, the shell tends to move adiabaticaly at maximum expansion.

5. Conclusions

Here we summarize the main results.

1. While the star is pulsating, the temperature, the effective temperature and the opacity are increasing upon compression.
2. The increase of the interior luminosity variation leads to a increase of h and T_{eff} variation and a decrease of κ variation.
3. The most powerfull influence on the thermal variation is that of the radiation (as expected).
4. If the shell thickness is increasing, the amplitude of h is decreasing (the local deviation from adiabaticity become smaller).
5. The stabilizing effect of rotation is reflected by the increase of κ variation (the growth of κ upon compression become more prononced), while h, T and T_{eff} are weakly influenced by rotation.
6. It seems that the effective temperature of equilibrium model has the greatest influence on the ∇ variation.

More interesting and useful information regarding thermal variation of a pulsating star and the way in which it is influenced by different factors, can be obtained from Figures 1-9. Finally, we conclude that this simple model can be used succesfully to shed much light upon thermal variation of the RR Lyrae type pulsating stars.

References

[1] Baker, N., 1966, *Stellar Evolution*, Eds. R. F. Stein and A. W. G. Cameron, Plenum Press, New York, p. 333

[2] Castor,J.I., 1968, *Ap.J.* **154**, 793

[3] Castor,J.I., 1971, *Ap.J.* **166**, 109

[4] Cox J.P., Giuli, 1968, *Principles of Stellar Structure*, Gordon and Breach, New York

[5] Cox, J.P., King, D.S., Stellingwerf, D.F., 1972, *Ap.J.* **171**, 93

[6] Goldreich and Weber, 1980, *Ap.J.* **238**, 991

[7] Iben,I.,Jr., 1971, *Ap.J.* **166**, 131

[8] Kippenhahn R.,Weigert A., 1991, *Stellar Structure and Evolution,* Springer-Verlag, Berlin

[9] Ledoux,P., Walraven,Th., 1958, *Hbd. Phys.* **50**, 353

[10] Lungu,N., 1982, *Stellar Pulsations*, Scientific and Encyclopedic Publishing House, Bucharest (in Romanian)

[11] Okamoto, I., Unno,W., 1967, *Publ. Astr.Soc. Japan*, **19**, 154

[12] Rudd, T.J., Rosenberg, R.M., 1970, *Astron&Astrophys.* **6**, 193

[13] Stellingwerf, R.F., 1972, *Astron&Astrphys.***21**, 91

[14] Stellingwerf, R.F., Gautshy, A., Dickens, R,J., 1987, *Ap.J.* **313**, 75

[15] Stellingwerf, R.F., Donohoe, M., 1987, *Ap.J.* **314**, 252

[16] Sweet, P.A., 1950, *M.N.R.A.S.* **110**, 548

[17] Usher, P.D., Whitney, C.A., 1968, *Ap.J.* **154**, 203

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EXPLANATION AND MORE PRECISE DEFINITION OF MOTION OF MERCURY'S PERIHELION WITHOUT USE OF GENERAL RELATIVITY

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It is implemented a Calculation of the Planets' Perihelion on the Ground of the Gravitation's Theory, that takes into account a Relative Speed of Gravitating Bodies. It is take into Consideration an Influence of Solar Photons and their Screening by an Planets' Atmosphere. It is showed that a Constant of Gravitation for Sun is a Variable Quantity and at Present in Vacuum it is equal to $5.026 \cdot 10^{-8} sm^3/g \cdot sec^2$.

KEY WORDS: Mercury, perihelion, photon, constant of gravitation, relativity

"Numbers decide all."

Max Planck

1. Introduction

A History of the Question is expounded in the Issue [1]. According to Newton a trajectory of the Planet is Ellipse

$$px = 1 + e\,cos\varphi. \tag{1}$$

Here $x = 1/r$, r - is a Distance of the Planet to the Sun. The rest Conventional Signs are determined in the Table. But according to astronomical Data there is century Parallax of a Perihelion of Planets $\Delta\varphi(n)_{exp}$ (see the Table). A Problem was springing up. Einstein [2] gave one of the Version of its Decision. He received a Formula for an Angle Parallax in Time of one Rotation of the Perihelion

$$\Delta\varphi_E(1) = 3\bar{\alpha}\pi, \tag{2}$$

$$\bar{\alpha} = \alpha/p, \qquad \alpha = 2GM_{\odot}/c^2 = 2.948\,km. \tag{3}$$

According to the Table Einstein's Prophecy is contrary to Facts with the Exception of the Mercury. Again a Problem arose. A Purpose of the present Work is a Decision of this Problem. But before to offer one Model of the Gravitation Author elucidates a Sense of Newton's and Einstein's Models.

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2. Newton's Model

A Gravitational Charge is analogous to an electrical Charge. Only it radiates Gravitons instead of Photons. This Analogy allows to use the Formula (14) of [3] for a Model of the Gravitational Field

$$-i\,\frac{\partial\rho}{\partial\tau} = a^2\Delta\rho - \chi\rho. \tag{4}$$

Here ρ - a Volumetric Density of the Number of Gravitons, τ - a Newton's Time [4], a^2 - a Constant of the Diffusion, χ - a Middle Frequency of the Absorption of Gravitons in a Medium. Let a Sphere with a Radius R is a limit of the Source with a mass M. If $\chi = 0$ and $\langle\partial\rho/\partial\tau\rangle = 0$ the Equation (4) assumes an air for an Empty Space $R < r < \infty$

$$\Delta\rho(\vec{r}) = 0, \qquad \rho(r) = C_0 - C_1/r. \tag{5}$$

With the regard for a limit Conditions $\rho(R) = \rho_R$; $\rho(\infty) = 0$

$$\rho(r) = \rho_R R/r. \tag{6}$$

Since $\rho_R \sim M$ then $\rho(r) \sim M/r$. Let a Center of Masses of the Target m is found at the Distance r from a Center of Masses of the Source and let N Gravitons interact with a Volume of the Target V_m. Since $V_m \sim m$ then $N \sim Mm/r$. If an Energy of own

Graviton is equal to $\hbar\omega$ then an Energy of the Stream of Gravitons is equal to $E_g \sim \hbar\omega Mm/r$ or

$$U_N = GMm/r. \tag{7}$$

Thus a potential Energy U_N is none other than a Kinetic Energy of Gravitons K_g which interact with a Target. Knowing U_N it is possible to solve a Problem about a Trajectory of the Material Point m under an Influence of the Central Forces. A Well-known Decision of this problem [5, §7] is given below.

$$\frac{m}{2}(v_r^2 + v_\varphi^2) = E - U, \tag{8}$$

$$mrv_\varphi = C, \tag{9}$$

$$U = U_N = -A/r, \qquad A = GMm, \tag{10}$$

$$v_r = \frac{\partial r}{\partial \tau}, \qquad v_\varphi = r\,\frac{\partial \varphi}{\partial \tau}, \tag{11}$$

$$d\tau = \frac{mr^2}{C}d\varphi, \tag{12}$$

$$d\varphi = dr/(\ r^2\sqrt{\frac{2mE}{C^2} + \frac{2mA}{C^2 r} - \frac{1}{r^2}}\ ),$$

$$d\varphi = \frac{-dx}{\sqrt{(\frac{e}{p})^2 - (x - \frac{1}{p})^2}}, \tag{13}$$

$$(\frac{e}{p})^2 = \frac{2mE}{C^2} + \frac{m^2A^2}{C^4}, \qquad \frac{1}{p} = \frac{mA}{C^2}, \tag{14}$$

$$\varphi(x) = \arcsin\frac{p}{e}(x - \frac{1}{p}) + \frac{\pi}{2}. \tag{15}$$

Parameters of the Ellipse p and e are calculated on the Basis of the Astronomical Data for the Apogee and the Perihelion of the Planet

$$x_{min} = \frac{1-e}{p}, \qquad x_{max} = \frac{1+e}{p}. \tag{16}$$

Further Constants of Integration A, C will be calculated by means of Formulas (14) always if Values e and $p = a(1 - e^2)$ are known.

3. Einstein's Model

If only a gravitational Field U acts on the material Point then it is possible a Motion of the Perihelion only when U differs from U_N. For Example

$$U = U_N(1 + \delta). \tag{17}$$

A Problem consists of the Finding of the Amendment δ. According to (11), (12), (14)

$$\beta_r = \frac{v_r}{c} = (\frac{C}{mc})\frac{dx}{d\varphi}, \qquad \beta_\varphi = \frac{v_\varphi}{c} = (\frac{C}{mc})x,$$

$$(\frac{C}{mc})^2 = \frac{\alpha p}{2}, \ \frac{2mE}{C^2} = \frac{e^2-1}{p^2}, \ \frac{2U_N}{mc^2} = -\alpha x. \tag{18}$$

Therefore

$$\beta_r^2 = \frac{\alpha p}{2}(\frac{dx}{d\varphi})^2, \qquad \beta_\varphi^2 = \frac{\alpha p}{2}x^2, \tag{19}$$

$$\beta_r^2 = \frac{2E}{mc^2} - \beta_\varphi^2 + \alpha x(1 + \delta)$$

or

$$(\frac{dx}{d\varphi})^2 = \frac{e^2-1}{p^2} + \frac{2}{p}\,x - x^2 + \alpha x(\frac{mc^2}{C})^2\delta.$$

Einstein derived an analogous Equation [2]

$$(\frac{dx}{d\varphi})^2 = \frac{e^2-1}{p^2} + \frac{2}{p}x - x^2 + \alpha x^3. \tag{20}$$

Therefore Einstein's Amendment has a Shape

$$\delta_E = (\frac{C}{mc})^2 x^2 = \beta_\varphi. \tag{21}$$

and Einstein's potential Energy is equal to

$$U_E = U_N(1 + \beta_\varphi^2). \tag{22}$$

Taking into account (22) a Deduction of the Formula (2) is trivial. Let

$$z = px, \qquad z_1 = 1 - e, \qquad z_2 = 1 + e. \tag{23}$$

Then

$$(\frac{dz}{d\varphi})^2 = f(z), \qquad \varphi = \int_{z_1}^{z_2}\frac{dz}{\sqrt{f(z)}}, \tag{24}$$

So

$$f(z) = R_N + \bar{\alpha}z^3, \qquad R_N = a + bz + cz^2, \tag{25}$$

$$a = e^2 - 1, \qquad b = 2 \qquad c = -1, \tag{26}$$

$$\frac{1}{\sqrt{f(z)}} \approx \frac{1}{\sqrt{R_N}} - \frac{\bar{\alpha}z^3}{2\sqrt{R_N^3}}, \tag{27}$$

$$\int_{z_1}^{z_2}\frac{dz}{\sqrt{R_N}} = \pi \qquad \int_{z_1}^{z_2}\frac{z^3}{\sqrt{R_N^3}} = -3\pi. \tag{28}$$

$$\Delta\varphi_E(1) = 2(-\frac{\bar{\alpha}}{2}(-3\pi) = 3\bar{\alpha}\pi.$$

A Coincidence $\Delta\varphi_E(n)$ with an Experiment is contented for Mercury only. But if a theory has a Coincidence only in one Point of the Massive of the Experimental Data then it isn't evidence of the Good of the theory. No Theory may be decided on the Ground of its Coincidence in the only Point of the Massive of the Experimental Data. A Very unpleasant Moment in Einstein's Prophecies lies in that Fact that Experimental data for Earth and Mercury exceed Einstein's Data. Again a problem was springing up.

4. Author's Model

In 2001 Author [6] published without a Deduction a following Generalization of the Formula (22)

$$U_T = U_N(1 + \beta_\varphi^2 + 2s\beta_r + \beta_r^2), \qquad s = \pm 1. \tag{29}$$

Further it is given a Deduction of this Formula. Let a material Point m revolves on a fixed Center. For the present Case formula (29) is transformed in Formula (22). If we will turn into a revolving System with a circular Speed v_φ then Formula (22) is transformed in Formula (7). This means that in Fact Newton's Formula (7) is put down for a revolving System with a Center in a Source! Here the Source and a Target are fixed between itself and Gravitons are a Speed $\vec{c_r}$. Then Impulses of these Particles are equal to

$$\vec{P_m} = 0, \qquad \vec{P_g} = m_g\vec{c_r}. \tag{30}$$

Gravitons and a Target are found in one common immovable Point $A(\vec{r})$ at a Moment of the Interaction. But in an initial System a material point $A(\vec{r})$ moves with a Speed $\vec{v}(t)$. Therefore in this System Impulses of the Particles are equal to

$$\vec{P_m} = -m\vec{v}, \qquad \vec{P_g} = m_g(\vec{c_r} - \vec{v}). \tag{31}$$

A Kinetic Energy of the graviton is equal to

$$K_g = \frac{m_g}{2}(\vec{c_r} - \vec{v})^2. \tag{32}$$

Since $\vec{v} = \vec{v_r} + \vec{v_\varphi}$ then a scalar Product of the Vectors $\vec{v_g}$ is equal to

$$v_g^2 = (\vec{c_r} - \vec{v})(\vec{c_r} - \vec{v}) =$$

$$(\vec{c_r} - \vec{v_r})(\vec{c_r} - \vec{v_r}) + v_\varphi^2 =$$

$$\vec{c_r} - 2\, c_r v_r \cos(\vec{c_r}, \vec{v_r}) + v^2,$$

$$K_g^0 = \frac{m_g}{2}\, c_r^2,$$

$$K_g/K_g^0 = 1 - 2\beta_r\ \cos(\vec{c_r}, \vec{v_r}) + \beta^2. \tag{33}$$

Since $U_N = K_g^0$ then by $c_r = c$

$$U_T = U_N(1 + 2s\beta_r + \beta^2), \qquad s = \cos\,(\vec{c_r}, \vec{v_r}). \tag{34}$$

Now it is necessary to calculate a Trajectory of the Target m by Chance an Energy of the Gravitational Interaction (29). Our Amendment in Formula (17) is equal to

$$\delta_T = 2s\beta_r + \beta^2. \tag{35}$$

Therefore

$$\beta_r^2 = \frac{2E}{mc^2} - \beta_\varphi^2 + \xi(1 + \delta_T), \qquad \xi = \alpha x, \tag{36}$$

$$(1 - \xi)\beta_r^2 - 2s\xi\beta_r = F(x),$$

$$F(x) = \frac{2E}{mc^2} - \beta_\varphi^2 + \xi\beta_\varphi^2, \tag{37}$$

$$\beta_r = \frac{s\xi}{1 - \xi} + \sqrt{\frac{F(x)}{1 - \xi} + (\frac{s\xi}{1 - \xi})^2}. \tag{38}$$

With the regard for (19)

$$\frac{dx}{d\varphi} = \sqrt{\frac{2}{\alpha p}}\frac{s\xi}{1 - \xi} + \sqrt{\frac{2}{\alpha p}(\frac{F(x)}{1 - \xi} + (\frac{s\xi}{1 - \xi})^2)}, \tag{39}$$

$$\frac{dz}{d\varphi} = \sqrt{f(z)} + s\varepsilon, \qquad \varphi = \int_{z_1}^{z_2} \frac{dz}{\sqrt{f(z)} + s\varepsilon}, \tag{40}$$

$$f(z) = \frac{R_N(z) + \bar{\alpha}z^3}{1 - \bar{\alpha}z} + \frac{2}{\bar{\alpha}}(\frac{\bar{\alpha}z}{1 - \bar{\alpha}z})^2,$$

$$\varepsilon = \sqrt{2\bar{\alpha}}\frac{z}{1 - \bar{\alpha}z}. \tag{41}$$

The Integral (40) is calculated by an analogy with (24)

$$\frac{1}{\sqrt{f(z)} + s\varepsilon} \approx \frac{1}{\sqrt{f(z)}} - \frac{s\varepsilon}{f(z)},$$

$$\varphi = \int_{z_1}^{z_2} \frac{dz}{\sqrt{f(z)}} - \int_{z_1}^{z_2} \frac{\varepsilon dz}{f(z)} +$$

$$+ \int_{z_2}^{z_1} \frac{-dz}{\sqrt{f(z)}} + \int_{z_2}^{z_1} \frac{\varepsilon(-dz)}{f(z)},$$

$$\varphi = 2\int_{z_1}^{z_2} \frac{dz}{\sqrt{f(z)}}. \tag{42}$$

$$f(z) = R_N + \varepsilon_G, \tag{43}$$

$$\varepsilon_G = (e^2 - 1)\bar{\alpha}z + 4\bar{\alpha}z^2 + O(\bar{\alpha}^2), \tag{44}$$

$$\varphi = 2\phi + 4\bar{\alpha}\pi \tag{45}$$

$$\Delta\varphi_G(1) = 4\bar{\alpha}\pi. \tag{46}$$

We would remind you that $\Delta\varphi_E(1) = 3\bar{\alpha}\pi$. In the Table are adduced Results of the Calculation $\Delta\varphi_G(n)$ by Formula (46). It is obvious that Forces of the non-gravitational Origin influence on a Motion of the perihelion of Planets also. It is necessary to take into account this Forces.

Table 1: Results of the Calculation $\Delta\varphi_G(n)$ by Formula (46)

Planet	Mercury	Venus	Earth	Mars
$m, 10^{24}\ kg$	0.330	4.869	5.974	0.642
$a, 10^6\, km$	57.91	108.21	149.60	227.9
e	0.2056	0.0068	0.0167	0.0933
$p, 10^6\, km$	55.46	108.20	149.56	225.92
$T, 24\, hours$	87.969	227.701	365.256	686.98
n	415.210	160.410	100.	53.168
$\Delta\varphi_E(n)$	42".906	8".496	3".832	1".341
$\Delta\varphi_G(n)$	57".210	11".329	5".109	1".798
$\Delta\varphi_{calc}(n)$	43".110	8".537	5".109	1".798
$\Delta\varphi_{exp}(n)$	43".110	3".4±4".8	5".0±1".2	8".094±8".3

Conventional Signs:
m - a mass of the planet,
a - a Big half-axis of the Orbit,
e - an Eccentricity of the Ellipse,
p - a Focal Parameter,
T - a period,
n - a Number of the revolution of planets around the Sun at a Century,
$M_\odot = 6.67 \cdot 10^{30} kg$ - a Mass of the Sun.

5. Mystery of Constant of Gravitation for Sun

The Sun reradiates always gravitons with a permanent Power W_g. Here Author brings into Use an Analogy with the Electron [3]. As long as are proceeding thermonuclear Reactions at the Sun it radiates Photons with a variable Power $W_\gamma(\tau)$. In contrast to Gravitons Photons repulse a Planet from the Sun and hence Photons diminish a Displacement of the Perihelion of Planets. An Energy of the Interaction of photons with a Planet has a Shape (34) also. But a Constant of the Interaction is equal to $G_\gamma(\tau)$. Therefore a Constant of Gravitation for the Sun is equal to

$$G_\odot = G - G_\gamma(\tau) = K(\tau)G. \tag{47}$$

$K(\tau)$ - a Coefficient of the Diminution of the Constant of Gravitation G at the Expense of the Radiation of Solar Photons W_γ. It is obvious that for a Satellite that rotates around a planet or around the "cold" Sun in Vacuum

$$\Delta\varphi(1) = 4\bar{\alpha}\pi \sim G. \tag{48}$$

But for a planet that rotates around the "hot" Sun

$$\Delta\varphi_\odot(1) = 4\bar{\alpha}_\odot\pi \sim G_\odot. \tag{49}$$

Author considers that the observed Value $\Delta\varphi_{exp}(1)$ for Mercury is an absolute truth. Then today

$$K(\tau) = \frac{\Delta\varphi_{exp}(1)}{4\bar{\alpha}\pi} = \frac{5.0335 \cdot 10^{-7}}{6.6797 \cdot 10^{-7}} = 0.75355,$$

$$\frac{G_\gamma}{G} = 0.24645.$$

A Critical power of the Radiation is equal to

$$W_{crit} = K_W\, W_\gamma \qquad K_W = G/G_\gamma. \tag{50}$$

For the Sun $K_{W,\odot} = 4.0576$. If $W_\gamma(0) > W_{crit}$ then Planets don't attracting to the Sun and it is impossible a Formation of the solar System. It is evident that the Sun comes into existence thus that a Power of the Radiation was $W_\gamma(0) < W_{crit}$ and $G_\odot(0) < G_\odot(\tau)$. After a finishing of the "Combustion" of the Sun its Constant of gravitation is equal to $G = 6.67 \cdot 10^{-8}\ cm^3/g\, s^2$. But just right now at present this Value is equal to $G_\odot = 5.026 \cdot 10^{-8} cm^3/g\ s^2$. Author considers that Photons don't screening by an Atmosphere of the Mercury and Venus. Therefore the Formula (49) is correct for the Mercury and Venus. Author considers that Photons are screening practically by an Atmosphere of the Earth and Mars. Therefore the Formula (48) is true for the Earth and Mars. Final Results for $\Delta\varphi_{calc}(n)$ are given in the Table. But a last Word belongs by right to an Experiment!

6. Conclusion

∘ A Situation concerning a motion of the Perihelion of Planets is more intricate than Einstein imagined it to himself.

∘ With this Purpose Author proposed a Theory of Gravitation that takes into account a relative Speed of Gravitating Bodies.

∘ Author take into Consideration an Influence of Solar Photons on a motion of the Perihelion of Planets.

∘ Author take into Consideration an Influence of the Atmosphere of Planets an a Decrease of the Flow of Solar Photons.

∘ Author showed that a Constant of Gravitation is a Variable Quantity and in Vacuum at present it is equal to $5.026 \cdot 10^{-8}\ cm^3/g\ s^2$. A Generally accepted Constant of Gravitation is just only to Objects that don't radiate Photons.

References

[1] N.T. Rouseveare, "Mercury's perihelion from le Verrier to Einstein." Clarendon Press-Oxford, 1982.

[2] A. Einstein, *Sitzungsber. preuss. Akad. Wiss.*, 47, 2, pp.831-839 (1915).

[3] V.R. Terrovere. Spacetime & Substance J. No.2(22), p.78 2004, http//spacetime.narod.ru.

[4] V.R. Terrovere. Spacetime & Substance J. No.1(21), p.29 2004, http//spacetime.narod.ru.

[5] V.K. Semenchenko. Selected Chapters of the Theoretical Physics. "Uchpedgiz", Moskow, 1960 (in Russian).

[6] V.R. Terrovere. Spacetime & Substance J. No.4(9), p.147 2001, http//spacetime.narod.ru.

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References

[1] F.W. Stecker, K.J. Frost, *Nature*, **245**, 270 (1973).

[2] V.A. Brumberg, "Relativistic Celestial Mechanics", Nauka, Moskow, 1972 (in Russian).

[3] S.W. Hawking, in: "General Relativity. An Einstein Centenary Sutvey", eds. S.W. Hawking and W. Israel, *Cambr. Univ. Press*, Cambridge, England, 1979.

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