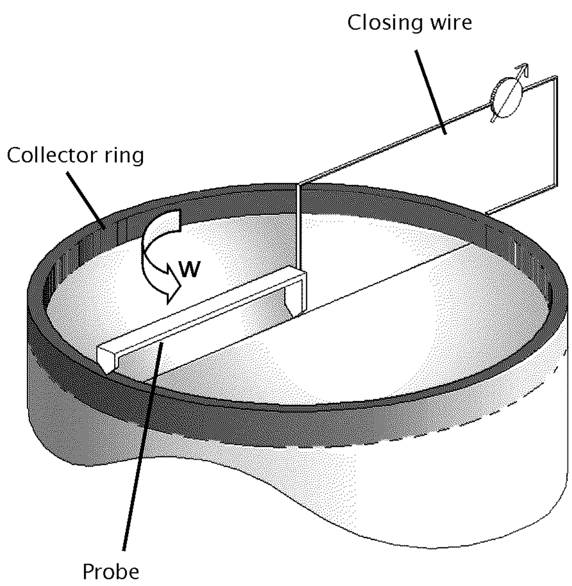
U K R A I N E ISSN 1726-4499

Spacetime & Substance

International Physical Journal



Volume 3, No. 4 (14), 2002

U K R A I N E ISSN 1726-4499

Spacetime & Substance

International Physical Journal

Certificate of the series AB, No. 4858, issued by the State Committee for Information Policy, TV and Broadcasting of Ukraine (February 12, 2001).

The Journal is published by Research and Technological Institute of Transcription, Translation and Replication, JSC(Kharkiv, Ukraine).

It is a discussion journal on problems of theoretical and experimental physics in the field of research of space, time, substance and interactions. The Journal publishes:
— the theories combining space, time, gravitation and others interactions (including the Einstein's SR and GR);
— application of theories for description and/or explanations of properties of the Universe and microcosmos;
— mathematical models and philosophical bases, which touch the description of a physical reality;
— description of set-ups aimed at the realization of fundamental physical experiments and the forthcoming results;
— discussion of published materials, in particular, those questions, which still have not a correct explanation.

The volume of one issue is 48 pages. Format is A4. Periodicity: 5 issues per one year during 2000–2002; monthly since 2003. The language is English. The equivalent versions: paper and electronic (*.TEX, *.PS, *.PDF).

Subscription information:

The price of one paper unit is $2.0 in Ukraine, $2.5 in NIS* states, $10.0 in all other countries (plus $1, $1.5 or $10 postage and handling accordingly). The electronic version price is 25% of the paper version price.

*) NIS (New Independent States without Ukraine) are Azerbaijan, Armenia, Byelorussia, Georgia, Kazakhstan, Kirghizia, Moldova, Russia, Tadjikistan, Turkmenistan, Uzbekistan.

Accounts:

In US Dollars

Correspondent: THE BANK OF NEW YORK
Eastern Europe Division
One WAll Street, New York, NY 10286
Account No. 890-0260-610
Beneficialry Bank: UKRSIBBANK of Ukraine
In favour of ZEMELNY BANK JSC
Account No. 1600-8-50174-01-00
SWIFT: KHAB UA 2K
Beneficiary: NTI TTR JSC
Account No. 26009011415

In UA Hryvnyas

Account No. 26009011415
in KHAB ZEMELNY BANK,
MFO 351652,
AO NTI TTR,
Cod 24473039,
Kharkov, Ukraine
(*for Ukraine subscribers, at the rate of the National Bank*)

The corresponding confermation as to the paying should be sent to the Editorial Office by E-mail.

Editorial Office: Zhuck N.A., RTI TTR, 3 Kolomenskaya St., Kharkov 61166, Ukraine
Tel.: +38 (0572) 19-55-77, (044) 265-79-94. Tel./fax: +38 (0572) 409-298, 409-594, 141-164, 141-165
E-mail: zhuck@ttr.com.ua, spacetime@ukr.net, krasnoh@iop.kiev.ua. http://spacetime.narod.ru

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 145–151

GRAVITY AND MOTION

Angelo Loinger[1]

Dipartimento di Fisica, Università di Milano Via Celoria, 16 - 20133 Milano, Italy

Received November 4, 2002

Several arguments concerning the relativistic vexatae quaestiones of the gravity field of a point mass and of the wavy gravity fields.

Introduction

I give a concise analysis, with essential historical references, of two critical subjects of relativistic astrophysics: the gravity field of a point mass and the wavy gravity fields.

First Part: On the gravity field of a point mass

1.– The solution of the problem of the Einsteinian gravitational field, which is generated by a point mass M at rest, is given – if r, θ, φ are spherical polar coordinates – by the following expression of the spacetime interval ([1], [2]):

$$\begin{aligned} \mathrm{d}s^2 \quad &= \quad \left[1 - \frac{2m}{f(r)}\right] c^2 \mathrm{d}t^2 - \\ &- \left[1 - \frac{2m}{f(r)}\right]^{-1} [\mathrm{d}f(r)]^2 - f^2(r)\mathrm{d}\omega^2, \qquad (1) \end{aligned}$$

where: $m \equiv GM/c^2$; G is the gravitational constant and c the speed of light *in vacuo*; $\mathrm{d}\omega^2 \equiv \mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2$; $f(r)$ is any regular function of r.

In the *original* solution form of the above problem given by Schwarzschild in 1916 [2], the function $f(r)$ is as follows:

$$f(r) \equiv \left[r^3 + 2m^3\right]^{1/3}; \qquad (2)$$

thus Schwarzschild's $\mathrm{d}s^2$ holds, physically and mathematically, in the *entire* spacetime, with the only exception of the origin $r = 0$, seat of the mass M, where we have a singularity.

If one chooses simply

$$f(r) \equiv r, \qquad (3)$$

one obtains the so-called *standard* form of solution, which is usually, *and erroneously*, named "by Schwarzschild". It was deduced *ex novo*, by integrating the Einstein equations, by Hilbert [3], by Droste [4], and by Weyl [5], independently.

Another interesting form, first investigated by M. Brillouin [6], is obtained by putting in (1)

$$f(r) \equiv r + 2m; \qquad (4)$$

it holds in the *whole*spacetime, with the only exception of the origin.

On the contrary, the *standard form is physically valid* **only for** $r > 2m$, because within the spatial surface $r = 2m$ (which is a singular locus) the time coordinate takes the role of the radial coordinate, and *vice versa*, the solution becomes *non*-static, and the $\mathrm{d}s^2$ loses its essential property of physical "appropriateness", according to the expressive Hilbert's terminology [3]. Further, I emphasize with Nathan Rosen that the radial coordinate of the standard solution has been initially chosen in such a way that the area of spatial surface $r = k$ is given by $4\pi k^2$. Accordingly, it is difficult to admit that the coordinate r can transform itself into a time coordinate. We ask ourselves: does the restriction $r > 2m$ imply a *physical* limitation? Not at all! Indeed, as the classic Authors knew, the exterior part $r > 2m$ of the standard form is *diffeomorphic* to the Schwarzschild's and Brillouin's forms, which hold for $r > 0$. One can say that the "*globe*" $r = 2m$ of the standard form shrinks into the *point* $r = 0$ of Schwarzschild's and Brillouin's forms, which is a singular point with an associate superficial area $4\pi(2m)^2$.

An odd reflection on the "*globe*" $r = 2m$ generated the notion of black hole: it would not have come forth if the treatises had expounded the forms of Schwarzschild or of Brillouin, in lieu of the standard form.

In a review article on the black holes [7], a true manifesto of scientific policy, we find some amazing results, e.g. the following evaluation of the average density of a black hole (*sic*): mass M divided by $(4/3)\pi r_0^3$, where $r_0 \equiv 2GM/c^2$; thus the *point* mass M is ideally distributed within the "*globe*" $r = 2m$; accordingly, the

[1] e-mail: angelo.loinger@mi.infn.it

average density of a black hole is inversely proportional to the square of its mass M. A marvellous consequence: if M is equal to the mass $M_{\odot}$ of the Sun, the average density is $\approx 10^{16}\mathrm{g}\cdot\mathrm{cm}^{-3}$, whereas for a mass $M = 10^8 M_{\odot}$, the average density is $\approx 1\mathrm{g}\cdot\mathrm{cm}^{-3}$, i.e. equal to water density. And a black hole of vanishing density has an infinitely large mass, and *vice versa*.

I remark that as far back as 1922 the competent scientists knew the right interpretation of the standard solution. Indeed, in 1922 a meeting was held at Collège de France, which was also attended by Einstein; the physical meaning of the "*globe*" $r = 2m$ was discussed and definitively clarified. See the lucid paper by Marcel Brillouin quoted in [6]. This author shows that it is not permitted to extend the radial coordinate of Schwarzschild's and Brillouin's forms to the negative values of the interval $-2m < r < 0$, and proves simultaneously that the attribution of a physical meaning to the interval $0 < r < 2m$ of the standard form is pure nonsense. Furthermore, let us recall that in a second fundamental memoir [8], Schwarzschild determined the Einsteinian gravitational field generated by an incompressible fluid sphere; now, if one computes the limit of his solution when the sphere contracts into a material point of a finite mass M, one finds anew the Schwarzschildian solution for a point mass: this is another proof of the "physicality" of the origin. Moreover, we remember that a fluid sphere of uniform density and *given mass* cannot have a radius smaller than $(9/8)(2m)$.

Quite similar considerations can be made for the gravitational fields generated by an electrically charged particle and by the spinning particle of the well-known Kerr's solution. In regard to the solution form – nonstatic and "maximally extended" – of Schwarzschild problem due to Kruskal [9] and Szekeres [10], we can declare its *physical superfluity*, because already the static forms of Schwarzschild and Brillouin, *in particular*, are "maximally extended".

Question concerning the *continued gravitational collapse*: it is almost evident that if we bear in mind, e.g., Schwarzschild's and Brillouin's forms of solution, no continued collapse can generate a black hole – and this was just Einstein's opinion. (See also Appendix A). On the other hand, it is physically clear that the real gravitational collapses cannot continue indefinitely, but end finally in astronomical objects of finite, relatively small, dimensions. (See my article "Relativistic spherical symmetries", http://xxx.lanl.gov/abs/physics/0107071 (July 28th, 2001) (misclassified: proper class.: gr-qc)).

I remark at last that the most cautious among the *observational* astrophysicists have always called in question the very notion of black hole. They know perfectly that the *observed* "black holes" do not coincide with the theoretical black holes, but are only large, or enormously large, masses concentrated in relatively small volumes.

1bis.– It can be shown (see e.g. P.A.M. Dirac, *General Theory of Relativity* – Wiley and Sons, New York, *etc.*, 1975 – p.32 and foll.) that the singular surface $r = 2m$ of the *standard* form of solution to Schwarzschild problem has the following properties:

i) A material point falling into the central body takes an infinite time to reach the critical surface $r = 2m$;
ii) If the above particle is emitting light of a certain frequency and is being observed by someone at a *large* value of r, its light is red-shifted by a factor $(1-2m/r)^{-1/2}$. This factor becomes infinite as the particle approaches the singular surface $r = 2m$. All the physical processes on the particle will be observed to be going more and more slowly as it approaches $r = 2m$;
iii) Let us consider an observer travelling *with* the particle; it reaches $r = 2m$ after the lapse of a *finite proper time* for the observer, who has aged only a finite amount when he and the particle reach $r = 2m$;
iv) The spatial region $r < 2m$ cannot communicate with the space for which $r > 2m$. Any signal, even a light signal, would take an infinite time to cross the boundary $r = 2m$; thus we cannot have a direct observational knowledge of the region $r < 2m$. Such a region is called a *black hole*, because things may fall into it (taking an infinite time by our clocks, to do so), but nothing can come out.

I have reproduced almost literally some significative sentences of sect. **19** of the cited Dirac's booklet. The proofs of properties *i*), *ii*), *iii*) are quite rigorous. On the contrary, the proof of property *iv*) rests on a paralogism: indeed, if one *hides* (as Dirac and many authors do) the singularity $r = 2m$ in the connection between the coordinates (r, t) and *suitable* new coordinates (ρ, τ), it is possible to extend the *transformed* solution to the region $r < 2m$ (where the roles of r and t are interchanged!). Then, one can "prove" formally property *iv*).

I remark finally that properties *i*), *ii*), *iii*) hold also for Schwarzschild's and Brillouin's forms (e.g.) if we substitute the critical surface $r = 2m$ of the standard form with the point $r = 0$ of the above forms.

In lieu of property *iv*), which characterizes the odd notion of black hole, the mentioned forms have the following property: any signal which starts from $r = 0$ will take an infinite time to reach any finite distance from the origin: it seems that one must be content with a black *point*. However, the story is not concluded, because it is possible to show that there exist infinite, non-trivial forms of solution to Schwarzschild problem which are *regular everywhere for* $r \geq 0$ (see my article "Regular solutions of Schwarzschild problem", http://xxx.lanl.gov/abs/physics/0104064 (April 20th, 2001) (misclassified: proper class.: gr-qc))).

2.– Many physicists think that the notion of black hole is only a relativistic generalization of a Newtonian notion, created by Michell (1784) and Laplace (1796).

Now, as it has been proved by McVittie (see *The Observatory*, **78** (1978) 272), this idea is based only on "a play of words in expressions such as *the velocity of escape or the escape from a body*".

Let us indeed consider a celestial spherical body of radius R and mass M. According to Newtonian dynamics, the velocity of escape w of a particle, which is projected radially outwards from the body's surface, is given by

$$w^2 = 2GM/R. \tag{5}$$

If the particle is projected with a velocity u smaller than w, i.e. if

$$u^2 < 2GM/R, \tag{6}$$

it will arrive at a finite distance from the celestial body, and then will fall on its surface again.

By employing the Newtonian corpuscular theory of light, which says that light is composed of corpuscles obedient to Newton's law of gravitation and travelling with a given velocity c, Michell and Laplace remarked that if $c < w$ the light corpuscles cannot go away indefinitely from the celestial body. Only if the radius R of the celestial body is such that

$$R = 2GM/c^2, \tag{7}$$

they can escape from the gravitational attraction exerted by the mass M. Then, if

$$R < 2GM/c^2, \tag{8}$$

the light corpuscles will attain to a finite distance D from the celestial body, and an observer situated at an intermediate distance between R and D will see the celestial body, owing to the light corpuscles which arrive at his eyes.

If only the spherosymmetrical black hole of general relativity existed, it ought to have the fundamental property that neither the material particles nor the light corpuscles can leave its surface (see sect.**1bis**). Therefore such an object would be invisible to any observer, however near he may be. None of the phenomena observed by the experimentalists in the region surrounding a "black hole" of Michell-Laplace is present in the neighbourhood of a black hole of general relativity.

The imagined connexion with the Newtonian formula (7) comes forth in this way: if for the determination of the Einsteinian gravitational field generated by a point mass M (Schwarzschild problem) one chooses the *standard* form – see (3) – as Droste, Hilbert and Weyl (*not Schwarzschild*) did, the radial coordinate r_0 of the points of the space surface $r = r_0$ (r_0 is the "radius of the black hole") is given by

$$r_0 \equiv 2GM/c^2; \tag{9}$$

this formula resembles the Newtonian formula (7), which regards a velocity of escape c. But eq.(7) implies obviously that all observers – including those at an infinitely great distance from the celestial body – can see it. As it is clear, the "Newtonian black hole" of Michell-Laplace is not a black hole!

The well-known scepticism of McVittie regarding the existence of relativistic black holes appears clearly from the final considerations of the cited Note, where he emphasizes, in particular, that there is "no way of asserting through some analogy with Newtonian gravitational theory that a black hole could be a component of a close binary system or that two black holes could collide. An existence theorem would first be needed to show that Einstein's field equations contained solutions which described such configurations".

3.– A question: why did the standard form (Droste-Hilbert-Weyl) prevail – in the scientific literature – over the original form of Schwarzschild? This is an interesting problem for an unprejudiced historian of our science. Here I limit myself to mention three reasons: *i*) the mathematical deduction of the standard form is simpler than that of Schwarzschild's form; *ii*) the influence implicitly exerted by Hilbert, the greatest mathematician and mathematical physicist of past century; *iii*) the premature death of Schwarzschild, due to a rare illness contracted at the German-Russian front.

4.- See in [24] a complete list of my papers concerning the subject of previous sects.$\mathbf{1 \div 3}$ published in Los Alamos Archive.

Second Part: On the wavy gravity fields

5. - During an epistolary discussion a known relativist wrote to me: "Without gravity waves, one would have to explain an instantaneous propagation of a change in the metric over the whole universe simply by changing the distribution of stress or mass in a system". This conviction is quite widespread, but is wrong. It is an incontestable *fact* that the physical non-existence of the gravitational waves is quite consistent with the fundamental principles of relativity theory: the Einsteinian field equations are time-symmetrical – and therefore it is perfectly legitimate to discard formal solutions which are time-asymmetrical. Analogously, Maxwell equations of the electromagnetic field are time-symmetrical: the existence of the electromagnetic waves is only a *theoretical possibility*, ***not*** *a theoretical necessity*: the *physical* existence of the e.m. waves is an ***experimental*** fact.

6.– ***A question***: what is the behaviour of the metric tensor when, e.g., a *supernova* explodes? ***Answer***: A sufficiently near apparatus would register a variation of the Einsteinian gravitational field which would be approximately similar to the corresponding variation of the Newtonian field. However, no gravitational *wave* – i.e. no physical entity endowed with a "life" *independent* of the source - would be emitted.

7.– There is an enormous number of papers concerning the gravitational waves. Here I limit myself to quote two manifestos by Schutz [11]; the first of them includes an ample bibliography.

Originally, the emission of gravitational waves was hypothesized and calculated in *analogy to the electromagnetic case*, starting from the **linear** approximation of the Einsteinian field equations, whose spatiotemporal substrate is "rigid" and coincides simply with Minkowski's spacetime. This "rigidity" tells us that there is a *primary conceptual difference* between exact theory and linearized theory, which is simply a theory of a *weak* gravitational field in a *flat* spacetime, having an invariant character with respect to the Lorentz transformations. Its formalism resembles the Minkowskian formalism of Maxwell theory, and – under suitable boundary conditions – allows the theoretical emission of undulating fields. Einstein did not like this result – and for many reasons. In spite of the innumerable computations that were performed since the Twenties of past century, he doubted always about the physical reality of the gravitational radiation. In particular, Einstein thought it is likely that only the *time-symmetrical* solutions of his field equations can represent physical phenomena.

I remark that the usual computations concerning the emission of gravitational waves by moving bodies are of a *perturbative* character and have the linearized version of the theory as a first approximation. Accordingly, they do not yield a true ***existence theorem***. On the other hand, it is possible to prove *rigorously* that no motion of point masses can generate gravitational waves [12]. A very simple proof is the following. Let us suppose that at a given instant t of its motion a given point mass M begins to send forth a gravitational wave and let us assume to know the *kinematical characteristics* of the motion between t and $t + |dt|$. It is indisputabile that we can reproduce *these* characteristics in a gravitational motion of the mass M in a suitable "external" gravitational field, within a time interval equal to $|dt|$, conveniently chosen. But in this case the mass M moves along a ***geodesic*** - and therefore it cannot emit any gravitational radiation: indeed, the geodesic motions are "free" motions, they are the perfect analogues of the rectilinear and uniform motions of an electric charge of the usual Maxwell-Lorentz theory. *Q.e.d.*

Conclusion: since *no "mechanism" exists for the generation of gravitational waves* (the restriction to motions of mass points is conceptually inessential), all the formal solutions of the Einsteinian field equations having an undulatory character do *not* describe *physical* waves. (See also Appendix B_1).

There are, however, other arguments which demonstrate the physical non-existence of the gravity radiation. Consider, for instance, that in the *exact* theory a gravitational wave would be an entity destitute of a *true* energy and a *true* momentum: consequently it cannot interact with any whatever apparatus or with an e.m. field: otherwise the energy-momentum account would not balance.

Many years ago, in the end of a paper [13] Pirani proposed to the reader and to himself the following problem: "Suppose for example that a Schwarzschild particle is disturbed from static spherical symmetry by an internal agency, radiates for some time, and finally is restored to static spherical symmetry. Is its total mass necessarily the same as before?" We have here a typical *Scheinproblem*: if the gravitational radiation existed, it would have only a pseudo (false) energy, therefore the final mass would be identical to the initial mass.

Furthermore, the undulatory character and the propagation velocity of a metric tensor *depend* on the reference system: with a suitable choice of the frame the undulatory character disappears, with a suitable choice of the frame the propagation velocity can take any value between zero and the infinite. (In general relativity we do not have a class of physically privileged frames of reference ...).

8.– Several authors have avoided intentionally the basic problem concerning the emission "mechanism" of the gravity waves and have looked for undulating solutions of the Einsteinian equations with a mass tensor equal to zero. Some exact solutions and others of a perturbative nature have been found. This is not surprising because the theory of the characteristics of the Einstein equations (Levi-Civita [14]) yields a rigorous proof of the existence of *wave fronts*; of what *kind* of waves? *Electromagnetic* waves, according to Levi-Civita – for several reasons, *in primis* because general relativity (analogously to special theory) must contain the geometrical optics.

Einstein, Møller, Scheidegger [15] and Rosen [16], but particularly Infeld e Plebanski [17] had serious reasons against the physical existence of the gravity waves, see Appendix B_2.

9.– According to a diffuse belief, the time decrease of the revolution period of the famous binary radiopulsar PSR1913+16 gives an experimental *indirect* proof of the physical reality of the gravitational radiation.

Owing to the observational data yielded by the "regular clock" of the pulsar, the interesting orbital parameters and the masses of the two stars (regarded as point objects) have been perturbatively computed. Then, the *perturbative* quadrupole formula gave a decrease of the revolution period, which agreed very well with the observations.

I emphasize the following points. In the *exact* theory the quadrupole formula loses any meaning because the hypothesized gravity waves do *not* have a *true* energy. Therefore, the *true* mechanical energy which is lost during the revolution motion ought to transform itself into the *pseudo* energy of the hypothetical gravity radiation: evidently the energy account does not balance.

Devil's advocate could object: if we restrict ourselves to the *linear* approximation of general relativity (as the experimentalists do), which has Minkowski spacetime as its substrate, the physical existence of the gravitational waves is surely a theoretical possibility. *Answer*: the energy-momentum of such gravitational waves has a tensor character only under Lorentz transformations, not under general transformations. Therefore it is always possible to find (and we remain, of course, in the ambit of the linear approximation) a general frame for which the above energy-momentum is equal to *zero*. But a wave with no energy and no momentum is not a physical object, even if it is formally endowed with a curvature tensor different from zero.

In the second place, there are realistic explanations of the decrease of the revolution period – as it is well known to the observational astrophysicists; for instance, viscous losses of the pulsar companion would give a time decrease of the revolution period of the same order of magnitude of that given by the hypothesized emission of gravity radiation.

Finally, the empirical success of a theory – or of a given computation – is not an *absolute* guaranty for its *conceptual* adequacy. Consider for instance the Ptolemaic theory of cycles and epicycles, which explained rather well the planetary orbits (with the only exception of Mercury's). As it was emphasized by Truesdell [20], the heliocentric theory would have been rejected if people of 17th century had had the modern computers.

10.– See in [25] a complete list of my papers concerning the subject of previous sects.**5**$\div$**9** published in Los Alamos Archive.

"Ti par che farrebe male un che volesse
mettere sotto sopra il mondo rinversato?"
("Do you think it is wrong
to reverse a reversed world?")
Giordano Bruno

APPENDIX A

All the Great Spirits who created and developed the general relativity (Einstein, Levi-Civita, Schwarzschild, Hilbert, Weyl, Eddington, Pauli, Fock,...) rejected always the very notion of black hole. In 1939 Einstein wrote a remarkable article [20], which was efficaciously summarized by Bergmann [21] with the following sentences, where the phrase "Schwarzschild singularity" means *more solito* (and improperly!) the critical surface of the *standard* form of solution to Schwarzschild problem. "Einstein investigated the field of a system of many mass points, each of which is moving along a circular path, $r =$ const. , under the influence of the field created by the ensemble. If the axes of the circular paths are assumed to be oriented at random, the whole system or cluster is spherically symmetric. The purpose of the investigation was to find out whether the constituent particles can be concentrated toward the center so strongly that the total field exhibits a Schwarzschild singularity. The investigation showed that even before the critical concentration of particles is reached, some of the particles (those on the outside) begin to move with the velocity of light, that is, along zero world lines. It is, therefore, impossible to concentrate the particles of the cluster to such a degree that the field has a singularity. (The singularities connected with each individual mass point are, of course, not considered.)

Einstein chose this example so that he would not have to consider thermodynamical questions, or to introduce a pressure, for the particles of his cluster do not undergo collisions, and their individual paths are explicitly known. In this respect, Einstein's cluster has properties which are nowhere encountered in nature. Nevertheless, it appears reasonable to believe that Einstein's result can be extended to conglomerations of particles where the motions of the individual particles are not artificially restricted as in Einstein's example." [21].

(N.B. – In reality, Einstein [20] employed the so-called isotropic coordinates in lieu of the standard coordinates. Of course, the validity of his argument is independent of this choice.).

APPENDIX B$_1$

i) Any particle of a continuous, incoherent "cloud of dust", characterized by the mass tensor

$$T^{jk} = \rho \frac{dx^j}{ds}\frac{dx^k}{ds}, \quad (j,k = 0,1,2,3), \tag{10}$$

where ρ is the invariant mass density, describes a *geodesic* line, and therefore cannot emit gravitational waves (see the first paper quoted in [12]). A simple application: the gravitational motions of the members of solar system.

ii) A well-known Fermi's geometrical theorem [22] as generalized by Eisenhart [23] affirms: For a manifold endowed with a *symmetric* connection it is possible to choose a coordinate system with respect to which the components Γ^i_{jk} $(= \Gamma^i_{kj})$ of the connection are *zero* at all points of a curve (or of a portion of it).

For a Riemann-Einstein spacetime this means that there exists a coordinate system with respect to which the first derivatives of the components $h_{jk}, (j,k = 0,1,2,3)$, of the metric tensor are *zero* at all points of a curve (or of a portion of it) – in particular, at all points of a time-like world line.

iii) Let us now consider a continuous medium (for instance, a perfect fluid) characterized by a certain mass tensor T_{jk} , and let $g_{jk}(x)$ be the solutions of Einstein equations

$$R_{jk} - \frac{1}{2}g_{jk}R = -\kappa T_{jk} \tag{11}$$

corresponding to a generic motion of our medium with respect to a given reference frame $(x) \equiv (x^0, x^1, x^2, x^3)$. Let us suppose to follow the motion of a given mass element describing a certain world line L. If we refer this motion, from the initial time t_0 on, to a Fermi's reference system $(z) \equiv (z^0, z^1, z^2, z^3)$, the components $h_{jk}(z)$ of the metric tensor will be equal to some constants for *all* points of line L. In other words, the gravitational field *on* L has been obliterated. Consequently, no gravitational wave has been sent forth. Now, line L is quite generic, and therefore *no motion of the continuous medium can give origin to a gravitational radiation.*

APPENDIX B$_2$

By means of approximation methods for the treatment of gravitational motions of the bodies Scheidegger in 1953 [15], and Infeld and Plebanski in 1960 [17] arrived at negative conclusions about the *physical* existence of a gravity radiation.

Scheidegger showed that all the computed radiation terms can be destroyed by suitable coordinate transformations. Infeld and Plebanski showed that "... it is hardly possible to connect any physical meaning with the flux of energy and momentum tensor defined with the help of the pseudo-energy-momentum tensor. Indeed, the radiation can be annihilated by a proper choice of the coordinate system. On the other hand, if we use a coordinate system in which the flux of energy may exist, then it can be made whatever we like by the addition of proper harmonic functions ...".

The common conclusion of the arguments of Appendices B$_1$ and B$_2$, in particular, is that there is no "mechanism" apt to produce gravitational waves. A conclusion which is in full accord with Einstein's ideas and Levi-Civita's conviction.

References

[1] Cfr. A.S. Eddington, *The Mathematical Theory of Relativity*, Second Edition (Cambridge University Press, Cambridge) 1960, p.94; W. Pauli, *Teoria della Relatività* (Boringhieri, Torino) 1958, p.245.

[2] Formula (1) holds also for the *exterior* part of any spherosymmetrical distribution of matter at rest. Moreover, it holds also if this distribution pulsates rhythmically, with any law, keeping the spherical symmetry.

[3] K. Schwarzschild, *Berl. Ber.*, (1916) 189; for an English version of this memoir, see http://xxx.lanl.gov/abs/physics/9905030 (May 12th, 1999).

[4] D. Hilbert, *Gött. Nachr.*, zweite Mitteilung, vorgelegt am 23. Dez. 1916; Idem, *Math. Annalen*, 92 (1924) 1.

[5] J. Droste, *Ned. Acad. Wet.*, S.A., **19** (1917) 197.

[6] H. Weyl, *Ann. Phys. (Leipzig)*, **54** (1917) 117; see, in particular, sect.*4*, and L. Flamm, *Phys. Z.*, **27** (1916) 448. Flamm and Weyl gave a topological interpretation of the standard form of solution to Schwarzschild problem, which – beginning from the Sixties of past century – provoked blamelessly the birth of the "wormholes".

[7] M. Brillouin, *Journ. Phys. Rad.*, **23** (1923) 43.

[8] A. Celotti, J.C. Miller and D.W. Sciama, *Class. Quantum Grav.*, **16** (1999) A3. The phrase "quantum gravity" is the typical product of a wishful thinking: it characterizes indeed only *vain efforts* to create a rational theory – *et pour cause*!

[9] K. Schwarzschild, *Berl. Ber.*, (1916) 424; for an English version of this memoir, see http://xxx.lanl.gov/abs/physics/9912033 (December 16th, 1999).

[10] M. Kruskal, *Phys. Rev.*, **119** (1960) 1743.

[11] G. Szekeres, *Publ. Mat. Debrecen*, **7** (1960) 285.

[12] B.F.Schutz, *Class. Quantum Grav.*, **16** (1999), A131; Idem, http://xxx.lanl.gov/abs/gr-qc/0003069 (March 16th, 2000).

[13] A. Loinger, http://xxx.lanl.gov/abs/astro-ph/0003230 (March 16th, 2000); and *Nuovo Cimento* B, **115** (2000) 679; http://xxx.lanl.gov/abs/physics/0106052 (June 17th, 2001) [misclassified (proper classification: gr-qc or astro-ph)].

[14] F.A.E. Pirani, *Phys. Rev.*, **105** (1957) 1089.

[15] T. Levi-Civita, *Rend. Acc. Lincei*, **11** (s.6a) (1930) 3 and 113.

[16] A.E. Scheidegger, *Revs. Mod. Phys.*, **25** (1953) 451.

[17] N. Rosen, *Gen. Rel. Grav.*, **10** (1979) 351.

[18] L. Infeld and J. Plebanski, *Motion and relativity* (Pergamon Press, Oxford, etc.) 1960, pp. 200 and 201.

[19] C. Truesdell, *An Idiot's Fugitive Essays on Science* (Springer-Verlag, New York, etc.) 1982, p.620 and foll.

[20] A. Einstein, *Ann. Math.*, **40** (1939) 922.

[21] P.G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Inc., Englewood Cliffs, N.Y.) 1960, p. 204.

[22] E. Fermi, *Rend. Acc. Lincei*, **31**^{1} (1922) 21 and 51; T. Levi-Civita, *Lezioni di calcolo differenziale assoluto* (Stock, Roma) 1925, p.190; Idem, *Math. Ann.*, **97** (1926) 291.

[23] L.P. Eisenhart, *Non-Riemannian Geometry*, (Am. Math. Soc., New York) 1927, p.64.

[24] A. Loinger, http://xxx.lanl.gov/abs/astro-ph/9801167 (October 11st, 1998);
http://xxx.lanl.gov/abs/gr-qc/9908009 (August 3rd, 1999);
http://xxx.lanl.gov/abs/gr-qc/9911077 (January 5th, 2000);
http://xxx.lanl.gov/abs/astro-ph/0001453 (January 26th, 2000);
http://xxx.lanl.gov/abs/gr-qc/0006033 (June 10th, 2000);
http://xxx.lanl.gov/abs/physics/0104064 (April 20th, 2001) – misclassified (proper classification: gr-qc);
http://xxx.lanl.gov/abs/physics/0107071 (July 28th, 2001) – misclassified (proper classification: gr-qc);
http://xxx.lanl.gov/abs/physics/0109055 (September 21th, 2001) – misclassified (proper classification: gr-qc).

[25] A. Loinger, http://xxx.lanl.gov/abs/astro-ph/9810137 (October 8th, 1998);
http://xxx.lanl.gov/abs/astro-ph /9904207 (April 20th, 1999);
http://xxx.lanl.gov/abs/astro-ph/9906058 (June 3rd, 1999);
http://xxx.lanl.gov/abs/gr-qc/9909091 (September 30th, 1999);
http://xxx.lanl.gov/abs/astro-ph/9912507 (December 23rd, 1999);
http://xxx.lanl.gov/abs/astro-ph/0002267 (February 12th, 2000);
http://xxx.lanl.gov/abs/astro-ph/0003230 (March 16th, 2000);
http://xxx.lanl.gov/abs/gr-qc/0007048 (July 19th, 2000);
http://xxx.lanl.gov/abs/physics/0102011 February 6th, 2001) – misclassified (proper classification: gr-qc);
http://xxx.lanl.gov/abs/physics/0105010 (May 3rd, 2001) – misclassified (proper classification: gr-qc);
http://xxx.lanl.gov/abs/physics/0106052 (June 17th, 2001) – misclassified (proper classification: gr-qc).

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 152–160

A "FREELY COASTING" UNIVERSE

Savita Gehlaut, A. Mukherjee,† S. Mahajan & D. Lohiya[1]‡

† *Department of Physics and Astrophysics, University of Delhi, Delhi–110 007, India*

‡ *Inter University Centre for Astronomy and Astrophysics, Postbag 4, Ganeshkhind, Pune 411 007, India*

Received December 4, 2002

A strictly linear evolution of the cosmological scale factor is surprisingly an excellent fit to a host of cosmological observations. Any model that can support such a coasting presents itself as falsifiable model as far as classical cosmological tests are concerned. This article discusses the concordance of such an evolution in relation to several standard observations. Such evolution is known to be comfortably concordant with the Hubble diagram as deduced from current supernovae Ia data. It passes constraints arising from the age and gravitational lensing statistics and just about clears basic constraints on nucleosynthesis. Such an evolution exhibits distinguishable and verifiable features for the recombination era. The overall viability of such models is discussed.

1. Introduction

Large scale homogeneity and isotropy of matter and radiation observed in the universe suggests the following [Friedmann-Robertson-Walker (FRW)] form for the space-time metric:

$$ds^2 = dt^2 - a(t)^2[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)].(1)$$

Here $K = \pm 1, 0$ is the curvature constant. In standard "big-bang" cosmology, the scale factor $a(t)$ is completely determined by the model for the equation of state of matter and Einstein's equations. The scale factor, in turn, determines the response of a chosen model to cosmological observations. Four decades ago, the main "classical" cosmological tests were (1) The galaxy number count as a function of red-shift; (2) The angular diameter of "standard" objects (galaxies) as a function of red-shift; and finally (3) the apparent luminosity of a "standard candle" as a function of red-shift. Over the last two decades, other tests that have been perfected, or are fast approaching the state of perfection, are: the early universe nucleosynthesis constraints, estimates of age of the universe in comparison to ages of old objects, statistics of gravitational lensing and finally, the physics of recombination as deduced from cosmic microwave background anisotropy.

In this article we explore concordance of the above observations with a FRW cosmology in which the scale factor evolves linearly with time: $a(t) \propto t$, right from the creation event itself. The motivation for such an endeavour comes from several considerations. First of all, such a cosmology does not suffer from the horizon problem. Horizons occur in models with $a(t) \approx t^\alpha$ for $\alpha < 1$ [see eg. [1, 2]]. As a matter of fact, a linearly evolving model is the only power law model that has neither a particle horizon nor a cosmological event horizon. Secondly, linear evolution of the scale factor is supported in alternative gravity theories where it turns out to be independent of the matter equation of state [3, 4, 5]. The scale factor in such theories does not constrain the matter density parameter. This contrasts with the Standard FRW model where the Hubble parameter determines a critical value of density which turns out to be a dynamical repeller. This is the root cause of the "flatness" or fine tuning problem. Finally, such a linear coasting cosmology, independent of the equation of state of matter, is a generic feature in a class of models that attempt to dynamically solve the cosmological constant problem [6, 4, 5]. Such models have a scalar field non-minimally coupled to the large scale scalar curvature of the universe. With the evolution of time, the non-minimal coupling diverges, the scale factor quickly approaches linearity and the non-minimally coupled field acquires a stress energy that cancels the vacuum energy in the theory.

There have been other gravity models that also account for a linear evolution of the scale factor. Notable among such models is Allen's [7] in which such a scaling results in an $SU(2)$ cosmological instanton dominated universe. Yet another possibility arises from the Weyl gravity theory of Mannheim and Kazanas [8]. Here again the FRW scale factor approaches a linear evolution at late times.

Although any of the above are good enough reasons for exploring the concordance of a linear coasting, we add to this list the following reason of our own.

[1]e-mail: dlohiya@iucaa.ernet.in

The averaging problem in General Relativity has never been properly addressed, let alone solved [9, 10]. This is in contrast with the corresponding problem in classical electromagnetic theory [11]. There one can (i) start with multi-singular solutions to the Laplace equation, (ii) smear each charge over a large enough sphere, and (iii) if the overall distribution satisfies Dirichlet / Neumann boundary conditions at infinity, the average potential can be defined and coincides with the solution to the Poisson equation. In General Relativity the corresponding construction has not been carried out. All precision tests of General Relativity involve vacuum (source free region) solutions of Einstein equations. Strictly speaking, there are no tests of Einstein theory with matter. In the interior of all astrophysical sources, either the weak field (Newtonian) limit is put to test or, where the weak field limit is expected to break down, one assumes General Relativity to parametrize the equation of state (eg. for neutron / quark stars etc.).

On the other hand, the above problems could be circumvented by taking Einstein's equations with the source terms as the *defining* equations for a gravity theory. The justification for such an approach could rely on its correct Newtonian limit. Such an attitude comes with its own problems.

First of all, one encounters a related *averaging* problem again when one applies the theory to cosmology. Is it justified to assume the large scale behaviour of the lumpy universe to be the same as that predicted by the smoothed out FRW models ? The essential issue is that averaging the metric does not commute with determining the local connection followed by the determination of the local Ricci tensor and finally forming the field equations to determine the metric. There have been several attempts to resolve this issue [9, 10], but with limited success. Moreover, reliance on an ansatz just because of its Newtonian limit may in fact be flawed. Newtonian gravity does not offer unique cosmological solutions in the continuum limit for an open cosmology [12].

All studies on the averaging problem and the continuum limit have not considered the retarded effects in their full generality. Newtonian cosmology, applied to an exploding *Milne ball* in a flat space-time [see eg. [13, 14]] gives a unique linear coasting cosmology viz. the FRW [Milne] metric with $a(t) = t$.

Finally, we recall an approach to General Relativity starting from a spin two field interacting with a source in a flat space-time. Incorporating back reaction on the source in a gauge invariant manner and to all orders of perturbations yields Einstein's theory [15, 16, 17, 18]. However, the entire analysis relies on canonical propagation of gravity and fails for a distribution of particles across horizons if one has a cosmological creation event. The equivalence Principle tells us that the natural way to describe a distribution of particles just after a creation event, in case one demands gravity not to have globally set in on account of event horizons, is a distribution in a flat space-time. This again takes one back to Milne cosmology.

Indeed, consider the universe just after its "creation event", defined at $t = 0$, at a small enough time $t = \epsilon$. In a classical description, let the matter be distributed as a swarm of particles in a Riemannian manifold. One may accept Einstein's theory as a local theory and invoke Einstein's equations at the location of each particle, viz.: $G_{\mu\nu} = -8\pi T_{\mu\nu}$. In the inter-particle spaces, the equations read: $G_{\mu\nu} = 0$. For ϵ small enough, there is no reason to expect the global space-time dynamics to be governed by an average stress energy distribution: $< G_{\mu\nu} >= -8\pi < T_{\mu\nu} >$. This is particularly unreasonable on account of horizons in the theory. There is absolutely no dynamical reason to expect an *average* gravity, described by Einstein's equations on the average, to have globally "set in". It is much more reasonable to expect gravity *not* to have set in globally on account of *retarded effects*. Matter distribution on large scales, in the absence of global gravitation, is naturally described as a distribution in a flat space-time. Such a general homogeneous and isotropic distribution of matter in a flat space-time, described in co-moving coordinates, is just the Milne ball. This reduces to an open FRW universe with the scale factor $a(t) = t$.

We may take any of the above as the basis for our linear coasting conjecture. In what follows, we assume that a homogeneous background FRW universe is born and evolves as a Milne Universe about which a matter distribution and standard General Relativity would determine the growth of perturbations. Thus we conjecture that the Einstein equations give a correct *microscopic* description of gravitation. This being so, the global dynamics of a FRW Universe, at a small time ϵ after a creation event, is not described by the averaged Einstein equations but as a freely coasting Milne Universe.

Interestingly, a universe born as a Milne model provides just the right initial condition required to sort out the cosmological constant problem. It is straightforward to formulate an action principle for gravity where the determinant of the metric is not a dynamical quantity. The trace of the stress tensor of any matter field does not contribute to the dynamics of gravitation [6]. Although this solves the naturalness problem of the cosmological constant, an effective cosmological constant appears as an integration constant in this formulation. What is needed is some physical reason that demands a flat space-time solution to describe cosmology at any instant of time and our conjecture does precisely that.

The following section reviews the concordance of linear evolution in relation to standard cosmological observations.

2. A linearly coasting cosmology

2.1. Classical Cosmology tests

To our knowledge, the first exploration of concordance of a linearly evolving scale factor with observations was conducted by Kolb [19]. Kolb obtained a linear evolution by a judicious choice of "K-matter" that makes the universe curvature dominated at low red-shifts. At sufficiently high red-shifts, normal matter becomes increasingly dominant. One could thus manage to have a linear coasting at low red-shifts without giving up several nice results of standard cosmology such as cosmological nucleosynthesis. Kolb demonstrated that data on galaxy number counts as a function of red-shift as well as data on angular diameter distance as a function of red-shift do not rule out a linearly coasting cosmology. Unfortunately, these two tests are marred by effects such as galaxy mergers and galactic evolution. For these reasons these tests have fallen into disfavour as reliable indicators of a viable model.

The variation of apparent luminosity of a "standard candle" as a function of red-shift is referred to as the Hubble test. The discovery of Type Ia-Supernovae [SNe Ia] as reliable standard candles raised hopes of elevating the status of this test to that of a precision measurement that could determine the viability of a cosmological model. The main reasons for regarding these objects as reliable standard candles are their large luminosity, small dispersion in their peak luminosity and a fairly accurate modeling of their evolutionary features. Recent measurements on 42 high red-shift SNe Ia's reported in the supernovae cosmology project [20] together with the observations of the 16 lower red-shift SNe Ia's of the Callan-Tollolo survey [21, 22] have been used to determine the cosmological parameters Ω_Λ and Ω_M for the *standard model*. The data eliminates the "minimal inflationary" prediction defined by $\Omega_\Lambda = 0$ and $\Omega_M = 1$. The data can however, be used to assess a "non-minimal inflationary cosmology" defined by $\Omega_\Lambda \neq 0$, $\Omega_\Lambda + \Omega_M = 1$. The maximum likelihood analysis following from such a study has yielded the values $\Omega_M = 0.28 \pm 0.1$ and $\Omega_\Lambda = 0.72 \pm 0.1$ [23, 24, 26, 27].

To explore the concordance of a linear coasting cosmology, it is convenient to consider a power law cosmology with the scale factor $a(t) = \bar{k}t^\alpha$, with $\bar{k}$, α arbitrary constants. It is straightforward to discover the following relation between the apparent magnitude $m(z)$, the absolute magnitude M and the red-shift z of an object for such a cosmology:

$$m(z) = \mathcal{M} + 5\log H_o + 5\log(\frac{\alpha}{H_o})^\alpha(1+z)\bar{k}S\times$$

$$\times[\frac{1}{(1-\alpha)\bar{k}}(\frac{\alpha}{H_o})^{1-\alpha}(1-(1+z)^{1-\frac{1}{\alpha}})]. \quad (2)$$

Here $S[X] = X$, $\sin(X)$ or $\sinh(X)$ for $K = 0, \pm 1$ respectively, and $\mathcal{M} = M - 5log(H_o) + 25$. The best fit turns out to be $\alpha = 1.001 \pm .0043$, $K = -1$. [28]. The minimum χ^2 per degree of freedom turns out to be 1.18. This is comparable to the corresponding value 1.17 reported by Perlmutter et al for non-minimal inflationary cosmology parameter estimations. The concordance of linear coasting with SNe Ia data finds a passing mention in the analysis of Perlmutter [20] who noted that the curve for $\Omega_\Lambda = \Omega_M = 0$ (for which the scale factor would have a linear evolution) is "practically identical to **best fit** plot for an unconstrained cosmology". More recently, new high redshift supernovae sightings have been reported. Linear coasting is as accommodating even for the largest red-shift supernova (1997ff) as the standard non-minimal inflationary model [25].

The age estimate of the ($a(t) \propto t$) universe, deduced from a measurement of the Hubble parameter, is given by $t_o = (H_o)^{-1}$. The low red-shift SNe Ia data [21, 22] gives the best value of 65 km sec^{-1} Mpc^{-1} for the Hubble parameter. The age of the universe turns out to be 15×10^9 years. This is $\approx 50\%$ greater than the age inferred from the same measurement in standard (cold) dark matter dominated cosmology (without the cosmological constant). Such an age estimate is comfortably concordant with age estimates of old clusters.

A study of consistency of linear coasting with gravitational lensing statistics has recently been reported [29]. The expected frequency of multiple image lensing events is a sensitive probe for the viability of a given cosmology. A sample of 867 high luminosity optical quasars projected in a power law FRW cosmology gives an expected number of five lensed quasars for a power $\alpha = 1.09 \pm 0.3$. This indeed matches observations. Thus a strictly linear evolution of the scale factor is comfortably concordant with gravitational lensing statistics.

2.2. "The precision" tests

a) The Nucleosynthesis Constraint What makes linear coasting particularly appealing is a recent demonstration that primordial nucleosynthesis is not an impediment for a linear coasting cosmology [30, 31]. A linear evolution of the scale factor may be expected to radically effect nucleosynthesis in the early universe. Surprisingly, the following scenario goes through.

Energy conservation, in a period where the baryon entropy ratio does not change, enables the distribution of photons to be described by an effective temperature T that scales as $a(t)T =$ constant. With the age of the universe estimated from the Hubble parameter being $\approx 1.5 \times 10^{10}$ years, and $T_0 \approx 2.7$K, one concludes that the age of the universe at $T \approx 10^{10}$K would be some four years [rather than a few seconds as in standard cosmology]. The universe would take some 10^3 years to cool to 10^7K. With such time periods being large in comparison to the free neutron life time, one would hardly expect any neutrons to survive. However, with such a low rate of expansion, weak interactions remain in equilibrium for temperatures as low as 10^8K. The neutron - proton ratio keeps falling as $n/p \approx \exp[-15/T_9]$. Here T_9 is the temperature in units of 10^9K and the factor of 15 comes from the n-p mass difference in these units. There would again hardly be any neutrons left if nucleosynthesis were to commence at (say) $T_9 \approx 1$. However, as weak interactions are still in equilibrium, once nucleosynthesis commences, inverse beta decay would replenish neutrons by converting protons into neutrons and pumping them into the nucleosynthesis channel. With beta decay in equilibrium, the baryon entropy ratio determines a low enough nucleosynthesis rate that can remove neutrons out of the equilibrium buffer at a rate smaller than the relaxation time of the buffer. This ensures that neutron value remains unchanged as heavier nuclei build up. It turns out that for baryon entropy ratio $\eta \approx 5 \times 10^{-9}$, there would just be enough neutrons produced, after nucleosynthesis commences, to give $\approx 23.9\%$ helium and some 10^8 times the metallicity produced in the early universe in the standard scenario. This metallicity is of the same order of magnitude as seen in lowest metallicity objects.

The only problem that one has to contend with is the significantly low yields of deuterium in such a cosmology. Though deuterium can be produced by spallation processes later in the history of the universe, it is difficult to produce the right amount without a simultaneous over production of lithium [32] However, as pointed out in [30], the amount of helium produced is quite sensitive to η in such models. In an inhomogeneous universe, therefore, one can have the helium to hydrogen ratio to have a large variation. Deuterium can be produced by a spallation process much later in the history of the universe. If one considers spallation of a helium deficient cloud onto a helium rich cloud, it is easy to produce deuterium as demonstrated by Epstein [32] - without overproduction of lithium.

Interestingly, the baryon entropy ratio required for the right amount of helium corresponds to $\Omega_b \approx 0.2$. Here Ω_b is the ratio of the baryon density to a "density parameter" determined by the Hubble constant: $\Omega_b \equiv \rho_b/\rho_c = 8\pi G\rho_b/3H_o^2$. $\Omega_b \approx 0.2$ closes dynamic mass estimates of large galaxies and clusters [see eg [33, 34]]. In standard cosmology this closure is sought to be achieved by taking recourse to non-baryonic cold dark matter. Thus in a linearly scaling cosmology, there would be no need of non-baryonic cold dark matter at all.

b) The recombination epoch

We describe this in some detail as the peculiarities of the recombination epoch in a linearly coasting cosmology are not covered in any standard (curvature dominated) cosmology description.

The salient features of a linear coasting cosmology at the recombination epoch can be deduced by making a simplifying assumption of thermodynamic equilibrium just before recombination. As in standard cosmology, a recombination process that directly produces a hydrogen atom in the ground state releases a photon with energy $B = 13.6$ eV in each recombination. $n_\gamma(B)$, the number density of photons in the background radiation with energy B, is given by [see eg. [35, 34]]:

$$\frac{n_\gamma(B)}{n} = \frac{16\pi}{n}T^3\exp(\frac{-B}{T}) \approx \frac{3\times10^7}{\Omega_B h^2}\exp(\frac{-13.6}{\tau}), \quad (3)$$

where τ is the temperature in units of eV. This ratio is unity at $\tau \approx 0.8$ for $\Omega_B h^2 \approx 1$ and decreases rapidly at lower temperatures. Any 13.6 eV photons released due to recombination have a high probability of ionizing neutral atoms formed a little earlier. [In the following, we shall quote all results for our favoured values $\Omega_b \approx 0.2$ and the Hubble parameter 65 km/sec/Mpc.] This process is therefore not very effective for producing a net number of neutral atoms. The dominant recombination process proceeds through an excited state: $(e + p \longrightarrow H^* + \gamma_1;\ H^* \longrightarrow H + \gamma_2)$. This produces two photons, each having lesser energy than the ionization potential of the hydrogen atom. The 2p and 2s levels provide the most rapid route for recombination. The 2p decay produces a single photon, while the decay from the 2s is by two photons. As the reverse reaction occur at the same rate, recombination is a non-equilibrium process that proceeds at a much slower rate. The thermally averaged cross section for the process of recombination $(p + e \leftrightarrow H + \gamma)$ is given by [33, 34]:

$$\frac{<\sigma v>}{c} \approx 4.7\times10^{-24}(\frac{T}{1\mathrm{eV}})^{1/2}\ \mathrm{cm}^2. \quad (4)$$

This gives the reaction rate:

$$\Gamma = n_p <\sigma v> = 2.374\times10^{-10}\tau^{7/4}\times$$
$$\times\exp(-6.8/\tau)(\Omega_b h^2)^{1/2}\ \mathrm{cm}^{-1}. \quad (5)$$

This is to be compared to the Hubble expansion rate at that epoch, $H = H_0(T/T_0)$. Given the Hubble constant ($H_0 = 100h$ km/sec/Mpc) and CMB effective temperature $T_0 = 2.73$K now, the Hubble parameter at any temperature turns out to be: $H = 4.7\times10^{-25}h\tau\ \mathrm{cm}^{-1}$. This equals Γ at

$$\tau^{-3/4}\exp(6.8/\tau) = 1.96\times10^{15}(\Omega_\mathrm{b})^{1/2}. \quad (6)$$

A straightforward iteration gives:

$$\tau^{-1} \approx 5.17 - 0.11\ln(\tau^{-1}) + 0.074\ln(\Omega_\mathrm{b}) \approx (0.2)^{-1} \quad (7)$$

corresponding to a red-shift given by:

$$1 + z \approx 874.5[1 + 0.015\ln(\Omega_\mathrm{b})]^{-1}. \quad (8)$$

The residual fraction of electrons turns out to be [34]:

$$x_e \approx (\frac{\pi}{4\xi(3)\sqrt{2}})^{\frac{1}{2}}\eta^{-\frac{1}{2}}(\frac{T}{m_e})^{-\frac{3}{4}}\exp(-\frac{6.8}{\tau}). \quad (9)$$

From eqn.(6), we have

$$x_e \approx 7.9 \times 10^{-9} \frac{\tau^{-3/2}}{\Omega_b h}. \tag{10}$$

For the red-shift range $800 < z < 1200$, the approximate fractional ionization is:

$$x_e = \frac{2.4 \times 10^{-3}}{\Omega_b h^2} (\frac{z}{1000})^{12.75}. \tag{11}$$

After decoupling at $\tau = 0.2$, this gives a residual ionization:

$$x_{e,res} \approx 9 \times 10^{-8} (\Omega_b h)^{-1}. \tag{12}$$

The only process that may still be effective at such low temperatures is the Thomson scattering with a cross section $\sigma_T = 6.7 \times 10^{-25}$ cm^2. The optical depth for photons would be:

$$\tau_\gamma = \int_0^t n_b(t) x_e(t) \sigma_T dt =$$

$$= - \int_0^z n_b(z) x_e(z) \sigma_T (\frac{dt}{dz}) dz. \tag{13}$$

With $n_b(z) = \eta n_\gamma(z) = \eta \times 421.8(1+z)^3$ cm^{-3}, and

$$\frac{dt}{dz} = - \frac{1}{H_0 (1+z)^2} \tag{14}$$

one can find the red-shift at which the optical depth goes to unity.

If one considers the residual ionization $x_{e,res}$, we get

$$\tau_\gamma = 4.7 \times 10^{-2} \times (\frac{z}{1000})^2. \tag{15}$$

From this optical depth, we can compute the probability that a photon was last scattered in the interval $(z, z+dz)$. This is given by:

$$P(z) = e^{-\tau_\gamma} \frac{d\tau_\gamma}{dz} \approx .94 \times 10^{-5} (\frac{z}{1000}) \times$$

$$\times \exp[-0.047 (\frac{z}{1000})^2] \tag{16}$$

τ_γ becomes unity at $z \approx 4610$. This implies that the *residual ionization* has insufficient optical depth to scatter photons from the decoupling epoch. From the expression for fractional ionization eqn(11), the optical depth of the last scattering surface can be deduced to be:

$$\tau_\gamma = 170 \times (\frac{z}{1000})^{14.75}. \tag{17}$$

This gives:

$$P(z) \approx 2.5 (\frac{z}{1000})^{13.75} \exp[-170 (\frac{z}{1000})^{14.75}] \tag{18}$$

τ_γ goes to unity at $z_R \approx 703$. This $P(z)$ can be approximated by a Gaussian centered at $z_R \approx 703$ with a width $\Delta z \approx 51.8$.

An important scale that determines the nature of CMB anisotropy is the curvature scale which is the same as the Hubble radius for the linear coasting. The angle subtended today, by the Hubble radius at $z_R = 703$, is determined by

$$\frac{1+z_R}{2} \frac{\theta}{2} = \sinh[\frac{d(\theta)(1+z_R)}{2a_0}]. \tag{19}$$

Here $d(\theta) = d_H(t_R) = H(t_R)^{-1} = [H_0(1+z_R)]^{-1}$. This gives:

$$(\frac{1+z_R}{2}) \frac{\theta}{2} = \sinh(\frac{1}{2}) \tag{20}$$

or $\theta_H \approx 10$ minutes.

In standard cosmology, the *sound horizon* is of the same order as the Hubble length at recombination. The Hubble length determines the scale over which physical processes can occur coherently. Thus one expects all acoustic signals to be contained within an angle of the order of the angle subtended by the Hubble length at recombination. In a linear coasting cosmology, the Hubble length is precisely the inverse of the curvature scale. However, the sound horizon (s^*) is much larger. Strictly speaking, the particle as well as the sound horizon are infinite for a linear coasting cosmology. For our purpose, it suffices to take the epoch of birth of pressure waves as the epoch of baryon production. We take this to be the QGP phase transition epoch $T_{QGP} \approx 10^{12} K$. The distance a sound wave travels from this epoch till recombination, projected transverse to the line of sight on the last scattering surface [LSS], subtends an angle at the current epoch which can be refered to as the *sound horizon angle*:

$$\theta^* \approx \frac{1}{\sqrt{3}} \ln(\frac{T_i}{T_f}) \times \frac{2}{1+z^*}. \tag{21}$$

This is $\approx 2^o$ for $T_i = T_{QGP}$ and $T_f \approx 10^3 K$ corresponding to $z^* \approx 705$. The angle subtended by the sound horizon scale is thus roughly 12 times that subtended by the curvature length scale of ten minutes. The photon diffusion scale is determined by the thickness of the LSS. With $z^* \approx 705$ and $\Delta z \approx 51$, the photon diffusion scale projected on the LSS corresponds to an angular size roughly one fourteenth of the Hubble length at the LSS. This subtends an angle of 43" at the current epoch.

The above scales in principle determine the nature of CMB anisotropy. The CMB effectively ceases to scatter when the optical depth to the present drops to unity. After last scattering, the photons effectively free stream. On the LSS, the photon distribution may be locally isotropic while still possessing inhomogeneities i.e. hot and cold spots, which will be observed as anisotropies in the sky today [see eg. [36, 37]. As described in the Appendix, temperature fluctuations, determined by the potential and density perturbations, are expressible by an expansion in terms of eigenmodes of the generalized Laplace operator ∇^2 with eigenvalues $-k^2$. The phase of oscillation is frozen in at last scattering. The critical wave number $k_A \equiv \pi/s^*$ corresponds to the sound horizon at that time. Longer wavelengths will not have evolved from the initial conditions and possess $\Phi/3$ gravitational potential fluctuations after gravitational red-shift [36, 37]. This combination of the intrinsic temperature fluctuation and the gravitational red-shift is the "Sachs - Wolfe effect". Shorter wavelengths can be frozen at different phases of the $\cos(ks^*)$ oscillation for adiabatic perturbative modes and as $\sin(ks^*)$ for isocurvature fluctuation modes. For adiabatic modes as a function of k there will be a harmonic series of temperature fluctuation peaks with $k_m = mk_A = m\pi/s^*$ for the mth peak ($m = 1, 2...$). Odd peaks represent compression phase (temperature crests), whereas even peaks represent the rarifaction phase (temperature troughs), inside potential wells. In the isocurvature case, just as in the adiabatic case, the self gravity of the photon baryon fluid essentially drives the oscillations. Unlike the adiabatic case, it is the sine rather than the cosine oscillations that are driven now. Peaks occur at $k = (m + 1/2)k_A$ with all even peaks being enhanced by the baryon drag. More exotic models might produce a phase shift leading to a fluctuation $\cos(ks^* + \phi)$. This would shift the location of the first peak while leaving the spacing between the peaks the same: $k_m - k_{m-1} = k_A$. Thus the sound horizon at last scattering should be measurable from the CMB.

Subtle complications that arise in our CMB anisotropy study can be tackled in the same manner that deals with them in the standard model. For example, in the total variance of temperature fluctuation, it can be seen that the photon density and potential fluctuations would cancel the velocity (Doppler) fluctuations were the sound speed exactly $c_s = 1/\sqrt{3}$. However, for $c_s < 1/\sqrt{3}$, the locations of the peaks for the temperature variance coincides with those of the photon density and potential fluctuations [see eg [37]]. The wave number $k = 1$, in units of the curvature scale, would correspond to a length on the LSS that subtends an angle of $10'$ today. It is straightforward to determine the peak location for the adiabatic and isocurvature perturbations for the primary SW effect. For adiabatic modes, compression peaks occur for odd values of m at angles $\theta_m^{ad} = 120/m\pi$ minutes. For isocurvature modes they occur at even m at $\theta_m^{iso} = 120/(m + \frac{1}{2})\pi$ minutes. Fluctuations would have a decreasing amplitude for smaller angles due to photon diffusion that makes the coupling between the baryon - photon fluid bleed for small scales as it vanishes at $43''$.

All modes corresponding to angles greater than 10 minutes correspond to eigenmodes $0 < k < 1$. These are supercurvature modes. The location of the largest (adiabatic) wavelength peak is $k = \pi/12 \approx 1/4$ corresponding to an angle of $\approx 40'$. As explained in the appendix [39, 38], the eigenfunctions of supercurvature modes are suppressed for open models. For $k = 1/4$ the eigenfunction is suppressed by a factor of the order unity. The relative amplitudes of the k modes is determined by an initial power spectrum that is set by an *ab initio* ansatz. The suppression of the supercurvature mode with $k \approx 1/4$ can be countered by a corresponding change in the initial power spectrum. With the mode amplitude increasing with decreasing k below the curvature scale and the modes suppressed beyond the curvature scale, it is in principle possible to ensure the location of a primary peak at roughly 20 - 25 minutes by suitable choice of the initial power spectrum.

The exact profile of the anisotropy would be determined by the choice of the nature of initial conditions (adiabatic or isocurvature), the chosen initial power spectrum, and the growth of perturbations after z^* (decoupling). These determine the late or the *integrated SW effect*, aspects of reionization etc.

The main point we make in this article is that in spite of a significantly different evolution, the recombination history of a linearly coasting cosmology gives the location of peaks for the primary acoustic peaks in the same range of angles as that given in Standard Cosmology. Given that none of the alternative anisotropy formation scenarios provide a compelling *ab initio* model [41] , it is perhaps best to keep an open mind to all possibilities. As the large scale structure and CMB anisotropy data continue to accumulate, one could explore the general principles for an open coasting cosmology to aid in the empirical reconstruction of a consistent model for structure formation.

Finally, we are tempted to mention that a linear coasting cosmology presents itself as a falsifiable model. It is encouraging to observe its concordance. In standard cosmology, falsifiability has taken the backstage - one just constrains the values of cosmological parameters subjecting the data to Bayesian statistics.

Acknowledgements

The authors thank G. F. R. Ellis and J. Ehlers for clarifying aspects of the averaging problem. Fruitful discussions with T. Saurodeep and S. Engineer are gratefully acknowledged.

Appendix

Subsequent to decoupling, perturbations of the last scattering surface [LSS] and the intervening space, leave an imprint on the streaming microwave background photons observed at the present epoch. To describe the *gross* features of perturbations of the model we start by writing the background line element as

$$ds^2 = {}^{(0)}g_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j =$$

$$= a^2(\eta)(d\eta^2 - \gamma_{ij}dx^i dx^j), \quad (A.1)$$

where η is the conformal time $d\eta \equiv a^{-1}dt$.

$$\gamma_{ij} = \delta_{ij}[1 + \frac{1}{4}K(x^2 + y^2 + z^2)]^{-2}, \quad (A.2)$$

where $K = -1$ for the $\eta =$ constant hypersurface describing an open model's space-like section.

Assuming the perturbations to be described by the perturbed Einstein Equations: $\delta G_{\mu\nu} = \delta T_{\mu\nu}$, the metric can be expanded as usual in terms of the scalar, vector and tensor modes [see eg. [42]]. The gauge invariant scalar perturbation equations are:

$$\nabla^2\Phi - 3H\Phi' - 3(H^2 - K)\Phi = 4\pi G a^2 \delta\epsilon^{gi}; \quad (A.3a)$$

$$(a\Phi)'_{,i} = 4\pi G a^2(\epsilon_o + p_o)\delta u_i^{gi}; \quad (A.3b)$$

$$\Phi'' + 3H\Phi' + (2H' + H^2 - K)\Phi = 4\pi G a^2 \delta p^{gi}. \quad (A.3c)$$

Here, $\nabla^2\Phi \equiv \gamma^{i,j}\Phi_{;i;j}$, is the wave operator for the open model. $H \equiv a'/a$, where $'$ is a derivative with respect to conformal time, and finally the $\delta\epsilon^{gi}$, δu_i^{gi} and δp^{gi} are the gauge invariant density, velocity and pressure parameters respectively [42]. These equations are valid whenever linear perturbation theory is valid. This requires $|\Phi| << 1$ but not necessarily $|\delta\epsilon/\epsilon| << 1$. The above equations combine to give:

$$\Phi'' + 3H(1+c_s^2)\Phi' - c_s^2\nabla^2\Phi + [2H' + (1+3c_s^2)(H^2 - K)]\Phi =$$

$$= 4\pi G a^2 \tau \delta S. \quad (A.4)$$

Here the parameters c_s, τ are determined in terms of the matter, radiation and entropy densities ϵ_m, ϵ_γ, S and are given by:

$$c_s^2 = \frac{1}{3}(1 + \frac{3}{4}\frac{\epsilon_m}{\epsilon_\gamma})^{-1}, \quad \tau = \frac{c_s^2\epsilon_m}{S}. \quad (A.5)$$

Entropy perturbations, δS, also called isocurvature perturbations, can be generated if the different matter components are distributed non-uniformly in space but with uniform total energy density and hence uniform curvature at the beginning.

For a radiation dominated epoch, the evolution of adiabatic perturbations ($\delta S = 0$)is given by putting $c_s \approx 1/\sqrt{3}$ when eqn(A.4) reduces to:

$$\Phi'' + 4\Phi' + \frac{k^2}{3}\Phi + 4\Phi = 0, \quad (A.6)$$

where we define $-k^2$ as the eigenvalue for ∇^2. A straightforward solution to this equation is: $\Phi \longrightarrow t^{-2}\exp(ik\eta/\sqrt{3})$. This form for Φ, together with eqn(A.3a) determine the density perturbations in the radiation dominated epoch provided we have an ansatz for an initial power spectrum. It is also straightforward to solve the potential equations in the matter dominated epoch as well.

In general [see eg [36]] it is convenient to expand cosmological perturbations in a series of eigenfunctions of the Laplacian. Firstly, each mode (each term in the series) evolves independently with time. This makes is easy to evolve a given initial perturbation forward in time. Secondly, by assigning a Gaussian probability distribution to the amplitude of each mode, one can generate a homogeneous Gaussian random field. Such a field consists of an ensemble of possible perturbations. It is supposed that the perturbations seen in the observable universe is a typical member of the ensemble. The stochastic properties of a Gaussian random field are determined by its two point correlation function $< f(1)f(2) >$, where f is the perturbation and the brackets denote the ensemble average. For a homogeneous field, the correlation depends only on the distance between the two points.

For the expansion of perturbations in terms of the Laplacian with eigenvalues $-k^2/a^2$, modes with real $k^2 > 1$ provide a complete orthonormal basis for L^2 functions [40, 39]. They vary appreciably on scales less than the curvature scale a and are called subcurvature modes. A related wave number and a related radial coordinate are defined as:

$$q^2 \equiv k^2 - 1, \quad \chi \equiv \sinh^{-1} r.$$

A typical expansion of the wave mode is:

$$f(\chi, \theta, \phi, t) = \int_0^\infty dq \sum_{lm} f_{klm}(t) Z_{klm}(\chi, \theta, \phi), \quad (A.7)$$

where $Z_{klm} \equiv \Pi_{kl}(\chi)Y_{lm}(\theta, \phi)$, and the radial functions are:

$$\Pi_{kl} = \frac{\Gamma(l + 1 + iq)}{\Gamma(iq)} \frac{1}{\sqrt{\sinh\chi}} P^{-l-1/2}_{iq-1/2}(\cosh\chi) \quad (A.8)$$

normalized as:

$$\int_0^\infty \Pi_{kl}(\chi)\Pi_{k'l'}(\chi)\sinh^2\chi d\chi = \delta(q - q')\delta_{ll'};$$

$$\int Z^*_{klm} Z_{k'l'm'} d\mathcal{V} = \delta(q - q')\delta_{ll'}\delta_{mm'}. \quad (A.9)$$

The constant non-zero phase of Π_{kl} can be dropped by defining the real function:

$$\Pi_{kl} \equiv N_{kl}\hat{\Pi}_{kl};$$

$$\hat{\Pi}_{kl} \equiv q^{-2}(\sinh\chi)^l(\frac{-1}{\sinh\chi}\frac{d}{d\chi})^{l+1}\cos(q\chi);$$

$$N_{kl} \equiv \sqrt{\frac{2}{\pi}} q^2 [\Pi^l_{n=0}(n^2 + q^2)]^{-1/2}. \quad (A.10)$$

The problems with these modes is that they are inadequate to describe perturbations over scales larger than the curvature scale. For this purpose, while considering perturbations in an open universe, one should retain not only the subcurvature modes (defined as eigenfunctions of the Laplacian with eigenvalues less than -1 in units of curvature scale), but also the supercurvature modes whose eigenvalues lie between 0 and -1. All modes must be included to generate the most general homogeneous Gaussian random field even though they may not be linearly independent. The reason for this is the following:

With cosmological perturbations assumed to be Gaussian in the regime of linear evolution, a Gaussian perturbation is defined as one whose probability distribution functions are multivariate Gaussians and its stochastic properties are completely determined by its correlation function. The perturbation turns out to be homogeneous with the correlation function depending only on the distance between the points.

If one merely includes the subcurvature modes, it is easy to deduce the form for the correlation function [39, 40]:

$$\xi_f = \int_1^\infty \frac{dk}{k} P_f(k) \frac{\sin(qr)}{q\ \sinh r}. \qquad (A.11)$$

Setting $r = 0$ gives the mean square value:

$$\xi_f(0) \equiv < f^2 >= \int_1^\infty \frac{dk}{k} P_f(k). \qquad (A.12)$$

Therefore, by expanding a perturbation in terms of subcurvature modes, the correlation is bounded by:

$$\frac{\xi_f(r)}{\xi_f(0)} < \frac{r}{\sinh r} \qquad (A.13)$$

$q \longrightarrow 0$ does not correspond to infinitely large scales, but to scales of the order of the curvature scale.

Thus including only the subcurvature modes generates a Gaussian perturbation whose correlation function necessarily falls off faster than r/sinhr. This reflects the fact that each supercurvature mode varies strongly on a scale bigger than the curvature scale. A random superposition of such modes will hardly ever be nearly constant on a scale much bigger than the curvature scale. This is precisely what the lack of correlation on large scales tells us.

One could consider correlation on arbitrarily large scales by including the super curvature modes. For $-1 < q^2 < 0$ the analytic continuation of the radial function Π_{kl} gives the supercurvature modes:

$$\Pi_{kl} \equiv N_{kl}\hat{\Pi}_{kl};$$

$$\hat{\Pi}_{kl} \equiv |q|^{-2}(\sinh r)^l\left(\frac{-1}{\sinh r}\frac{d}{dr}\right)^{l+1}\cosh(|q|r);$$

$$N_{k0} \equiv \sqrt{\frac{2}{\pi}}|q|;$$

$$N_{kl} \equiv \sqrt{\frac{2}{\pi}}|q|[\Pi_{n=1}^l(n^2+q^2)]^{-1/2}, \qquad (l > 0). \quad (A.14)$$

These supercurvature modes go as $exp[-(1-|q|)r]$ at large r. With the volume element $d\mathcal{V} = \sin h^2 r \sin\theta d\theta dr d\phi$ the integral over all of space of a product of any two of them diverges. The modes are therefore not orthogonal let alone orthonormal. In a finite region of space they are not linearly independent of the subcurvature eigenfunctions. None of this matters for the purpose of generating a Gaussian perturbation. The supercurvature modes add to the expansion (A.7), an additional:

$$f^{SC}(r,\theta,\phi,t) = \\ = \int_0^1 d(iq)\sum_{lm} f_{klm}(t) Z_{klm}(r,\theta,\phi). \quad (A.15)$$

From this, the supercurvature contribution to the correlation function is seen to be [39]:

$$\xi_f^{SC}(r) = \int_0^1 \frac{dk}{k} P_f(k) \frac{\sinh(|q|r)}{|q|\sinh r}. \qquad (A.16)$$

Consider a supercurvature mode corresponding to a peak at $k \approx 1/3$ or $q = 2\sqrt{2}i/3$ in units of curvature scale. For such a mode, the correlation function is suppressed by a factor $\sinh(|q|r)/(|q|\sinh r) \approx 2/3$. This is a suppression by a factor of the order unity and can be compensated by an appropriate initial power spectrum.

The spectrum of initial fluctuations can be characterized by a power law $|\delta_k|^2 = VAk^n$ where n is a spectral index and A is the amplitude at very early epochs. The values of these parameters should emerge from the physical model which describes the the production of the initial spectrum. In the absence of any reliable theoretical prediction for A and n, it is best to treat them as free parameters which can be determined by comparison with observations.

References

[1] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Company, 1990).

[2] G. Borner, *The Early Universe: Facts and Fiction* (Springer Verlag, 1992).

[3] M. Sethi, A. Batra, D. Lohiya, *Phys. Rev* **D60**, 108301 (1999).

[4] A. D. Dolgov in *The Very Early Universe*, eds. G. Gibbons, S. Siklos, S. W. Hawking, C. U. Press (1982); *Phys. Rev.* **D55**, 5881 (1997).

[5] L. H. Ford, *Phys Rev* **D35**, 2339 (1987).

[6] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).

[7] R. E. Allen, *Cosmo-98*, Second Int. Workshop on Particle Physics and Early Universe, eds. D.O. Caldwell (American Institute of Physics, Newyork, 1999), *astro-ph/9902042*.

[8] P. Mannheim & D. Kazanas, *Gen. Rel. & Grav.* **22**, 289 (1990).

[9] G. F. R. Ellis, *Gen. Rel. & Grav.* **32**, 1135 (2000).

[10] J. Ehlers, *Gen. Rel. & Grav.* **25**, 1225 (1993).

[11] J. D. Jackson, *Classical Electrodynamics* (John Wiley, New York, 1975).

[12] T. Buchert, J. Ehlers, *MNRAS* **264**, 375 (1993).

[13] E. A. Milne, *Relativity, Gravitation and World Structure* (Oxford, 1935).

[14] W. Rindler, *Essential Relativity* (Springer Verlag, 1985).

[15] S. Deser, *Gen. Rel. & Grav.* **1**, 9 (1970).

[16] R. Kraichnan, *Ph. D. Thesis, Massachusetts Institute of Technology* (1947); and *Phys. Rev.* **98**, 1118 (1955).

[17] R. P. Feynman, *Chapel Hill Conference* (1956).

[18] S. N. Gupta, *Proceedings of the Physical Society of London* **A65**, 608 (1952).

[19] E. W. Kolb, *ApJ* **344**, 543 (1989).

[20] S. Perlmutter, et al., *ApJ* **517**, 565 (1999).

[21] M. Hamuy, et al., *Astron. J.* **112**, 2391 (1996).

[22] M. Hamuy, et al., *Astron. J.* **109**, 1 (1995).

[23] S. Perlmutter, et al., *Nature* **391**, 51 (1998).

[24] S. Perlmutter, et al., *ApJ* **483**, 565 (1997).

[25] E. Wright, http://www.astro.ucla.edu/wright/cosmology.html

[26] W. L. Freedman, J. R. Mould, R. C. Kennicutt & B. F. Madore, *astro-ph /9801080*.

[27] D. Branch, *Ann. Rev. of A & A* **36**, 17 (1998), *astro-ph/9801065*.

[28] Abha Dev, Meetu Sethi, Daksh Lohiya, *Phys. Lett. B* **504**, 207 (2001).

[29] Abha Dev, M. Safonova, D. Jain & D. Lohiya, *Phys. Lett. B* **548**, 12 (2002).

[30] A. Batra, D. Lohiya, S. Mahajan, A. Mukherjee, *Int. J. Mod. Phys.* **D9**, 757 (2000).

[31] G. Steigman, in *Cosmic Abundances: Proceedings of the 6th Annual Astrophysics Conference in Maryland*, eds. Holt and G. Gonneborn, *astro-ph/9601126*, (1996).

[32] R. I. Epstein, J. M. Lattimer, and D. N. Schramm, *Nature* **263**, 198 (1976).

[33] P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, 1993).

[34] T. Padmanabhan, *Structure Formation in the Universe* (Cambridge University Press, 1993).

[35] S. Seager, D. D. Sasselov, D. Scott, *ApJ* **523**:L1-L5 (1999).

[36] Wayne Hu, *Ph. D. Thesis, Massachusetts Institute of Technology* (1995).

[37] M. Tegmark *ApJ* **514**, L69 (1999).

[38] W. Hu, N. Sugiyama, *ApJ* **471**, 542, (1996).

[39] D. H. Lyth, A. Woszczyna, *Phys. Rev. D* **52**, 3338 (1995).

[40] A. M. Yaglom, in *Proceedings of the Forth Berkeley Symposium Volume II*, edited by J. Neyman (University of California Press, Berkeley , 1961).

[41] W. Hu, N. Sugiyama, *ApJ* **444**, 489 (1995).

[42] V. F. Mukhanov, H. A. Feldman, R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992).

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 181–182

THE RESTUDY ON THE DEBATE BETWEEN EINSTEIN AND LEVI-CIVITA AND THE EXPERIMENTAL TESTS

Fangpei Chen[1]

Department of Physics, Dalian University of Technology, Dalian 116024, China

Received November 12, 2002

After making a thorough investigation on the debate between Einstein and Levi-Civita at 1917-1918 and on the Einstein's doubt, the conservation laws of Lorentz and Levi-Civita is reaffirmed. Some new specific properties of gravitational field or gravitational wave are deduced from these laws. These new specific properties are distinct from the prevalent views, for example: the gravitational field is possessed of only zero or negative energy density; the deflection and the delay of echo pulses for gravitational waves acted by external gravitational field in vacuum do not exist; the background gravitational waves are not similar to the radiations of black-body in spectrum type; the gravitational bremsstrahlung with positive energy is not existent; etc. These specific properties are expounded in detail and by using these specific properties some experiments or observations to test the conservation laws of Lorentz and Levi-Civita are offered.
PACS number: 04.20.Cv , 04.30.-w , 04.80.-y

1. Introduction

The conservation laws of Lorentz and Leve-Civita are one kind of conservation laws of energy-momentum tensor density for gravitational system including matter fields and gravitational field. The main contents of these laws are represented by the following two relations:

$$\mathfrak{T}^{\mu}_{(M)\nu}(x) + \mathfrak{T}^{\mu}_{(G)\nu}(x) = 0; \tag{1}$$

$$\frac{\partial}{\partial x^{\mu}}(\mathfrak{T}^{\mu}_{(M)\nu}(x) + \mathfrak{T}^{\mu}_{(G)\nu}(x)) = 0, \tag{2}$$

which are deduced successively and advocated vigorously by Lorentz and Levi-Civita in 1916–1917 [1]. They define the energy-momentum tensor density for gravitational field by $\mathfrak{T}^{\mu}_{(G)\nu}(x) \stackrel{def}{=} 2\frac{\delta W_G}{\delta g_{\mu\alpha}} g_{\nu\alpha}$.This definition is similar to the definition of energy-momentum tensor density for matter field: $\mathfrak{T}^{\mu}_{(M)\nu}(x) \stackrel{def}{=} 2\frac{\delta W_M}{\delta g_{\mu\alpha}} g_{\nu\alpha}$. In the above definitions, $W_M = \int \mathfrak{L}_M(x) d^4x$, $\delta W_M = \int \frac{\delta W_M}{\delta g_{\mu\nu}} \delta g_{\mu\nu}$; $W_G = \int \mathfrak{L}_G(x) d^4x$, $\delta W_G = \int \frac{\delta W_G}{\delta g_{\mu\nu}} \delta g_{\mu\nu}$; $\mathfrak{L}(x) = \mathfrak{L}_M(x) + \mathfrak{L}_G(x)$ is the Lagrangian density of whole system; $\mathfrak{L}_M(x)$ and $\mathfrak{L}_G(x)$ are the matter field part and the pure gravitational field part of $\mathfrak{L}(x)$ respectively. In general relativity, $\mathfrak{T}^{\mu}_{(G)\nu}(x) = \frac{c^4}{8\pi G}\sqrt{-g}\,(R^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} R)$; so according to Lorentz and Levi-Civita's formulation, the Einstein field equations

$$\sqrt{-g}(R^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} R) = -\frac{8\pi G}{c^4}\mathfrak{T}^{\mu}_{(M)\nu} \tag{3}$$

are interpreted as both field equations and conservation laws. Some people think that $\mathfrak{T}^{\mu}_{(G)\nu}$ is a pure geometric quantity and it can not be used as the definition of energy-momentum tensor density for gravitational field. This view might be incorrect; because the metric tensor $g_{\mu\nu}$ is both geometric quantity and dynamic quantity in the theory of gravitation, so $\mathfrak{T}^{\mu}_{(G)\nu}$ does be also.

Previously, Einstein had proposed another conservation laws for gravitational system in 1914 [1, 2]:

$$\frac{\partial}{\partial x^{\mu}}(\mathfrak{T}^{\mu}_{(M)\nu}(x) + t^{\mu}_{(G)\nu}(x)) = 0, \tag{4}$$

$t^{\mu}_{(G)\nu}(x)$ is a pseudo tensor density, which is used to represent the energy-momentum tensor density for gravitational field by Einstein. Eq. (4) can be derived from the local translation symmetry of the gravitational system [3, 4]. There exist the relations [4]

$$t^{\mu}_{(G)\nu}(x) = 2\frac{\delta W_G}{\delta g_{\mu\alpha}} g_{\alpha\nu} - \frac{\partial}{\partial x^{\sigma}} v^{\mu\sigma}_{(G)\nu};$$

$$\frac{\partial}{\partial x^{\sigma}} v^{\mu\sigma}_{(G)\nu} = -\frac{\partial}{\partial x^{\sigma}} v^{\sigma\mu}_{(G)\nu}, \tag{5}$$

where $v^{\mu\sigma}_{(G)\nu}$ is determined by the Lagrangian density $\mathfrak{L}_G$. It should be reminded that many quantities could be taken as $t^{\mu}_{(G)\nu}$ to satisfy Eq.(4); for if $t'^{\mu}_{(G)\nu}$ satisfies Eq.(4), then $t''^{\mu}_{(G)\nu}$ also satisfies Eq.(4), provided that there exist the relations: $t''^{\mu}_{(G)\nu} - t'^{\mu}_{(G)\nu} = \partial_{\sigma} u^{\mu\sigma}_{\nu}$, and $\partial_{\sigma} u^{\mu\sigma}_{\nu} = -\partial_{\sigma} u^{\sigma\mu}_{\nu}$. ($\partial_{\sigma} = \frac{\partial}{\partial x^{\sigma}}$)

$\mathfrak{T}^{\mu}_{(G)\nu}$ and Eq.(2) has marked advantages as compared with $t^{\mu}_{(G)\nu}$ and Eq. (4): $\mathfrak{T}^{\mu}_{(G)\nu}$ is similar to $\mathfrak{T}^{\mu}_{(M)\nu}$

[1] e-mail: chenfap@dlut.edu.cn

in definition, but $t^{\mu}_{(G)\nu}$ is at odds with $\mathfrak{T}^{\mu}_{(M)\nu}$; only one quantity can be taken as $\mathfrak{T}^{\mu}_{(G)\nu}$ to satisfy Eq.(1) and Eq.(2), but many quantities can be taken as $t^{\mu}_{(G)\nu}$ to satisfy Eq.(4); and as it is emphasized by Levi-Civita, the more important difference is that $\mathfrak{T}^{\mu}_{(G)\nu}$ and Eq.(2) have, but $t^{\mu}_{(G)\nu}$ and Eq.(4) "lack the invariant character it should have in the spirit of general relativity" [1, 5]. Owing this criticism an important debate was evoked about the definitions of energy-momentum tensor density for gravitational field and the related conservation laws in 1917–1918, Einstein is on the one side of that debate, his opponents are Levi-Civita and others [1]. This debate had not reached unanimity; because Einstein had raised the doubt of rationality about Eq.(1) (for detail, see section 2), but Levi-Civita and others could not reply Einstein's doubt with plenty reasons; in addition, due to Einstein enjoyed great prestige among academic circles and many scholars followed him; therefore the propositions of Einstein became gradually the prevalent views now in the gravitational theory.

In the last few years, the author have engaged in studies on the energy-momentum tensor densities and the conservation laws for gravitational systems [4, 6], and have found that the doubt raised by Einstein in the debate of 1917–1918 is not difficult to clarify. It is also found that the conservation laws of Lorentz and Levi-Civita not only are rational perfectly but also have abundant physical contents. Some deductions of these laws are different very much from the prevalent views; these differences can be tested by some experiments or observations. These tests are conducive to understand deeply the specific properties of gravitational field; they are worth to research farther.

This article is confined to study the general relativity, the field variables are still the metric tensor $g_{\mu\nu}$ and the field equations are still the equations of Einstein. Moreover, the connection $\left\{ {\mu \atop \nu\sigma} \right\}$,the Lagrangian density $\mathfrak{L}_M(x)$ or $\mathfrak{L}_G(x)$,and the definition of energy-momentum tensor density for matter field: $\mathfrak{T}^{\mu}_{(M)\nu}$ all are not altered. We only change the definition of energy-momentum tensor density for gravitational field, therefore, except the relations and conclusions associated with the energy-momentum tensor densities for gravitational field, the other relations and conclusions in general relativity remain unchanged. For example, the singularity theorems and Robertson-Walker metrics of the Universe are still tenable in our theory. But the different definitions of energy-momentum tensor density for gravitational field might change the properties of gravitational wave and change the contents of the cosmology. The influences by the conservation law of Lorentz and Levi-Civita on the cosmology require to study specially, the influences on the properties of gravitational wave are studied in this paper.

In order to understand thoroughly the conservation law of Lorentz and Levi-Civita, let us recount the doubt of Einstein firstly.

2. Einstein's doubt and the rational clarification for it.

Although Einstein acknowledged that there is no logical objection against the definition $\mathfrak{T}^{\mu}_{(G)\nu}(x) \stackrel{def}{=} 2\frac{\delta W_G}{\delta g_{\mu\alpha}} g_{\alpha\nu}$ and one is not entitled to define a priori $t^{\mu}_{(G)\nu}$ as a quantity representing the energy-momentum tensor density for gravitational field [1], but Einstein said that Eg.(1) "does not exclude the possibility that a material system disappears completely, leaving no trace of its existence. In fact the total energy is zero (as shown in Eq.(1)) from the beginning; the conservation of this value of the energy does not guarantee the persistence of the system in any form" [1]. So he doubted the rationality of Eg.(1), this is just the so-called Einstein's doubt. Because it is recognized generally that a material system does not disappear completely in physics, so Einstein opposed to choose $\mathfrak{T}^{\mu}_{(G)\nu}$ as the energy-momentum tensor density for gravitation field, thus the conservation laws of Lorentz and Levi-Civita was led up to refusal among many physical scholars.

Will a material system be caused inevitably to disappear completely by Eq.(1)? To clarify this question is crucially important to reaffirm the conservation laws of Lorentz and Levi-Civita. We must point out that, merely using the conservation law of energy-momentum, it is impossible to determine whether a physical change would happen. The physical changes of a real system must yet obey many other laws. For example, the physical changes of a charged system (including gravitational field) must obey also the conservation law of electric charge, i.e. it will continue to preserve the total electric charge unchangeably. If the total electric charge does not equal zero, we may reasonably conclude that this system is definitely not possible to disappear completely.

Another example is concerned in entropy. Suppose a certain macroscopic state of a gravitational system (including matter and gravitational field) has N microscopic states. According to statistical mechanics the entropy of the system in this macroscopic state must obey the Boltzmann's relation $S = k \ln N$ [7]. Generally $N >> 1$, thus $S > 0$ usually. If the above system could disappear completely, then in the disappearing process N will decrease to N =1 gradually; at here we look upon the complete disappearance as a special state. There is no difference in the meaning between macroscopic state and microscopic state for the complete disappearance, so N=1. Therefore in the complete disappearance process of this system, its entropy should decrease to $S = 0$ from $S > 0$, this is contrary to the theorem of entropy increase, consequently a gravitational system can not "disappear completely, leaving no trace of its existence." It should emphasize that $\mathfrak{T}^{\mu}_{(M)\nu}(x)+\mathfrak{T}^{\mu}_{(G)\nu}(x)=0$ do not necessarily give $\mathfrak{T}^{\mu}_{(M)\nu}=0$ and $\mathfrak{T}^{\mu}_{(G)\nu}=0$ at the ending of changes, it only implies $\Delta\mathfrak{T}^{\mu}_{(G)\nu}(x)=-\Delta\mathfrak{T}^{\mu}_{(M)\nu}(x)$. Since Eg.(1) does not lead to "that a material system disappears completely" in reality, thus Einstein's argument to deny the conservation laws of Loventz and Levi-Civita is untenable.

It is easy to see that Eg.(2) is covariant owing to $D_{\mu}\left(\mathfrak{T}^{\mu}_{(M)\nu}(x)+\mathfrak{T}^{\mu}_{(G)\nu}(x)\right)=\frac{\partial}{\partial x^{\mu}}\left(\mathfrak{T}^{\mu}_{(M)\nu}(x)+\mathfrak{T}^{\mu}_{(G)\nu}(x)\right)$ $=0$ (D_{μ} is the symbol of covariant derivative), but Eg.(4) is not covariant; moreover, $\mathfrak{T}^{\mu}_{(G)\nu}$ is tensor density but $t^{\mu}_{(G)\nu}$ is not. Consequently $\mathfrak{T}^{\mu}_{(G)\nu}$ and Eg.(2) are more in line with the spirit of general relativity and they are superior to $t^{\mu}_{(G)\nu}$ and Eq.(4) theoretically.

On the other hand, $\mathfrak{T}^{\mu}_{(G)\nu}$ and Eg.(2) can be looked upon as the special case of $t^{\mu}_{(G)\nu}$ and Eg.(4) respectively; for if $t'^{\mu}_{(G)\nu}$ satisfy Eg.(4) and Eq. (5), when we choose $u^{\mu\sigma}_{\nu}$ such that $t''^{\mu}_{(G)\nu}-t'^{\mu}_{(G)\nu}=\partial_{\sigma}u^{\mu\sigma}_{\nu}=-\partial_{\sigma}u^{\sigma\mu}_{\nu}$, and $\frac{\partial}{\partial x^{\sigma}}\left(v^{\mu\sigma}_{(G)\nu}+u^{\mu\sigma}_{\nu}\right)=0$ then from Eg.(5) we get $t''^{\mu}_{(G)\nu}=2\frac{\delta W_G}{\delta g_{\mu\alpha}}g_{\nu\alpha}=\mathfrak{T}^{\mu}_{(G)\nu}$ and $\frac{\partial}{\partial x^{\mu}}\left(\mathfrak{T}^{\mu}_{(M)\nu}(x)+t''^{\mu}_{(G)\nu}(x)\right)=\frac{\partial}{\partial x^{\mu}}(\mathfrak{T}^{\mu}_{(M)\nu}(x)+\mathfrak{T}^{\mu}_{(G)\nu})=0$. So, if $t^{\mu}_{(G)\nu}$ and Eg.(4) are rational and correct, there is no reason to deny the rationality and the correctness of $\mathfrak{T}^{\mu}_{(G)\nu}$ and Eg.(2).

Therefore the author hold that in order to study the gravitational field deeply and explore it from all sides, it is necessary to reaffirm and to restudy the definition of $\mathfrak{T}^{\mu}_{(G)\nu}$ and the conservation laws of Lorenz and Levi-Civita.

The energy density obtained from $t^{\mu}_{(G)\nu}$ may be positive or negative in theory, but only positive values are taken for this energy density according to the prevalent view; since it is believed that the gravitational field is similar to the electromagnetic field, the energy density of the electromagnetic field is always positive, so some physicists consider that the energy density of the gravitational field is positive also. On the other hand, the energy density obtained from $\mathfrak{T}^{\mu}_{(G)\nu}$ must be negative in theory, this is because that the energy density of a matter field is always positive, then owing Eq.(1) the energy density of gravitational field must be negative. Besides, according to the classical idea of general relativity, in vacuum $\mathfrak{T}^{\mu}_{(M)\nu}(x)=0$, therefore $\mathfrak{T}^{\mu}_{(G)\nu}(x)=0$ also in there. Thus the energy density associated with $\mathfrak{T}^{\mu}_{(G)\nu}$ is zero in vacuum and negative in non-vacuum; but the energy density associated with $t^{\mu}_{(G)\nu}$ is supposed to be positive always in both of vacuum and non-vacuum, provided there exist gravitational field. This difference has important physical deductions for gravitational waves; we shall give a detailed discussion in next section.

People might ask the following question: if the energy density of gravitational field is negative and the energy density of matter field is positive, then the gravitational system would go on the self-accelerating motion [8]; this motion is so preposterous, can you rule it out? The reply is that the above self-accelerating motion does not exist, because that

(1), a self-accelerating system must be composed of two particles with equal and opposite mass which are separated by a distance [8], but Eq. (1) means that the energy density of gravitational field is equal and opposite to the energy density of matter field at every points, so these two fields are not separated by a distance;

(2), the theoretical foundation of self-accelerating motion is established on Newton's theory of gravitation which suppose that any one mass can gravitate any other mass always; but in the general relativity the situation is different, people always interpret the Einstein field equations Eq.(3) as that the energy-momentum tensor density of matter field $\mathfrak{T}^{\mu}_{(M)\nu}(x)$ is the source of gravitational force (or gravitational field), however it is quite evident from Eq.(3) that the energy momentum tensor density of gravitational field $\mathfrak{T}^{\mu}_{(G)\nu}(x)$ should not be considered as the source of gravitational field.

The reasons cited by (2) may be explained more clearly by using the following formulas:

$$\frac{dp^{\mu}_{M}}{d\lambda}+\left\{ {\mu \atop \nu\sigma} \right\} p^{\nu}_{M}\frac{dx^{\sigma}}{d\lambda}=0; \tag{6}$$

$$\frac{dp^{\mu}_{G}}{d\lambda}+\left\{ {\mu \atop \nu\sigma} \right\} p^{\nu}_{G}\frac{dx^{\sigma}}{d\lambda}=0, \tag{7}$$

where

$$p^{\mu}_{M}\overset{def}{=}\frac{1}{c}\int_{v}\mathfrak{T}^{\mu 0}_{(M)}(x)dv \qquad (\mathfrak{T}^{\mu\nu}_{(M)}=g^{\alpha\nu}\mathfrak{T}^{\mu}_{(M)\alpha})$$
$$p^{\mu}_{G}\overset{def}{=}\frac{1}{c}\int_{v}\mathfrak{T}^{\mu 0}_{(G)}(x)dv \qquad (\mathfrak{T}^{\mu\nu}_{(G)}=g^{\alpha\nu}\mathfrak{T}^{\mu}_{(G)\alpha})$$

are the definitions of 4-momentum for the matter field and the gravitational field respectively in the volume v. Eq. (6) and Eq. (7) can be derived according to the Papapetrou's method [9] in single-pole approximation from the covariant conservation laws $\mathfrak{T}^{\mu\nu}_{(M);\nu}=0$ and $\mathfrak{T}^{\mu\nu}_{(G);\nu}=0$. Owing Eq. (1) it is evident that $p^{\mu}_{M}(x)+p^{\mu}_{G}(x)=0$. In Eq. (6) and Eq. (7), $\left\{ {\mu \atop \nu\sigma} \right\} p^{\nu}_{M}\frac{dx^{\sigma}}{d\lambda}$ and $\left\{ {\mu \atop \nu\sigma} \right\} p^{\nu}_{G}\frac{dx^{\sigma}}{d\lambda}$ represent the four dimensional gravitational forces acting on the matter field and the gravitational field in the volume v respectively. The source of these gravitational forces is only the energy-momentum tensor density of matter field in the whole system, since $\left\{ {\mu \atop \nu\sigma} \right\}$ are calculated from $g_{\mu\nu}$, which are determined by $\mathfrak{T}^{\mu}_{(M)\nu}$. Eq. (6) and Eq. (7) tell us that the material of matter field in a volume is gravitated by the material of matter field in other volume, and the material of gravitational field is gravitated by the material of matter field, but the material of matter field is not gravitated by the material of gravitational field; at here "A is gravitated by B" means that B is the source (or part of the source) of gravitational force acting on A. These properties and the reason cited by (1) can assure that the above self-accelerating motion does not exist.

3. Two theories for gravitational waves

In this section we shall discuss two theories for gravitational waves, the one is the theory of Einstein which use the conservation laws of Einstein as the foundation, and the other is the theory of Lorentz and Levi-Civita which use the conservation laws of Lorentz and Levi-Civita as the foundation.

In the theory of Einstein for gravitational waves, owing it is supposed that the energy density of the gravitational field is always positive, so the gravitational wave will carry positive energy in its propagation as the electromagnetic wave does.

But in the theory of Lorentz and Levi-Civita, the gravitational wave will carry only negative energy or zero energy in its propagation; we shall explain this conclusion with the relations:

$$-\frac{\partial}{\partial t}\int_v (\mathfrak{T}^0_{(M)0}(x) + \mathfrak{T}^0_{(G)0}(x))dv =$$
$$= c\oint_S (\mathfrak{T}^i_{(M)0}(x) + \mathfrak{T}^i_{(G)0}(x))dS_i = 0, \qquad (8)$$

which can be derived from Eq. (2) after using Eq.(1). This formula indicates that the total energy flowing out from (or into) the volume v is identically equal to zero, no matter whether the surface S is located wholly in vacuum or not. Two cases will be discussed in the following:

(1), The surface S is located wholly in vacuum.

At the surface S, $\mathfrak{T}^i_{(M)0}(x) = 0$, therefore $\mathfrak{T}^i_{(G)0}(x) = 0$ from Eq.(1); consequently there is always $\oint_S \mathfrak{T}^i_{(G)0}(x)dS_i = 0$. It indicates that no gravitational energy flows across the surface S even though there exists gravitational wave. This result means that the gravitational wave does not carry energy whenever it propagates through vacuum. When the gravitational wave arrives, the metric tensor $g_{\mu\nu}(x)$ and the Ricci curvature tensor $R^{\mu}_{\nu}(x)$ should take place change even in vacuum, for example, at a certain point $\{x^i\}$ in the three dimension space, the metric tensor changes from $g_{\mu\nu}\ (x^i,t)$ to $g'_{\mu\nu}(x^i,t')$ and the Ricci curvature tensor changes from $R^{\mu}_{\nu}(x^i,t)$ to $R'^{\mu}_{\nu}(x^i,t')$. Why does not the gravitational wave carry energy in vacuum?

This is because that there are $R^{\mu}_{\nu} - \frac{1}{2}g^{\mu}_{\nu}R = 0$ and $R'^{\mu}_{\nu} - \frac{1}{2}g'^{\mu}_{\nu}R' = 0$ always in vacuum, so the energy-momentum tensor density of gravitational field retains the zero value persistently.

(2), Part of the surface S is not located in vacuum but there is energy flux (or particle flux) of matter field across the surface

In these case $\oint_S \mathfrak{T}^i_{(G)0}(x)dS_i = -\oint_S \mathfrak{T}^i_{(M)0}(x)dS_i \neq 0$, it indicates that the gravitational wave does carry negative energy (because the energy of matter field is positive always) in its propagation.

The above different results about the energy carried by gravitational wave can be tested by experiments and observations; we shall give a detailed discussion in section 5.

It is well known that the quantum theory of electromagnetic field has been well established; the quantum of electromagnetic field is called photon. The four dimensional momentum of photon satisfies the relation

$$p^{\mu} = \hbar k^{\mu}, \qquad (9)$$

k^{μ} is the four dimensional wave vector. This relation can be derived from the theory of quantum field[10]. The propagation of electromagnetic wave can be interpreted as the propagation of a bundle of real photons and each photon carries positive energy $\varepsilon = cp^0 = \hbar\omega = h\nu$, where ν is the frequency of wave.

The theory of Einstein considers that gravitational field and gravitational wave are similar to electromagnetic field and electromagnetic wave on many sides; corresponding to the photon, the graviton is considered to be the quantum of gravitational field, and satisfies Eq. (9) also. The propagation of gravitational wave can be also interpreted as the propagation of a bundle of real gravitons and every graviton carries positive energy $\varepsilon = h\nu$ [11]. But on account of a complete and consistent quantum theory of gravitational field has not been constructed yet till now, these similarities and the relation Eq. (9) for gravitational field are merely suppositions. The above suppositions perhaps could be consistent with the theory of Einstein for gravitational wave without leading obvious trouble, but they cannot be consistent with the theory of Lorentz and Levi-Civita and shall lead theoretical contradictions. As having explained before, in the latter theory the gravitational wave carries negative or zero energy in its propagation, this means that if graviton exists, its energy must be negative or zero, these conclusions is not fit to Eq. (9) which means ν and $\varepsilon = h\nu$ are always positive. Besides, $\nu \neq 0$ for gravitational wave even in vacuum. Thus if there exist graviton, before the perfect quantum theory of gravitational field being constructed, we can only affirm that its energy must be negative or zero in the theory of Lorentz and Levi-Civita. Moreover, in this theory there is no gravitational force existed between any two gravitons because that $\mathfrak{T}^{\mu}_{(G)\nu}(x)$ do not be the source of gravitational force. We shall use these properties of graviton in section 5.

The observation of the orbital period decay rate for the binary pulsar PSR1913+16 has been widely interpreted as the verification for energy radiation of gravitational wave [12], and this verification is regarded as the affirmation to the theory of Einstein for gravitational wave. The theoretical basis of this verification is

$$-\frac{\partial}{\partial t}\int_v (\mathfrak{T}^0_{(M)0}(x)+t^0_{(G)0}(x))dv =$$

$$= c\oint_S (\mathfrak{T}^i_{(M)0}(x)+t^i_{(G)0}(x))dS_i, \qquad (10)$$

which is derived from Eq. (4). The energy density of PSR1913+16 can be divided into two parts: $\mathfrak{T}^0_{(M)0}(x) = obit\mathfrak{T}^0_{(M)0}(x) + oth\mathfrak{T}^0_{(M)0}(x)$; $obit\mathfrak{T}^0_{(M)0}(x)$ is the density of the orbital energy which includes the kinetic energy and gravitational potential energy (see section 4), $oth\mathfrak{T}^0_{(M)0}(x)$ is the density of the other energy for the matter.

If the surface S surrounded the volume v is located in vacuum and the sum of $\int_v\ oth\mathfrak{T}^0_{(M)0}(x)dv$ and $\int_v t^o_{(G)0}(x)dv$ can be neglected, then Eq. (10) is reduced to

$$-\frac{\partial}{\partial t}\int_v obit\mathfrak{T}^0_{(M)0}dv = c\oint_S t^i_{(G)0}dv, \qquad (11)$$

which is interpreted that the radiation energy of gravitational wave is transformed from the decreasing orbital energy of PSR1913+16. The orbital period decay rate can be calculated by using Eq. (11), the coincidence between the calculation and the observation for orbital period decay rate is looked upon as the above verification. Because Eq. (11) is only an approximate relation and the coincidence between the calculation and the observation is bad for the another binary pulsar PSR1534+12, so it makes us doubt. On the other hand, the orbital energy decrease of PSR1913+16 can be also explained with the theory of Lorentz and Levi-Civita. From Eq. (1) we can get

$$-\Delta obit\mathfrak{T}^0_{(M)0}(x) = \Delta oth\mathfrak{T}^0_{(M)0}(x) + \Delta\mathfrak{T}^0_{(G)0}(x) \quad (12)$$

Eq. (12) might be interpreted as while the orbital energy decrease the other matter energy and the pure gravitational field energy increase correspondingly. Moreover, there exists the relation Eq. (5), the orbital period decay rate calculated by using Eq. (8) is equivalent to that calculated by using Eq. (10). Therefore the author believes that the theory of Einstein for gravitational wave is not yet affirmed [13].

4. The energy transformations in the mechanical resonant antennas for detecting gravitational waves

The methods used to probe gravitational waves in the theory of Lorentz and Levi-Civita are just the same as that in the theory of Einstein, the mechanical resonant antennas and the laser interferometers all can be used for detection. This is because that the basic principle of these methods are all founded on the equation of geodesic deviation, but this equation has no relations with how energy-momentum tensor density is defined and whether gravitational wave carries energy and what sign of this energy is.

Peoples might ask that if the gravitational wave does carry negative energy or no energy, then what is the originate of the positive vibration energy in the mechanical resonant antennas after receiving gravitational wave? Before replying this question we shall ask another question: when the mechanical resonant antenna receives gravitational wave, whether or not its vibration energy would be transferred wholly from the wave? We shall give a minute analysis in the following.

Weber had used a simplified model to deduce the four dimensional equations of motion [14] for an arbitrary small part with mass m in the mechanical resonant antenna:

$$m\frac{d^2x^\mu}{ds^2} + m\left\{{\mu \atop \alpha\beta}\right\}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = \frac{F^\mu}{c^2}. \qquad (13)$$

In Eq. (13) the right hand represents the four dimensional forces exerted on m by the other parts of the antenna, the left hand can be obtained by the variation of the action $I = mc\int ds$[15]. It is well known in the theories of gravitation that the total action W_T can be decomposed into two parts: $W_T = W_M + W_G$, W_M is called the part of matter field and W_G is called the part of pure gravitational field, the interaction between matter field and gravitational field is included in W_M[16]. In general relativity, the action of pure gravitational field is always taken to be $W_G = \frac{c^4}{4\pi G}\int\sqrt{-g}Rd^4x$, the action I belongs to W_M. The gravitational potential energy with m being connected with the second item of left hand in Eq. (13) and being decided by I, it should be included in the energy of matter field accordingly.

When gravitational wave reaches, the energy of the pure gravitational field in antenna will change along with the change of the metric tensor $g_{\mu\nu}(x)$ of gravitational field. At the same time, owing to Eq. (1) or Eq. (2) the energy of the matter field must also change. The energy of matter field includes the gravitational potential energy, the vibration energy of the antenna, the thermal motion energy of the molecules in the antenna, etc. In the following we shall use (1),(2), (3)... as the top symbols to represent successively the gravitational potential energy, the vibration energy of the antenna, the thermal motion energy of the molecules in the antenna, etc. Now let us conduct a detailed discussion according the laws of Lorentz and Levi-Civita and the laws of Einstein respectively.

In the theory of Lorentz and Levi-Civita for gravitational wave, the following relation: $\Delta \overset{(1)}{\mathfrak{T}^0}_{(M)0}(x) +$

$\Delta \overset{(2)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta \overset{(3)}{\mathfrak{T}^0}_{(M)0}(x) + ... + \Delta\mathfrak{T}^0_{(G)0}(x) = 0$ can be obtained from Eq. (1), where $\Delta\mathfrak{T}^0_0(x) = \Delta\mathfrak{T}^0_0(x^i,t) = \mathfrak{T}^0_0(x^i,t+\Delta t) - \mathfrak{T}^0_0(x^i,t)$ represents the increment of energy density at 3-dimensional space point $\{x^i\}$ in time interval Δt. From the above relation we get

$$\Delta \overset{(2)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta \overset{(3)}{\mathfrak{T}^0}_{(M)0}(x) = = -\left(\Delta \overset{(1)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta\mathfrak{T}^0_{(G)0}(x) + ...\right). \quad (14)$$

Although the gravitational wave carries negative energy or no energy, Eq. (14) indicates that $\Delta \overset{(2)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta \overset{(3)}{\mathfrak{T}^0}_{(M)0}(x)$ might have positive value if $\left(\Delta \overset{(1)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta\mathfrak{T}^0_{(G)0}(x) + ...\right)$ is decreased. This analysis shows the possibility that the vibration (and the thermal motion) energy might be transformed from the gravitational potential energy and the energy of pure gravitational field inside the antenna. The cause of this possibility is that when gravitational wave arrives, through the change of $g_{\mu\nu}$(x) and the influences of Einstein field equations and other physics laws, a series of energy changes shall happen inside the antenna.

In the theory of Einstein for gravitational wave, $\frac{\partial}{\partial t}(\Delta \overset{(1)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta \overset{(2)}{\mathfrak{T}^0}_{(M)0}(x) + \Delta \overset{(3)}{\mathfrak{T}^0}_{(M)0}(x) + ... + \Delta t^0_{(G)0}(x)) = -c\frac{\partial}{\partial x^i}(\mathfrak{T}^i_{(M)0}(x) + t^i_{(G)0}(x))$ can be obtained from Eq. (4); after some calculations we get

$$\Delta \int_v \overset{(2)}{\mathfrak{T}^0}_{(M)0}(x)dv + \Delta \int_v \overset{(3)}{\mathfrak{T}^0}_{(M)0}(x)dv = = -\left(\Delta \int_v \overset{(1)}{\mathfrak{T}^0}_{(M)0}(x)dv + \Delta \int_v t^0_{(G)0}(x)dv + ... +c\Delta t \oint_S t^i_{(G)0}(x)dS_i\right). \quad (15)$$

In Eq. (15), $\Delta t = t' - t$ represents the time interval to receive gravitational wave, the other Δ represent the symbol of increments for some integrals in Δt; we have supposed that there is vacuum outside the antenna and the surface S in Eq. (15) is wholly located at vacuum. Eq. (15) indicates that the increments of the vibration energy and the molecules thermal motion energy are not only transferred from the gravitational wave, but also transformed from the gravitational potential energy and the energy of the pure gravitational field inside the antenna, etc. Therefore, even in the theory of Einstein, the increments of the vibration energy and the molecules thermal motion energy for the antenna do not be transferred wholly from the gravitational wave. If $\mathfrak{T}^0_{(G)0}(x)$, $t^0_{(G)0}(x)$ and $\oint_S t^i_{(G)0}(x)$ could be detected directly and independently, it is possible to determine which theory for gravitational wave is more correct through the comparison between Eq. (14) and Eq. (15). However there are not direct detections and comparisons yet now. The experimental tests and the comparisons between the theory of Einstein and the theory of Lorentz and Levi-Civita are via some indirect methods, which will be discussed in next section.

5. Experimental tests

Using the specific properties originated from different signs of energy carried with gravitational wave in the theory of Einstein and the theory of Lorentz and Levi-Civita, it is possible to design observations and experiments to test the above two theory and to determine which is correct. In this section we shall discuss

5.1. Observation of deflection and delay of echo pulses for gravitational waves acted by outer gravitational field in vacuum

The observation of deflection for light and delay of echo pulses for radar in gravitational field had played an important role for the experimental test of general relativity [11]. The formula of calculation for these two phenomena is $\frac{d^2x^\mu}{d\lambda^2} + \left\{ {\mu \atop \nu\sigma} \right\} \frac{dx^\nu}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0$, which is deduced from Eq. (6) by using the relation $p^\mu_{(M)} = \frac{h\nu}{c^2}\frac{dx^\mu}{d\lambda}$. Eq. (6) stems from the change of momentum when the photon is acted by gravitational force. It should point out that the deflection and the delay of echo pulses which can be observed for light waves occur always in vacuum, hence for comparison, we shall discuss these phenomena for gravitational wave in vacuum also.

If the momentum of graviton (or gravitational wave packet) changes also when it is acted by external gravitational field, then deflection and delay of echo pulses should also exist for gravitational wave. However, in the theory of Lorentz and Levi-Civita for gravitational wave, the four dimensional momentum P^μ_G of graviton (or gravitational wave packet) in vacuum is always equal to zero and does not change even it is acted by external gravitational field, therefore, for gravitational wave, the deflection and the delay of echo pulses shall not be observed in vacuum.

Some methods might be designed to observe the above two phenomena for gravitational waves acted by outer gravitational field (for example, gravitational field of the sun or other star) in vacuum. If it is determined that these two phenomena do not exist, then it would mean the theory of Lorentz and Levi-Civita might be correct; if these two phenomena do exist then it would mean the theory of Einstein might be correct.

5.2. The tests for excitations of atom or molecule acted by gravitational wave

It is well known that photon can excite atom or molecule. If graviton carries with positive energy, it will excite atom or molecule also; but if graviton carries with negative energy or no energy, it does not excite atom or molecule. This is because that to excite a body means that some energy in positive value has been transferred to this body and its energy state is raised. Therefore, in the theory of Einstein, the excitation of atom of molecule might exist; but in the theory of Lorentz and Levi-Civita, this phenomenon does not exist.

But on the other hand, in the theory of Lorentz and Levi-Civita, when a bundle of gravitons propagate, owing Eq. (1) some matter particles shall propagate together with the graviton along the same direction and carry positive energy. Atoms or molecules could be excited by these companying matter particles. However this excitation is still different from the excitation caused by photon. The excitation caused by photon is related to the frequency of electromagnetic wave owing to the quantum relation $\varepsilon = h\nu$. But in the theory of Lorentz and Levi-Civita,the excitation caused by the companying matter particles have no relation with the frequency of gravitational wave; since the frequency of gravitational wave is independently determined by the change rate of the metric tensor $g_{\mu\nu}(x)$ at the location of wave packet and does not relate to the energy carried with graviton or companying matter particle.

Therefore if we design some experiments to observe the states of atoms or molecules excited by gravitational wave, after analyzing the results it could be determined which of the above two theories for gravitational wave is more correct.

5.3. The comparison between the spectrum type of background gravitational waves and the spectrum type of black body radiations

According to the prevalent view it is believed that there exists background gravitational waves [17], which are similar to the cosmic micro waves background radiation and have also the spectrum type of black body radiations.

The similarity between the spectrum type of background gravitational waves and the spectrum type of black body radiations requires that the graviton have a positive quantum energy and it might exchange this quantum energy with matter particles. As being explained before, the theory of Einstein for gravitational wave satisfies this demand, but the theory of Lorentz and Levi-Civita is sharply contradictory with this demand. Hence the former theory affirms that the background gravitational waves have the spectrum type of black body radiation, but the latter theory does not.

Consequently through probing the background gravitational waves left over from the early epochs of our universe and observing its spectrum type; it might provide an experimental test to decide which is the correct theory for gravitational wave.

5.4. Probing gravitational bremsstrahlung of the sun

It is well known that photons can be produced by collisions between material particles; for example, the metal target may radiate x-ray photons as the nucleus of the metal target come into collision with a bundle of electrons projected at the target. (Actually, the so-called "collision" is the phenomenon of energy-momentum change owing to the Coulomb force acting between electron and nucleus) This radiation is called bremsstrahlung. The energy of bremsstrahlung is transformed from the decreasing energy at collisions between particles.

According to the prevalent view, it is believed that there also exist gravitational bremsstrahlung which is similar to light bremsstrahlung, i.e. besides photon, graviton with positive energy can be also produced by collisions between material particles. The microscopic particles in the sun impact each other unceasingly owing their thermal motion, hence the sun should have gravitational bremsstrahlung [18]. The emission power of gravitational bremsstrahlung from the sun has been calculated to be about 6×10^{14} erg/sec—5×10^{15} erg/sec [18] according to the theory of Einstein for gravitational wave; these values are greatly exceeding the sun's gravitational radiations aroused by other causes. The energy fluxes at the surface of the earth from the sun's gravitational bremsstrahlung are about 2×10^{-13} erg/cm^{2}sec—1.7×10^{-12} erg/cm^{2}sec. These energy fluxes approximately correspond to those energy fluxes at the surface of the earth transmitted from the gravitational waves, which are estimated to take place in the compact binaries of Virgo cluster or Vela pulsar. To probe the gravitational waves from these stars are the striving direction of some gravitational wave observatories whose detectors are being tried to improve sensitivities.

According to the theory of Lorentz and Levi-Civita for gravitational wave, the existence of gravitational bremsstrahlung composed of graviton with positive energy is not possible. Though there could produce gravitons (or gravitational waves) when material particles impact each other, but these gravitons carry negative energy; and owing Eq. (1) they are the companions of the bremsstrahlung photons produced in the collisions of material particles. Hence in the theory of Lorentz and Levi-Civita, the gravitational bremsstrahlung of the sun described above does not exist. This difference provides another experimental test.

The gravitational wave is not observed yet. If gravitational wave could be observed and could be emitted artificially, besides if the detectors are improved to have sufficient sensitivities, the above experimental tests are not too difficult to realize. They will determine which is the correct theory for gravitational wave; they will also judge which is the correct definition of energy-momentum tensor density for gravitational field and which is the correct formulation of conservation laws for gravitational system. What are the results? We shall wait and see.

References

[1] C. Cattani, M. De Maria, "Conservation laws and gravitational waves in general relativity." In: The Attraction of Gravitation, New studies in the history of general relativity. edited by Earman J, Janssen M and Norton J D. Boston: Birkhauser, 63, 1993.

[2] A. Einstein, "Die formale Grundlage der allgemeinen Relativitätstheorie." Königlich Preussische Akademie der Wissenschaften (Berlin). Sitzungsberichte, 1030, 1914.

[3] J.N. Goldberg, "Conservation laws in general relativity." *Phy. Rev.* **111**, 315 (1958).

[4] F.P. Chen, "Restudy in energy-momentum tensor density for gravitational field in general relativity." *Journal of Dalian University of Technology*, **37**, 33 (1997) (in Chinese).

[5] T. Levi-Civita, *Rendiconti accademia dei Lincei.* Ser. 5, **26**, 381 (1917).

[6] F.P. Chen, "The historic debate in the definition for energy-momentum tensor of gravitational field and its new study." *Journal of Hebei Normal University*, **24**, 326 (2000) (in Chinese).

[7] M. Planck, "Theory of Heat." Printed in Great Britain by Richard Glay & Sons, Limited, 227, 1932.

[8] W.B. Bonnor, "Negative mass in general relativity." *Gen. Rel. Grav.* **21**, 1143 (1989).

[9] A. Papapetrou, "Spining test particles in general relativity I." *Proc. Roy. Soc. A*, **209**, 248 (1951).

[10] D. Lurie, "Particles and Fields." London: John Wiley & Sons, 1968.

[11] S. Weinberg, "Gravitation and Cosmology." New York: John Wiley & Sons, 1972.

[12] J.H. Taylor, "Measurements of general relativistic effects in the binary pulsar PSR1913+16." *Nature*, **277**, 437 (1979).

[13] F.P. Chen, "Investigation on problem of coincidence between the observed and the predicted value of orbital period decay rate $\dot{P}_b$ for PSR 1534+12." *Ziran Zazhi (Journal of Nature)*, **20**, 178 (1998) (in Chinese).

[14] J. Weber, "General Relativity and Gravitational Waves." London: Interscience Publishers Ltd, 1961.

[15] L. Landau andE. Lifshitz, "The Classical Theory of Fields." Translated by Hamermesh M. Oxford: Pergamon Press, 1975.

[16] R. Utiyama, "Invariant theoretical interpretation of interaction." *Phy. Rev.* **101**, 1597 (1956).

[17] A.D. Jeffries, "Gravitational Wave Observatories." Scientific American, **256**(6), 50 (1987).

[18] G. Papini, S.R. Valluri, "Gravitons in Minkowski Space-time, interactions and results of astrophysical interest." *Phys. Rep.* **33C**, 51 (1977).

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 169–173

CONFORMALLY FLAT SPHERICALLY SYMMETRIC COSMOLOGICAL MODELS-REVISITED

Anirudh Pradhan[1]† and Om Prakash Pandey‡

† *Department of Mathematics, Hindu Post-graduate College (V. B. S. Purvanchal University), Zamania-232 331, Ghazipur, India*

‡ *Department of Mathematics, Hindu Post-graduate College (V. B. S. Purvanchal University), Zamania-232 331, Ghazipur, India*

Received November 1, 2002

A conformally flat spherically symmetric cosmological models representing a charged perfect fluid as well as a bulk viscous fluid distribution have been obtained. The cosmological constant Λ is found positive and is decreasing function of time which is consistent with the recent supernovae observations. The physical and geometrical properties of the models are also discussed.

1. Introduction

A considerable interest has been shown to the study of physical properties of spacetimes which are conformal to certain well known gravitational fields. The general theory of relativity is believed by a number of unknown functions - the ten components of g^{ij}. Hence there is a little hope of finding physically interesting results without making reduction in their number. In conformally flat spacetime the number of unknown functions is reduced to one. The conformally flat metrices are of particular interest in view of their degeneracy in the contex of Petrov classification. A number of conformally flat physically significant spacetimes are known like Schwarzschild interior solution and Lemaître cosmological universe.

At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is isotropic and homogeneous. Buchdahl [1] has obtained the conformal flatness of the Schwarzschild interior solution. Singh and Roy [2] have discussed the possibility of existence of electromagnetic fields conformal to some empty spacetime. Singh and Abdussattar [3] have obtained a non-static generalization of Schwarzschild interior solution which is conformal to flat spacetime. Roy and Bali [4] have obtained the solution of Einstein's field equations representing non-static spherically symmetric perfect fluid distribution which is conformally flat. Pandey and Tiwari [5] have discussed conformally flat spherically symmetric charged perfect fluid distribution. Reddy [6] and Rao and Reddy [7] discussed static conformally flat solutions in the Brans-Dicke and Nordtvedt-Barker scalar-tensor theories. Shanthi [8] has shown that the most general conformally flat static vacuum solution in the Nordtvedt-Barker scalar-tensor theory is simply the empty flat spacetime of general relativity. There has been a recent literature (Melfo and Rago [9], Mannhelm [10], Yadav and Prasad [11], Endean [12,13], Obukhov et al. [14], Mark and Harko [15]) which shows a significant interest in the study of conformally flat spacetime.

Most cosmological models assume that the matter in the universe can be described by 'dust'(a pressureless distribution) or at best a perfect fluid. Nevertheless, there is good reason to believe that - at least at the early stages of the universe - viscous effects do play a role [17]-[19]. For example, the existence of the bulk viscosity is equivalent to slow process of restoring equilibrium states [20]. The observed physical phenomena such as the large entropy per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [32]) for a review on cosmological models with bulk viscosity). The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity [21]-[31].

The purpose of this paper is to apply conformally flat spherically symmetric line element to charged perfect fluid and to bulk viscous fluid models in cosmology. This paper is organized as follows. The field equations are presented in Section 2. Section 3 includes the solu-

[1]e-mail: acpradhan@yahoo.com, pradhan@iucaa.ernet.in

tion of the field equations in presence of charged perfect fluid distribution. Section 3.1 contains some physical properties of the model. Finally in Section 4 the bulk viscous models are considered.

2. Field Equations

We consider the conformal metric in spherical polar coordinates

$$ds^2 = e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 - dt^2), \tag{1}$$

where λ is a function of r and t. We number the coordinates as $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$ and $x^4 = t$.

The energy momentum tensor for distribution of a charged perfect fluid has the form

$$T_{ij} = (\epsilon + p)v_i v_j + p g_{ij} + E_{ij}, \tag{2}$$

where E_{ij} is the electromagnetic field given by

$$E_{ij} = \frac{1}{4\pi}\left[F_{ai}F_{bj}g^{ab} - \frac{1}{2}g_{ij}F_{ab}F^{ab}\right]. \tag{3}$$

Here ϵ and p are the energy density and isotropic pressure respectively and v^i is the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1. \tag{4}$$

The electromagnetic field tensor F_{ij} satisfies Maxwell's equations

$$F^{ij}_{;j} = 4\pi\rho v^i, \tag{5}$$

$$F_{[ij;k]} = 0, \tag{6}$$

ρ being the current density. Here and henceforth a comma and a semicolon denote ordinary and covariant differentiation respectively. The Einstein field equations

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = -8\pi T_{ij}, \tag{7}$$

for the line element (1) has been set up as

$$8\pi[(\epsilon + p)v_1^2 + pe^{\lambda}] = \frac{3\lambda_1^2}{4} + \frac{2\lambda_1}{r} - \lambda_{44} - \frac{\lambda_4^2}{4}$$

$$+e^{-\lambda}(F_{14})^2 + \Lambda e^{\lambda}, \tag{8}$$

$$8\pi pe^{\lambda} = \lambda_{11} + \frac{\lambda_1^2}{4} + \frac{\lambda_1}{r} - \lambda_{44} - \frac{\lambda_4^2}{4} - e^{-\lambda}(F_{14})^2 + \Lambda e^{\lambda}, \tag{9}$$

$$8\pi[(\epsilon + p)v_4^2 - pe^{\lambda}] = \frac{3\lambda_4^2}{4} - \lambda_{11} - \frac{\lambda_1^2}{4} - \frac{2\lambda_1}{r}$$

$$-e^{-\lambda}(F_{14})^2 - \Lambda e^{\lambda}, \tag{10}$$

$$8\pi(\epsilon + p)v_1 v_4 = \frac{\lambda_1\lambda_4}{2} - \lambda_{14}. \tag{11}$$

Equation (4) gives

$$v_4^2 - v_1^2 = e^{\lambda}. \tag{12}$$

3. Solutions of the field equations

From eqs. (8) and (9) we have

$$8\pi[(\epsilon + p)v_1^2] - 2e^{-\lambda}(F_{14})^2 = \frac{\lambda_1^2}{2} + \frac{\lambda_1}{r} - \lambda_{11}. \tag{13}$$

Also eqs. (9) and (10) readily give

$$8\pi[(\epsilon + p)v_4^2] + 2e^{-\lambda}(F_{14})^2 = \frac{\lambda_4^2}{2} - \frac{\lambda_1}{r} - \lambda_{44}. \tag{14}$$

Combining eqs. (12), (13) and (14) we obtain

$$8\pi[(\epsilon + p)e^{\lambda}] + 4e^{-\lambda}(F_{14})^2 = \frac{\lambda_4^2}{2} - \frac{\lambda_1^2}{2} - \frac{2\lambda_1}{r}$$

$$-\lambda_{44} + \lambda_{11}. \tag{15}$$

Equations (9) and (15) together reduce to

$$8\pi\epsilon e^{\lambda} + 3e^{-\lambda}(F_{14})^2 = \frac{3}{4}\left(\lambda_4^2 - \lambda_1^2 - \frac{4\lambda_1}{r}\right). \tag{16}$$

In comoving coordinate system $v_1 = 0$, then eq. (13) reduces to

$$-e^{-\lambda}(F_{14})^2 = \frac{\lambda_1^2}{4} + \frac{\lambda_1}{2r} - \frac{\lambda_{11}}{2}. \tag{17}$$

From eq.(11) we obtain

$$2\lambda_{14} - \lambda_1\lambda_4 = 0. \tag{18}$$

The general solution of (18) is obtained as

$$e^{\lambda} = [\alpha(r) + \beta(t)]^{-2}, \tag{19}$$

where α and β are functions of r and t respectively.

Hence the geometry of the spacetime (1) reduces to the form

$$ds^2 = \frac{1}{(\alpha + \beta)^2}\left(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 - dt^2\right), \tag{20}$$

which is the model for a distribution of charged perfect fluid with the flow vector in t-direction. The pressure and density for the model (20) are given by

$$8\pi p - \Lambda = 3(\alpha_1^2 - \beta_4^2) + (\alpha + \beta)\left(2\beta_{44} - \alpha_{11} - \frac{3\alpha_1}{r}\right), \tag{21}$$

$$8\pi\epsilon + \Lambda = 3(\beta_4^2 - \alpha_1^2) + 3(\alpha + \beta)\left(\alpha_{11} + \frac{\alpha_1}{r}\right). \tag{22}$$

Let us assume that the fluid obeys an equation of state of the form

$$p = \gamma\epsilon, \tag{23}$$

where $\gamma(0 \le \gamma \le 1)$ is constant. Using eq. (23) in eqs. (21) and eq. (22), we get

$$\epsilon = \frac{(\alpha + \beta)}{8\pi(1 + \gamma)}(\beta_{44} + \alpha_{11}), \tag{24}$$

$$\Lambda = -\frac{(1-\gamma)}{(1+\gamma)}(\alpha+\beta)(\beta_{44}+\alpha_{11}) - 3(\alpha_1^2-\beta_4^2)$$

$$+(\alpha+\beta)(\beta_{44}-2\alpha_{11}). \tag{25}$$

If we put $\Lambda = 0$ in our solution, we recover the solution obtained by Pandey and Tiwari[5].

Particular cases:

Case (i):

If we consider $\beta(t) = \frac{a}{t^2}; \alpha, a > 0$, eqs. (24) and (25) reduce to

$$\epsilon = k_1\alpha t^{-4} + k_1 a t^{-6}, \tag{26}$$

$$\Lambda = k_2\alpha t^{-4} + a(k_2+12a)t^{-6}, \tag{27}$$

where $k_1 = \frac{3a}{4\pi(1+\gamma)}, k_2 = \frac{12a^2\gamma}{(1+\gamma)}$.

Case (ii):

If we consider $\beta(t) = \frac{a}{t}; \alpha, a > 0$, eqs. (24) and (25) reduce to

$$\epsilon = \alpha k_3 t^{-3} + a k_3 t^{-4}, \tag{28}$$

$$\Lambda = \alpha k_3 t^{-3} + a(3a+k_4)t^{-4}, \tag{29}$$

where $k_3 = \frac{a}{4\pi(1+\gamma)}, k_4 = \frac{2a\gamma}{(1+\gamma)}$. It is observed from eqs. (27) and (29) that the cosmological constant Λ, in both cases, is a decaying function of time and it approaches a small value as time progresses (i.e., the present epoch), which explains the small value of Λ at present. Additionally, Λ also comes out positive which is consistent with the recent supernovae observations (Perlmutter et al. [33], Riess et al. [34], Garnavich et al. [35], Schmidt et al. [36]).

3.1. Physical properties of the model

The non-vanishing component of the flow vector, v^4 is given by

$$v_4 = \frac{1}{(\alpha+\beta)}. \tag{30}$$

The reality conditions $(\epsilon+p) > 0$ and $(\epsilon+3p) > 0$ lead to

$$\beta_{44} + \alpha_{11} + \frac{\alpha_1}{r} > 0, \tag{31}$$

and

$$(\alpha_1^2 - \beta_4^2) + (\alpha+\beta)\left(\beta_{44} - \frac{\alpha_1}{r}\right) > 0. \tag{32}$$

Using eq. (19) in eq. (17) gives

$$F_{14} = \frac{\left(\frac{\alpha_1}{r} - \alpha_{11}\right)^{\frac{1}{2}}}{(\alpha+\beta)^{\frac{3}{2}}}. \tag{33}$$

From eqs. (5) and (33) the current density ρ is given by

$$\rho = -\frac{(\alpha+\beta)^3}{r^2}\frac{\partial}{\partial r}\left[\frac{r^2\left(\frac{\alpha_1}{r} - \alpha_{11}\right)^{\frac{1}{2}}}{(\alpha+\beta)^{\frac{3}{2}}}\right]. \tag{34}$$

The non-vanishing component of the acceleration vector

$$\dot{v}_1 = v_{i;j}v^j, \tag{35}$$

is given by

$$\dot{v}_1 = -\frac{\alpha_1}{(\alpha+\beta)}. \tag{36}$$

Thus the acceleration is always directed in radial direction and the fluid flow in $t-$direction is uniform. If $\alpha_1 < 0$, acceleration is positive and if $\alpha_1 > 0$, there will be deceleration.

The expression for kinematical parameter expansion θ is given by

$$\theta = 3\beta_4. \tag{37}$$

All components of rotation w_{ij} and shear σ_{ij} tensors are found to be zero. We observe that the expansion is time-dependent only. Hence the model (20) representing a distribution of charged perfect fluid is expanding with time but non-rotating and non-shearing.

4. Bulk viscous models

In this section bulk viscous models of the universe are discussed. Weinberg [16] has suggested that in order to consider the effect of bulk viscosity, the perfect fluid pressure should be replaced by effective pressure $\bar{p}$ by

$$\bar{p} = p - \xi\theta, \tag{38}$$

where p represent equilibrium pressure, ξ is the coefficient of bulk viscosity and θ is the expansion scalar. Here ξ is, in general, a function of time. Therefore, from eq. (21), we obtain

$$8\pi(p-\xi\theta) - \Lambda = 3(\alpha_1^2 - \beta_4^2)$$

$$+(\alpha+\beta)\left(2\beta_{44} - \alpha_{11} - \frac{3\alpha_1}{r}\right). \tag{39}$$

Thus, given $\xi(t)$ we can solve the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density [21]–[23].

$$\xi(t) = \xi_0\epsilon^n, \tag{40}$$

where ξ_0 and n are constants. If $n = 1$, eq. (40) may correspond to a radiative fluid. However, more realistic models [24] are based on lying in the regime $0 \le n \le \frac{1}{2}$.

4.1. $Model I : (\xi = \xi_0)$

In this case we assume $n = 0$ in eq. (40) which gives $\xi = \xi_0 =$ constant. By the use of eqs. (23) and (37) in eqs. (22) and (39), we obtain

$$4\pi(1+\gamma)\epsilon = 12\pi\xi_0\beta_4 + (\alpha+\beta)(\alpha_{11}+\beta_{44}) \tag{41}$$

$$(1+\gamma)\Lambda = 3(1+\gamma)(\beta_4^2 - \alpha_1^2)$$
$$+(\alpha+\beta)\left[(1+3\gamma)\alpha_{11} + 3(1+\gamma)\frac{\alpha_1}{r} - 2\beta_{44}\right]$$
$$-24\pi\xi_0\beta_4. \tag{42}$$

4.2. $Model II : (\xi = \xi_0\epsilon)$

In this case we assume $n = 1$ in eq. (40) which gives $\xi = \xi_0\epsilon$. By the use of eqs. (23) and (37) in eqs. (22) and (39), we obtain

$$4\pi\epsilon = \frac{(\alpha+\beta)(\alpha_{11}+\beta_{44})}{(1+\gamma-3\xi_0\beta_4)} \tag{43}$$

$$(1+\gamma)\Lambda = 3(1+\gamma)(\beta_4^2 - \alpha_1^2)$$
$$+(\alpha+\beta)\left[(1+3\gamma)\alpha_{11} - 2\beta_{44} + 3(!+\gamma)\frac{\alpha_1}{r}\right]$$
$$-\frac{6\xi_0\beta_{44}(\alpha+\beta)(\alpha_{11}+\beta_{44})}{(1+\gamma-3\xi_0\beta_4)} \tag{44}$$

These eqs. (41) - (44) will supply different viable models for suitable choices of $\beta(t)$.

5. Conclusions

We have obtained conformally flat spherically symmetric cosmological models in the presence of a charged perfect fluid where the acceleration is always directed in radial direction and the fluid flow in t-direction is uniform. We have also discussed two particular cases. In both cases we observe that the energy conditions hold good and the cosmological constant is found positive and is decreasing function of time which is consistent with the present observations. The model is expanding with time but non-rotating and non-shearing.

Assuming $\xi(t) = \xi_0\epsilon^n$, where ϵ is the energy density and n is the positive index, we have obtained exact solutions. The effect of the bulk viscosity is to produce a change in the perfect fluid.

Acknowledgements

One of the authors (A. Pradhan) would like to thank the Inter-University Centre for Astronomy and Astrophysics, India for providing facility where this work was carried out. We would like to thank R. G. Vishwakarma for many helpful discussions.

References

[1] H. A. Buchdahl, *Phys. Rev.* **115**, 1325 (1959).

[2] K. P. Singh and S. R. Roy, *Proc. Natn. Inst. Sci. India*, **32A**, 223 (1966).

[3] K. P. Singh and Abdussattar, *Gen. Rel. Grav.* **5**, 115 (1974).

[4] S. R. Roy and Raj Bali, *Indian J. pure appl. Math.* **9**, 871 (1978).

[5] S. N. Pandey and R. Tiwari, *Indian J. pure appl. Math.* **12**, 261 (1981).

[6] D. R. K. Reddy, *J. Math. Phys.* **20**, 23 (1979).

[7] V. U. M. Rao and D. R. K. Reddy, *Gen. Rel. Grav.* **14**, 1017 (1982).

[8] K. Shanthi, *Astrophys. Space Sc.* **162**, 163 (1989).

[9] A. Melfo and H. Rago, *Astrophys. Space Sc.* **193**, 9 (1992).

[10] Philip D. Mannheim, *The Astrophys. J.* **391**, 429 (1992).

[11] R. B. S. Yadav and U. Prasad, *Astrophys. Space Sc.* **203**, 37 (1993).

[12] G. Endean, *Astrophys. J.* **479**, 40 (1997).

[13] G. Endean, *J. Math. Phys.* **39**, 1551 (1998).

[14] V. V. Obukhov, S. D. Odintsov and L. N. Granda, *Europhys. Lett.* **46**, 268 (1999).

[15] M. K. Mak and T. Harko, *Int. J. Mod. Phys.* **D9**, 475 (2000).

[16] S. Weinberg, "Gravitation and Cosmology", J.Wiley and Sons, New York, 1972.

[17] W. Israel and J. N. Vardalas, *Lett. nuovo Cim.* **4**, 887 (1970).

[18] Z. Klimek, *Post. Astron.* **19**, 165 (1971).

[19] S. Weinberge, *Astrophys. J.* **168**, 175 (1971).

[20] L. Landau and E. M. Lifshitz, "Fluid Mechanics", Addison-Wisley, Mass. 1962, p. 304.

[21] D. Pavon, J. Bafaluy and D. Jou, *Class Quantum Gravit.* **8**, 357 (1991); "Proc. Hanno Rund Conf. on Relativity and Thermodynamics", Ed. S. D. Maharaj, University of Natal, Durban, (1996), p. 21.

[22] R. Maartens, *Class Quantum Gravit.* **12**, 1455 (1995).

[23] W. Zimdahl, *Phys. Rev.* **D53**, 5483 (1996).

[24] N. O. Santos, R. S. Dias and A. Banerjee, *J. Math. Phys.* **26**, 878 (1985).

[25] T. Padmanabhan and S. M. Chitre, *Phys. Lett.* **A120**, 433 (1987).

[26] V. B. Johri and R. Sudarshan, *Phys. Lett.* **A132**, 316 (1988).

[27] A. Pradhan, R. V. Sarayakar and A. Beesham, *Astr. Lett. Commun.* **35**, 283 (1997).

[28] A. Pradhan, V. K. Yadav and I. Chakrabarty, *Int. J. Mod. Phys.* **D10**, 339 (2001); *Int. J. Mod. Phys.* **D11**, 857 (2002).

[29] I. Chakrabarty, A. Pradhan and N. N. Saste, *Int. J. Mod. Phys.* **D10**, 741 (2001).

[30] A. Pradhan and V. K. Yadav, *Int. J. Mod. Phys.* **D11**, 857 (2002).

[31] A. Pradhan and Aotemshi I., *Int. J. Mod. Phys.* **D11**, (2002), in press.

[32] Ø. Grøn, *Astrophys. Space Sc.* **173**, 191 (1990).

[33] S. Perlmutter *et al.*, *Astrophys. J.* **483**, 565 (1997), Supernova Cosmology Project Collaboration (astro-ph/9608192); *Nature* **391**, 51 (1998), Supernova Cosmology Project Collaboration (astro-ph/9712212); *Astrophys. J.* **517**, 565 (1999), Project Collaboration (astro-ph/9608192).

[34] A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); Hi-Z Supernova Team Collaboration (astro-ph/9805201).

[35] P. M. Garnavich *et al.*, *Astrophys. J.* **493**, L53 (1998a), Hi-Z Supernova Team Collaboration (astro-ph/9710123); *Astrophys. J.* **509**, 74 (1998b); Hi-Z Supernova Team Collaboration (astro-ph/9806396).

[36] B. P. Schmidt *et al.*, *Astrophys. J.* **507**, 46 (1998), Hi-Z Supernova Team Collaboration (astro-ph/9805200).

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 174–178

CHEMICAL POTENTIAL OF MASSIVE NEUTRINOS IN AN EXPANDING UNIVERSE

R. Ruffini, M. Lattanzi, C. Sigismondi† and G. Vereshchagin‡

† *I.C.R.A. - International Center for Relativistic Astrophysics - University of Rome "La Sapienza", Physics Department, P.le A. Moro 5, 00185 Rome, Italy*

‡ *Belorussian State University, Theoretical Physics Department, Skorina ave. 4, 220050, Minsk, Republic of Belarus*

Received October 16, 2002

We discuss recent constraints on degeneracy parameters and neutrino masses, focusing on cosmological implications of neutrinos. When neutrinos are considered as Dirac particles with quite high values of degeneracy parameter adopted, the contribution of two flavors to the total density parameter of the Universe can be as high as $\Omega_\nu = 0.45$. In this case constraints on other cosmological parameters like Ω_{CDM} andΩ_Λ have to be reconsidered. Otherwise, if neutrinos are Majorana particles or neutrino oscillations are important at the early Universe, energy density of neutrinos is negligible. The Jeans mass and free streaming of degenerative neutrinos are considered.

1. Introduction

The status of neutrino in cosmology in the light of recent experimental data is being revised now. The cosmological data comes from Big Bang nucleosynthesis (BBN) together with cosmic microwave background (CMBR)anisotropy [1] and large-scale structure (LSS) [2-4]. The detection of neutrino oscillations [5] provides strong constraints as well [6].

After ruling out the Hot Dark Matter (HDM) models, mainly because of very low mass and free streaming [7] of relativistic neutrinos, the Cold Dark Matter (CDM) models are considered to be responsible for LSS formation. To be consistent with experimental data these models must include large enough Λ term. At the same time one has to admit that the nature of Λ term is still unknown. From the second hand,there is no experimental proof of CDM particles existence, in spite of huge number of hypothetical candidates.

Massive neutrinos still remain the natural candidate for DM since we know that neutrinos exist [8]. It's cosmological implications were widely discussed in literature about twenty years ago [9]. Since experimental limit on neutrino mass was quite undetermined, the neutrino abundance was considered to be as large as that to get closed cosmological model.

Today neutrino dominated Universe is no longer considered as a viable model. However, cosmological, as well as laboratory, constraints on neutrino mass and chemical potential were made independently, without taking into account influence of nonzero chemical potential on neutrino mass bounds and vice versa. Neutrino oscillations shows that it could not be a reasonable approximation.

Taking bounds on the chemical potential and mass of neutrinos from independent sources we show that neutrinos still can have a large enough density parameter.

In this paper we shall also discuss the behavior of the main cosmological quantities like neutrino free streaming and Jeans mass. Section 2 is devoted to cosmological and laboratory constraints on neutrino mass discussion. In the section 3 we make a review of recent bounds on the chemical potential of neutrinos. In section 4 neutrino oscillations and its role for these constraints are discussed. In section 5 the energy density of neutrinos is considered. We analyze Jeans mass and free streaming for degenerative massive neutrinos in section 6. Section 7 contains discussion and conclusions.

2. Neutrino mass

Cosmological constraints on neutrino mass, as a rule, are directly translated from constraints of neutrino abundance through Gerstein-Zeldovich limit [10] (see section 5). This limit, was derived assuming zero chemical potential. Most authors until today use this limit, i.e. direct relation between energy density and mass of neutrino.

Today, galaxy cluster abundance limits on neutrino mass [3,4], as well as 2dF Galaxy Redshift Survey [2], assuming zero chemical potential, give very small contribution from these particles to the energy density of the Universe. However, these estimations are based on measuring of redshifts with $z < 0.2$. Moreover, they require additional assumptions and bounds on other cosmological parameters. At the same time data from Lyman α forest [11] from distant quasars at $z < 3$ and compilations,including CMB, peculiar velocities and LSS [12] give significant contribution from neutrinos $\Omega_\nu = 0.2 - 0.3$. Moreover, the set of cosmological parameters determining CMB anisotropy spectrum is still large and the degeneracy between these parameters is present.

Recent laboratory limits on electronic neutrino mass comes from tritium β decay [13]. These data give limits

$$m_{\nu_e} < 2.5 \text{ eV}. \tag{1}$$

At the same time, no direct measurements or constraints on muonic and tauonic neutrino masses exist. Moreover, it is still unknown, whether neutrinos are Majorana or Dirac particles.

Very recent data from neutrinoless double β decay [14] give also lower bound on Majorana mass:

$$(0.05 \le m_{\nu_{ee}} \le 0.86) \text{ eV}. \tag{2}$$

3. Chemical potential

First constraints on neutrino degeneracy parameter from BBN were obtained in [15]. Recent data both from BBN and CMBR [1,16-18] strongly constrain neutrino degeneracy parameters. In paper [16] these constraints are surprisingly wide, $\eta_e < 1.4$ and $|\eta_{\mu,\tau}| < 40$. Other papers give essentially stronger constraints using additional assumptions, $\eta_e < 0.3$ and $|\eta_{\mu,\tau}| < 2.6$. In particular, [17] and [18] use very small range for variation of baryonic energy density $\Omega_b h^2$, but constraints on neutrino degeneracy are very sensitive to this parameter. In paper [1] very strong constrain on the helium abundance was used.

A successive approach when the chemical potential of particles (bosons and fermions) μ is nonzero was developed by Ruffini et al. [19]. The energy density of neutrinos in this case depends not only on it's mass but also on the chemical potential. This fact, as was shown, provides a possibility to consider the Universe, dominated by neutrinos with mass in range of few eV. The LSS formation in the model with nonzero chemical potential was examined in [20]. The cellular structure appearing in these models, as was shown, have a fractal nature. It was shown [21], that the small value of η_e and large values of $|\eta_{\mu,\tau}|$ simultaneously, can lead to BBN abundances which are consistent with observations.

The most important argument to neutrino chemical potential existence is conserved number of these particles. Since neutrino oscillations provide arguments to nonzero neutrino mass, massive Dirac neutrinos can have nonzero chemical potential. At the same time, pure Majorana neutrinos are their own antiparticles and the chemical potential for Majorana particles is zero. We consider both cases below.

4. Neutrino oscillations

When one consider different chemical potentials for all neutrino flavors at the epoch prior to BBN, neutrino oscillations equalize chemical potentials[22], if there is enough time to relaxation process [23]. On the basis of large mixing angle solution of the solar neutrino problem,which is favored by recent data [5], the BBN consideration constrains degeneracy parameters $|\eta \le 0.07|$ of all neutrino flavors [6]. However the situation when flavor equilibrium is not achieved before BBN is also possible. Thus in this paper we consider quite high values of the degeneracy parameter and assume it's positive value.

At the same time, data on solar and atmospheric neutrino oscillations allow to make not only upper but also a lower bound on the mass of electronic neutrino.

5. Energy density of massive degenerative neutrinos

Assuming that neutrinos decouple from matter when they were relativistic, their distribution function depending on momentum p and time t is given by

$$f(p,t) = [exp(\frac{p \pm \mu}{kT(t)}) + 1]^{-1}, \tag{3}$$

where k is Boltzmann's constant, μ is chemical potential and T is neutrino "temperature", which is now $T_{\nu 0} = 1.96$K. The sign "-" in(3) corresponds to particles and the sign "+" corresponds to antiparticles. This means that with positive value of the chemical potential one has exponential excess particles over antiparticles.

The energy density of one flavor of neutrinos today is [20]

$$\Omega_{\nu+\bar{\nu}} h^2 \simeq 10^{-1} g_\nu (\frac{m_\nu}{10eV}) A(\eta), \tag{4}$$

where $\Omega_{\nu+\bar{\nu}} = (\rho_\nu + \rho_{\bar{\nu}})/\rho_{crit}$ is the density parameter of neutrinos and antineutrinos in units of critical density

$$\rho_{crit} = 1.88\ 10^{-29} h^2 \text{ g/cm}^3, \tag{5}$$

m_ν is the mass of one flavor of neutrino, g_ν is the number of helicity states, $\eta = \mu/kT$ is degeneracy parameter and $A(\eta)$ is given by

$$A(\eta) = [4\eta_R(3)]^{-1}[\frac{1}{3}\eta^3 + 4\eta_R(2)\eta + \tag{6}$$

$$+4\sum_{n=1}^{\infty}(-1)^{n+1}\frac{e^{-n\eta}}{n^3}].$$

Here $\eta_R(k)$ is the Riemann eta function of index k.

When the chemical potential of neutrino is zero, the energy density of all neutrino flavors today is given by [10]:

$$\Omega_{\nu+\bar{\nu}}h^2 = \frac{1}{94eV}\sum_{i=e,\mu,\tau} g_i m_{\nu i}. \quad (7)$$

This relation is known as Gerstein-Zeldovich limit, and allow one to constrain neutrino mass directly from cosmological estimations of Ω_ν. The limit is even stronger when nonzero chemical potential is assumed.

Electronic neutrino with very small mass and degeneracy parameter cannot influence significantly on cosmological evolution because of small energy density. However, muonic and tauonic neutrinos are much less constrained. We shall then consider 2 flavors, assuming absence of sterile neutrinos, of Dirac ($g = 1$) neutrinos with nearly equal masses and chemical potentials. We use the symbol η for the common value of the two degeneracy parameters. Moreover, we assume that the Universe is flat, $\Omega = 1$. For energy density calculations we set $h = 0.7$.

It is necessary to note, that many authors (for references see [8]) express the relation between neutrino degeneracy parameter and energy density of one flavor by means of an effective number of additional neutrino species ΔN_ν:

$$\Delta N_\nu \equiv \frac{15}{7}[(\frac{\eta}{\pi})^4 + 2(\frac{\eta}{\pi})^2]. \quad (8)$$

This relation, however, is a consequence of energy density dependence on chemical potential and, moreover, is limited by condition $\Delta N \leq 1$. The energy density dependence on chemical potential can be used directly to constrain the chemical potential instead of eqn.(8).

This relation in the case of massless neutrinos in thermal equilibrium has the form:

$$\rho_{\nu+\bar{\nu}} = \frac{7}{8}\frac{\pi^2 T^4}{15}[1 + \frac{30}{7}(\frac{\eta}{\pi})^2 + \frac{15}{7}(\frac{\eta}{\pi})^4]. \quad (9)$$

It is easy to show, that in massive case the dependence on degeneracy parameter at early Universe is the same if m_ν is few eV, since $m_\nu \ll T$ and then the neutrinos are in their ultrarelativistic regime. Thus to constrain degeneracy parameter from BBN or CMB one can use relation(9).

One can see that quite high values of neutrino energy density is still possible. If one consider two Dirac neutrino flavors with equal masses and chemical potentials, $\eta_\mu = \eta_\tau \leq 2.6$ [18] and $m_{\nu_\mu} = m_{\nu_\tau} \leq 2.5$ eV [13], one gets the upper bound

$$\Omega_{\nu+\bar{\nu}} \leq 0.45. \quad (10)$$

In the case in which the masses are still equal, but only one flavor is degenerate, while the other has null chemical potential, the bound on the degeneracy parameter becomes $\eta \leq 3.1$. The corresponding bound on the density parameter is

$$\Omega_{\nu+\bar{\nu}} \leq 0.31. \quad (11)$$

From the other hand, if neutrino oscillations were important at the epoch of BBN, or neutrinos have Majorana mass, then the upper bound on neutrino energy density is

$$0.0033 \leq \Omega_\nu \leq 0.057. \quad (12)$$

if $\eta = 0$ and $(0.05 \leq m_\nu \leq 0.86)$eV for all three neutrino species.

6. Free streaming and Jeans mass of degenerative neutrinos

In neutrino dominated Universe the first possible structure occurs when these particles become nonrelativistic. At this epoch the cosmological redshift has the value

$$1 + z_{nr} \simeq 1.7\ 10^4\left(\frac{m_\nu}{10eV}\right)A(\eta)^{\frac{1}{2}}B(\eta)^{-\frac{1}{2}}, \quad (13)$$

where

$$B(\eta) = [48\eta_R(5)]^{-1}[\frac{1}{5}\eta^5 + 8\eta_R(2)\eta^3 + \quad (14)$$
$$48\eta_R(4)\eta + 48\sum_{n=1}^{\infty}(-1)^{n+1}\frac{e^{-n\eta}}{n^5}].$$

Assuming recent bounds on neutrino mass and chemical potential one can see, that neutrinos become nonrelativistic very close to recombination.

The basic mechanism for fragmentation of the initial inhomogeneities in an expanding Universe is governed by the Jeans instability. The characteristic scale associated with this instability is the Jeans length $\lambda_J = \sqrt{\pi v_s^2/G\rho}$ and the corresponding mass, the Jeans mass, is defined by $M_J = 4/3\pi\rho_m(\lambda_J/2)^3$, where v_s^2 is the velocity of sound and ρ_m is the mass density of the Universe. Note, that in nonrelativistic regime there is no difference between energy density and mass density.

However, in the calculation of Jeans' length of nonrelativistic collisionless neutrinos, we cannot use the velocity of sound as obtained by $v_s = \partial P/\partial\rho$. Instead, we have to make the substitution $v_s^2 \to < v^2 > /3$ [24], where the correct expression for $< v^2 >$ can be obtained by solving perturbed Vlasov (or collisionless Boltzmann)equation in the background of an expanding Universe. This gives at $z = z_{nr}$ is:

$$< v^2 >= 12\frac{\eta_R(5)}{\eta_R(3)}(\frac{kT_{\nu 0}}{m_\nu})^2\frac{B(\eta)}{A(\eta)}. \quad (15)$$

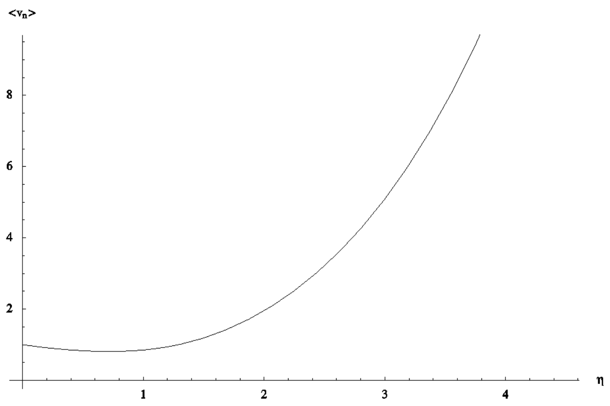


Figure 1: The normalized velocity dispersion $< v_n >= \frac{<v^2>^{1/2}(\eta)}{<v^2>^{1/2}(0)}$ dependence on degeneracy parameter with fixed value of energy density

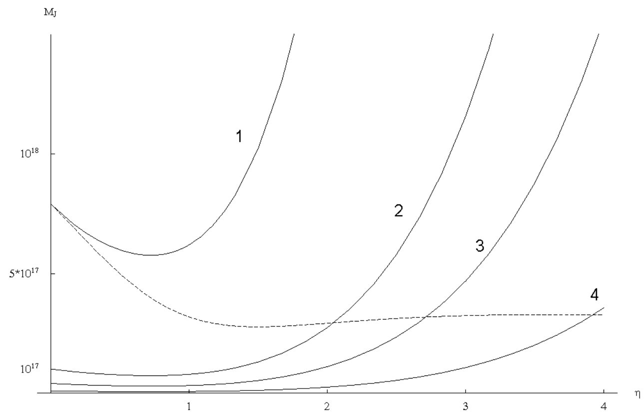


Figure 2: The Jeans mass dependence on degeneracy parameter with fixed value of energy density, curves (1-5). Curve (1) corresponds to energy density $\Omega_\nu = 0.11$. Curve (2)corresponds to $\Omega_\nu = 0.3$. Curve (3) represents neutrino energy density $\Omega_\nu = 0.5$ and, finally, curve (4) gives Jeans mass for $\Omega_\nu = 1$. The dashed line represents Jeans mass dependence on degeneracy parameter with fixed neutrino mass $m_\nu = 2.5$ eV

Finally, the Jeans mass (which has maximum at the moment (13)) is [24]

$$M_J(z_{nr}) = 1.475\,10^{17} M_\odot g_\nu^{-\frac{1}{2}} N_\nu^{-\frac{1}{2}} \times \times (\frac{m_\nu}{10eV})^{-2} A(\eta)^{-\frac{5}{4}} B(\eta)^{\frac{3}{4}}. \quad (16)$$

The Fig. 1 represents the dependence of the velocity dispersion on neutrino degeneracy parameter with fixed energy density. The neutrino mass falls approximately as η^{-2} (with small η) when the degeneracy parameter grows with constant energy density.

From this figure one can see that with low values of degeneracy parameter ($\eta \leq 1.5$) the velocity of neutrinos practically do not change. At the same time with sufficiently high $\eta \sim 4$ the velocity grows an order of magnitude being compared to the same velocity with zero degeneracy parameter.

This means that free streaming, which is determined by the neutrino velocity, grows significantly only with high values of degeneracy parameter. Since the free streaming length and the Jeans length are nearly equal [25], all the perturbations below the Jeans length are damped [7]. Thus, structure formation at scales less that the peak of Jeans mass at (13) suppressed in the Universe, filled only by neutrinos with equal masses. However, the scale of the Jeans mass peak could determine the cellular structure [20], which can be observed.

The peak of Jeans mass depending on degeneracy parameter for different fixed values of energy density as well as with constant mass $m_\nu = 2.5$ eV is shown at Fig. 2.

By comparing different curves with fixed value of η one can find the well known result, that the Jeans mass increases with decreasing neutrino mass. With growth of degeneracy parameter, however, neutrino mass decreases, and its different values correspond to different points at the same curve. For instance, when the degeneracy parameter equals to $\eta = 1$the neutrino mass decreases approximately 1.5 times with respect to$\eta = 0$; when $\eta = 2$ the mass decreases 3 times.

The region above the dashed line in Fig. 2 represents the region in which the neutrino mass is less than 2.5 eV. It's interesting to note, that this value of m_ν is still sufficient to obtain $\Omega_\nu = 1$ with$\eta \approx 4$.

7. Discussion and conclusions

The upper bound on energy density found in the paper, $\Omega_\nu \leq 0.45$, is very important so far as it provides so high neutrino contribution to the energy density of the Universe. It is higher than contribution from CDM particles. This means that we need to reconsider recent constraints on cosmological parameters like Ω_m and Ω_Λ if the current values of degeneracy parameter will be adopted. At the same time,when degeneracy parameter is not very high, $\eta \leq 1.5$, the Jeans mass practically do not depend on it (with fixed $\Omega\nu$).

From the other hand, if neutrino oscillations played significant role at the epoch of BBN, or neutrino is a Majorana particle, the upper bound on it density parameter is $\Omega_\nu \leq 0.05$.

The free streaming of neutrinos with acceptable values of degeneracy parameter is practically the same as in the case of $\eta = 0$. The effect of free streaming allows cellular structure formation on very large scales.

References

[1] S.H. Hansen, G. Mangano, A. Melchiorri, G. Miele and O. Pisanti, *Phys.Rev.* D **65** (2002) 023511.

[2] O. Elgaroy et al. astro-ph/0204152.

[3] N.A. Arhipova, T. Kahniashvili and V.N. Lukash, *Astron. Astrophys.* **386** (2002) 775.

[4] M. Fukugita, G.C.Liu and N. Sugiyama, *Phys. Rev. Lett.* **84** (2000)1082.

[5] Q.R. Ahmad et al. *Phys. Rev. Lett.* **87** 071301;
Recent results from SNO collaboration are abailable at http: // www.sno.phy.queensu.ca/sno/results_04_02.

[6] A.D. Dolgov et al. hep-ph/0201287.

[7] J.R. Bond and A.S. Szalay, *Ap. J.* **274** (1983) 443.

[8] A.D. Dolgov, hep-ph/0202122, to be published in *Phys. Repts. Criticism.*

[9] Ya.B. Zel'dovich, R.A. Syunayaev *Pis'ma Astron. Zh.* **6** (1980)451;
A.G. Doroshkevich and M.Yu. Khlopov, *Astron. Zh.* **58**(1981) 913;
P.J.E. Peebles, *Ap. J.* **258** (1982) 415;
J.R.Bond, G. Efstathiou, and J. Silk, *Phys. Rev. Lett.* **45** (1980)1980.

[10] S.S. Gerstein and Ya.B. Zeldovich, *Pis'ma ZhETF* **4** (1966)174; *JETF Letters* **4** (1966) 120.

[11] R.A.C. Croft, W. Hu and R. Dave, *Phys. Rev. Lett.* **83** (1999)1092.

[12] E. Gawiser and J. Silk, *Science.* **280** (1998)1405.

[13] C.V. Weinheimer et al. *Phys.Lett.* B **460** (1999), 219;
Lobashev V.M. et al., ibid, 227.

[14] H.V. Klapdor-Kleingrothaus et al. *Mod.Phys.Lett.* A **16** (2002),2409;
S. Pakvasa and P. Roy, hep-ph/0203188;
H.V. Klapdor-Kleingrothaus and U. Sarkar, *Mod.Phys.Lett.* A **16**(2001), 2449.

[15] A.G. Doroshkevish, I.D. Novikov, R.A. Siunaiev, Y.B.Zeldovich, in *Highlights of Astronomy,* de Jader ed., (1971) 318;
W.A.Fowler, *The astrophysical aspects of the weak interactions*, Academia Nazionale deiLincei, Roma, **157** (1971) 115;
G. Beaudet and P. Goret, *Astron. Astrophys.* **49** (1976) 415.

[16] M. Orito et al. astro-ph/0005446.

[17] J.P. Kneller et al. astro-ph/0105385.

[18] M. Orito et al. *Phys.Rev.* D 65 (2002) 123504, astro-ph/0203352.

[19] R. Ruffini, D.J. Song and L. Stella, *Astron. Astrophys.* **125** (1983) 265.

[20] R. Ruffini, D.J. Song, S. Taraglio, *Astron. Astrophys.* **190** (1988) 1.

[21] A. Bianconi, H.W. Lee and R. Ruffini, *Astron.Astrophys.* **241** (1991) 343.

[22] M. J. Savage, R.A. Malaney and G.M. Fuller, *Ap. J.* **368** (1991) 1.

[23] K.N. Abazajian et al. astro-ph/0203442.

[24] R. Ruffini, D.J. Song, *Gamow Cosmology: Enrico Fermi Course* 86 (1986) 370.

[25] C. Sigismondi, S. Filippi, R. Ruffini and L. A. Sanchez,*International Journal of Modern Physics* **10D** 5 (2001) 663.

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 179–180

SPACE-TIME IN THE CLASSICAL ELECTRODYNAMICS FROM THE VIEWPOINT OF QUANTUM MECHANICS

D.L. Khokhlov[1]

Sumy State University, R.-Korsakov St. 2, Sumy 40007, Ukraine

Received November 20, 2002

From the viewpoint of quantum mechanics, the classical electromagnetic field is considered as a dual particle-wave object the space coordinate and the momentum of which are defined separately. The coordinate space-time is defined by the Galilei transformation. The momentum space-time is defined by the Lorentz transformation. Since the coordinate space-time is defined by the Galilei transformation, observers in the rest and in the moving frames describe events in the same way.

The principle of relativity reads that events occurring in any inertial frame do not depend on the motion of the frame. This means that the physical laws are invariant under the transformation of space and time coordinates from one inertial frame to another. Such a transformation defines the space-time. Equations of the Newton mechanics are invariant under the Galilei transformation

$$x' = x - vt, \qquad y' = y, \qquad z' = z, \qquad t' = t, \tag{1}$$

where coordinates of space x, y, z and time t describe the rest frame, coordinates of space x', y', z' and time t' describe the moving frame, v is the speed of the moving frame. Thus the Galilei transformation defines the space-time in the Newton mechanics.

Equations of electrodynamics are not invariant under the Galilei transformation. Einstein [1] developed the theory of special relativity on the basis of two postulates. The first, the principle of relativity in the electrodynamics holds true. That is electromagnetic events occurring in any inertial frame do not depend on the motion of the frame. The second, the speed of light do not depend on the motion of the source of light, the postulate of invariance of the speed of light. To investigate the coordinate transformation in the electrodynamics one can use equation for the light interval

$$x^2 + y^2 + z^2 - c^2t^2 = 0, \tag{2}$$

where c is the speed of light. Einstein applied the principle of relativity and the postulate of invariance of the speed of light to eq. (2) and obtained the Lorentz transformation

$$\begin{aligned} x' &= \frac{x - vt}{(1 - v^2/c^2)^{1/2}}, \qquad y' = y, \qquad z' = z, \\ t' &= \frac{t - xv/c^2}{(1 - v^2/c^2)^{1/2}}. \end{aligned} \tag{3}$$

According to Einstein, the space-time in the electrodynamics is defined by the Lorentz transformation.

In the space-time defined by the Lorentz transformation, observers in the rest and in the moving frames describe events in the different way. According to Einstein, the true (physical) description of the event occurring in some frame is given by the observer in the same frame. However this sentence cannot be verified by the observers in the other frames. Therefore the space-time defined by the Lorentz transformation has not physical meaning.

In the electrodynamics, dynamical vectors and coordinate vectors are related as

$$\mathbf{p} \propto \frac{1}{\mathbf{r}}, \tag{4}$$

where $\mathbf{p}$ is the momentum, $\mathbf{r}$ is the radius vector. For the electromagnetic field, the energy and momentum are related as $E = pc$, the radius vector and time are related as $r = ct$. Then from eq. (4) it follows

$$E \propto \frac{1}{t}. \tag{5}$$

In view of eqs. (4,5), one can write eq. (2) in the form

$$\frac{1}{p_x^2} + \frac{1}{p_y^2} + \frac{1}{p_z^2} - \frac{c^2}{E^2} = 0. \tag{6}$$

When applying the principle of relativity and the postulate of invariance of the speed of light to eq. (6), one

[1] e-mail: khokhlov@cafe.sumy.ua

obtains the Lorentz transformation for the momentum and energy

$$p'_x = \frac{p_x(1 - v^2/c^2)^{1/2}}{1 - v/c}, \quad p'_y = p_y, \quad p'_z = p_z,$$
$$E' = \frac{E(1 - v^2/c^2)^{1/2}}{1 - v/c}. \qquad (7)$$

In the Einstein special relativity, eq. (2) describes the coordinate space-time, eq. (6) describes the momentum space-time. Both the coordinate space-time and the momentum space-time are defined by the Lorentz transformation.

Einstein considers electromagnetic field as a wave. Within the framework of quantum mechanics, electromagnetic field is conceived as a quantum object, photon, which has the dual particle-wave nature. The space coordinate $\mathbf{r}$ and the momentum of photon $\mathbf{p}$ are bound by the Heisenberg uncertainty principle

$$pr \geq \frac{\hbar}{2}, \qquad (8)$$

where $\hbar$ is the Planck constant. Therefore one cannot simultaneously define the space coordinate and the momentum of photon. When considering classical electrodynamics, one should take into account the constraints imposed by the quantum mechanics. That is to consider electromagnetic field as a dual particle-wave object the space coordinate and the momentum of which are defined separately. It is reasonable to define the coordinate space-time considering electromagnetic field as a massless particle and the momentum space-time considering electromagnetic field as a wave.

Consider electromagnetic field as a massless particle. Define the coordinate space-time with the use of equation for the motion of the massless particle

$$\Delta x - c\Delta t = 0. \qquad (9)$$

Apply the principle of relativity and the postulate of invariance of the speed of light to eq. (9). One can see that the Galilei transformation leaves eq. (9) invariant. Hence the coordinate space-time in the electrodynamics is defined by the Galilei transformation. That is coordinate vectors in the electrodynamics follow the Galilei transformation.

Consider electromagnetic field as a wave. Define the momentum space-time with the use of eq. (6). From the above consideration it follows that the momentum space-time is defined by the Lorentz transformation given by eq. (7). That is dynamical vectors in the electrodynamics follow the Lorentz transformation.

Thus, in the electrodynamics, coordinate vectors and dynamical vectors are transformed in the different way. Coordinate vectors follow the Galilei transformation, dynamical vectors follow the Lorentz transformation.

Let the electromagnetic wave move from the emitter to the receiver. Let the receiver be in the rest frame, and the emitter be in the frame moving with the speed v. In the moving frame, the momentum of the electromagnetic wave is defined with the Lorentz transformation

$$p' = \frac{p(1 - v^2/c^2)^{1/2}}{1 - v\cos\alpha/c}, \qquad (10)$$

where α is the angle (in the frame of the receiver) between the direction of the electromagnetic wave and the direction of the emitter. The first order term v/c is due to the inertial frame with the speed v. The second order term v^2/c^2 is due to the non-inertial frame with the gravitational potential of the inertial force v^2. Therefore, the first order effect is relative (depends on the angle α), and the second order effect is absolute (do not depend on the angle α).

Since the coordinate space-time is defined by the Galilei transformation, observers in the rest and in the moving frames describe events in the same way. Then both observers in the rest frame and in the moving frame determine the momentum of the electromagnetic wave in the rest frame as p and in the moving frame as $p \propto (1 - v^2/c^2)^{1/2}$. In this point, the presented theory and the Einstein theory diverge. According to Einstein, an observer in the moving frame determines the momentum of the electromagnetic wave in the moving frame as p, and an observer in the rest frame determines the momentum of the electromagnetic wave in the moving frame as $p \propto (1 - v^2/c^2)^{1/2}$. According to Einstein, the true (physical) momentum of the electromagnetic wave in the moving frame is determined by the observer in the moving frame and is given by p. The true (physical) momentum of the electromagnetic wave in the rest frame is determined by the observer in the rest frame and is also given by p. According to the presented theory, any observer determines the true (physical) values in any frame. So, the true (physical) momentum of the electromagnetic wave in the rest frame is given by p, the true (physical) momentum of the electromagnetic wave in the moving frame is given by $p \propto (1 - v^2/c^2)^{1/2}$. Note that since the second order effect is absolute the rest and the moving frame are defined here with respect to the inertial forces.

References

[1] W. Pauli, " Theory of Relativity ", Pergamon, New York, 1958.

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 181–182

WHY THE HUBBLE PARAMETER GROWS UP?

Alex M. Chepick[1]

Nizhni Novgorod, Russia

Received October 10, 2002

The increase during time of the Hubble parameter is explained from the point of view of the state Universe model. Also it is demonstrated why measurements give now and will give in future a decreasing of the Hubble constant.

Introduction

In 1995 when models of the Universe with decreasing rate of the scale factor's growth were still considered as the most probable, Tammann [1] has made a prediction, what by July, 1, 2007 exact value of the Hubble constant will be established: $H_0 = 55$ km/s/Mpc. This statement was based on restriction of age of the Universe, and on the noticed decreasing of size of the Hubble constant from measurement to measurement. The Hubble constant behaved incorrectly, not fitting to the accepted models of the Universe with zero cosmological constant.

But right at the end of XX century two independent groups of researchers headed by S. Perlmutter [2, (further P1998)] and W. Freedman [3, (further F2000)] have found out the phenomenon of the accelerated expansion of the Universe.

This unexpected result is deduced as consequence of the proved increase during time of Hubble parameter what has forced scientists to recollect Einstein's idea about nonzero cosmological constant and to search for its explanation.

The specified conclusion has been made on the basis of the Big Bang theory (BB). Meanwhile, the conclusion about such behavior of Hubble parameter follows easily from model of the state Universe.

1. Hubble constant in the expanding Universe

In tables 6, 9, 10 [F2000] it is possible to see, how Hubble parameter decreases with growth of distance up to objects, what corresponds to growth of Hubble parameter during time. We shall express Hubble parameter $H(t)$ through the scale factor $a(t)$: $H(t) = a'(t)/a(t)$, where t is time past from the moment of the Big Bang. The Hubble constant is value of Hubble parameter at the current moment $t0$: $H0 = H(t0)$. Growth $H(t)$ tells that the scale factor should increase not simply, but with acceleration. In turn, the accelerated growth of the scale factor tells about infinite expansion of the Universe, without transition in a stage of compression. It brings an attention to the question before cosmology not only what was up to and during the Big Bang but also what will be after that, because representation about possible yclicity, repeatability of existence of the Universe does not satisfy any more now.

Besides, the question about the reasons generating nonzero cosmological constant has appeared. Attempt to explain its action with the help of Kazimir Effect does not satisfy, because it is completely not clear, how Kazimir force influences on metric of the the space and increases the scale factor, so as it changes the metric distance between objects, but at the same time it not shift these objects from their places. And if the objects are moving, then these forces should have such energy, that the far galaxies would move concerning us with by almost light's speed. Besides there is no proof, what the greater "volume" of vacuum will give stronger the Kazimir Effect. It is what should be carried out for prospective action of the cosmological constant.

And one more remark is about stationarity of the orbit. In particular, orbit's stationarity of the Earth is necessary for existence of life on our planet. Clearly, that speed of change $a'(t)$ of the scale factor $a(t)$ is nonzero as Hubble parameter is nonzero. Hence, the scale factor is variable. If expansion of Universe is the comprehensive phenomenon, then electromagnetic and gravitational forces should correspond strictly to change of the scale factor $a(t) > 1$ for neutralization of this expansion in atoms, bodies, planetary systems and galaxies. Assume, for example, that at some moment we have stationary orbits in some planetary system, in which the scale factor $a(t) = const > 1$ operates. In that case the aspiration to increase of orbit radius is compensated to the greater value of gravitational constant G, than in a case $a(t) = 1$. But if the scale factor is variable then any value G cannot provide the

[1]e-mail: redshift0@narod.ru

stationarity of planet's orbit in this star system, as the variable increase in radius of an orbit results to variable decreasing of gravitational attraction force. And it would not been compensated by any constant value G. That is, the orbit of a planet have to be non-stationary. And as it isn't so, the scale factor can not act (as its operating will not be balanced) within the limits of a gravitational field, at least, there, where the attractive force is directed to one center of mass and there can be cyclical orbits (therefore we see the galaxies do not expanded).

In tables 1, 2 [P 1998] the redshifts and magnitudes of Supernovae *Ia* type at the top of their luminosity are given. These data have allowed to Perlmutter to announce about unexpected behavior of Hubble parameter depending on redshift. dependence of on. The similar data from the tables 6, 7, 9, 10 [F2000] have confirmed this conclusion. In particular, in P 1998 it has been noticed, that "*Supernovae in redshift* $z = (0.3 - 1.0)$ *give on the average on* $0.28 mag$ *the greater distance, than expected*" (it was implied $\Lambda = 0$). It is equivalent to reception of energy smaller than counted, from the object which is set on known distance or redshift.

Let's consider more in detail the formula of luminosity: $E = L / \left(R_0^2 \psi^2 (z) K (z) A (z)\right)$,
where E - accepted power of a stream of energy, L - luminosity of object, $R_0 = c/H_0$, $R_0 \psi (z)$ - metric function of distance (depends on the chosen model), $K (z)$ - K-correction, $A (z)$ factor of absorption.

As energy from far object comes less than counted (for given z) it is necessary to define, that distance $R_0 = c/H_0$ should be longer, but for this purpose it is necessary to reduce H_0. We received not only increasing $H(t)$, but also decreasing of H_0.

As we, aspiring to increase accuracy of definition H_0, try to increase data volumes for this purpose therefore it is necessary to take more and more far objects (in course of time opportunities of astronomers grow). But farther objects give smaller value H_0, it affects on an average results of measurements. Hence, this process of "decreasing of constant H_0" will proceed until models BB with $\Lambda = 0$ is used.

2. Hubble constant in the state Universe

The formula of dependence of distance R_S from redshift z in the state Universe has form:

$$R_S (z) = c/H_0 \ln (z + 1) .$$

The conclusion of this formula is given by many authors: Zwicky (1929) [4], Hubble (1932), Veinik (1969), LaViolette (1986) [5], Zhuck N.A. (1989) [6], etc., in their articles.

Let's analyze this dependence. In state Universe distance $R_S (z)$ is not limited. But for any model BB distance $R_{BB} (z)$ goes up to a limit: $R_{BB} (z) < c/H_0$. For $z = 0$ the ratio is executed: $R_S (0) = R_{BB} (0) = 0$. For small z the ratio is executed: $R_S (z) = R_{BB} (z) = zc/H_0$. Hence, $R_S (z) > R_{BB} (z)$ for $z > 0$.

Last inequality demonstrates, why visible luminosity of objects should increase in the state Universe in comparison with BB. Moreover, this conclusion is true for any model BB because of it is made without a concrete definition of size of the cosmological constant. Hence, the conclusion about "decreasing of constant H_0" will be true until any models BB with any Λ is used.

Calculation of increase in distance (in magnitudes) for *SNe Ia* in the state Universe in comparison with flat model BB with $\Lambda = 0$, $\Omega = 1$ under the Mattig metric formula: $R_{BB} (z) = 2 (c/H_0) \left[1 - (z + 1)^{-1/2}\right]$, has given for $z = (0.3 - 1.0)$ increase on the average on $0.26 mag$ the greater distance.

Conclusions

1) The counted increase in magnitudes *SNe Ia* in the state Universe in comparison with model BB with $\Lambda = 0$, $\Omega = 1$ for $z = (0.3 - 1.0)$ has coincided with actual.

2) Supernovae in redshift $z = (1.0 - 1.5)$ will give on the average on 0.13 mag the greater distance, than expected for $\Lambda > 0$ (on 0.43 mag the greater distance, than expected for $\Lambda = 0$).

3) The conclusion about "decreasing of constant H_0" will be true until any models BB with any Λ is used.

References

[1] Tammann G.A. "The Hubble Constant: A Discourse." 1996PASP. 108.1083T.

[2] Astro-ph/9812133, v1, 8 Dec, 1998. "MEASUREMENTS OF O AND Λ FROM 42 HIGH-REDSHIFT SUPERNOVAE", [S. Perlmutter et all], (**P1998**).

[3] Astro-ph/0012376 v1 18 Dec 2000. "Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant" [Wendy L. Freedman et all], (**F2000**).

[4] Zwicky, F., 1929, Proc. Nat. Ac. Sc., Washington, 15, 773.

[5] LaViolette, P. A., 1986301:544L.

[6] Zhuck, N.A., "Cosmology," Kharkiv: "Model Vselennoy," 2000.

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 183–183

DOPPLER EFFECT AGAINST SCALE-FACTOR

Alex M. Chepick[1]

Nizhni Novgorod, Russia

Received October 10, 2002

The impossibility of usage of the Doppler effect's relativistic formula for definition of a Hubble constant in the expanding Universe is demonstrated most shortly and simply.

The formula of Doppler effect does not fit for definition of the Hubble constant

When Hubble inferred his famous law $v = HD$, he with clean conscience has used the relativistic formula of Doppler effect for definition of a radial velocity v of object:

$$(z+1)^2 = (z+v) / (c-v)\,, \qquad (1)$$

where z is redshift, c is speed of light.

Many methods of definition of the Hubble constant have appeared since then, but for the Static Universe the definition of a radial velocity of objects and distance up to it lies in the basis of the most of it. The methods of definition of distance D can be miscellaneous. But the radial velocity$v$ is determining only under the relativistic formula of Doppler effect (1) or by its approaching for low speeds: $v - cz$.

For models of the expanding Universe the methods of definition of Hubble parameter are based in the majority on the formula $H(t) = a'(t) / a(t)$, where $a(t)$ is time-depending scale factor.

And as the formula of Doppler is deduced for static Universe and is tested only in it, then it is impossible to apply this formula in expanding Universe.

Really, let for some source of light we have measured redshift z. Guessing applicability of the formula (1) we determine speed v of moving of this object in our system of reference: $v = \left(2z + z^2\right) / \left(2 + 2z + z^2\right)$. Then under relativity theory the pace of time in a system of reference of the source grater in $\left(1 - (v/c)^2\right)^{-1/2}$ of times than pace of time in our system of reference.

But under the theory of expanding Universe the pace of time in a system of reference of this source grater in $(z+1)$ times concerning pace of time in our system of reference [1].

Compare two expressions for pace of time:

We obtain with the help of the formula (1), the equality (2) is executed only at $z = 0$, $v = 0$.

[1] e-mail: redshift0@narod.ru

Conclusions

1. The formula of Doppler effect deduced for the static Universe, it is defaulted in the expanding Universe.
2. The formula of Doppler effect in the expanding Universe should in addition depend on time of radiation, reception time and scale factor.

References

[1] K.A. Postnov. "Lectures on Astrophysics for Students of MSU," 2000; http:// info.phys.msu.su/ AstroPhysic/ Lecture5/ html/ lecture5.html.

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 184–185

MORE ON *FEYNMAN LECTURES* BY J. GUALA-VALVERDE

Fabio R. Fernández[1]

Ecogestion SRL, Pozo Hondo 129, 8300-Neuquén, Neuquén-Argentina

Received November 30, 2002

Guala-Valverde calculations are improved in order to get the correct sign for the induced emf.

PACS number: 41.20.Jb, 41.90.+e

After performing a series of epoch making experiments which definitively solved the homopolar induction issue [1], [2], [3], [4], [5] Guala-Valverde (G-V) was able to stress the hierarchy of the potential vector A in the realm of classical electrodynamics [3], [4]. Although essentially correct, G-V calculations can be improved in order to get the correct sign for the induced emf. Don't forget G-V gives $\varepsilon' = -\omega B\left(R^2/2\right)$, a *negative* number, instead of the measured and classically calculated $\varepsilon = \varepsilon'$ one [3], [4].

In fact, G-V performed an *indefinite* integration when solving Faraday's law in terms of the potential vector, $\varepsilon = -d/dt \oint_C A.dl$ and this oversimplifaction was the responsible for the minus sign appearing in the calculated emf.

In order to improve the above treatment we only need to specify the actual integration path involved in the above integration. In fact, Neumann's induction law [3], [6], [7], $\varepsilon = -d/dt \oint_C A.dl$, can be divided into two partial-path integrals,

$$\varepsilon = -d/dt\left[\int_{probe+collector} A.dl + \int_{CW} A.dl\right] =$$

$$= -d/dt \int_{probe+collector} A.dl \tag{1}$$

the first one evaluated on the (moving) probe plus collector (ring) path segments, the second on the closing wire CW. The latter being at rest in the lab, and its geometry remaining unchanged, the time derivative of the second integral vanishes. The collector path is time-dependent since it connects the moving probe with CW. Therefore, the integration path available for the *emf* calculation starts ar $r = 0$ on the probe and reaches the CW through the collector ring (Fig. 1). The line element becomes $dl = (dr)\, i_1 - (r\omega dt)\, i_2$ when expressed in circular cylindrical coordinates and here, i.e.

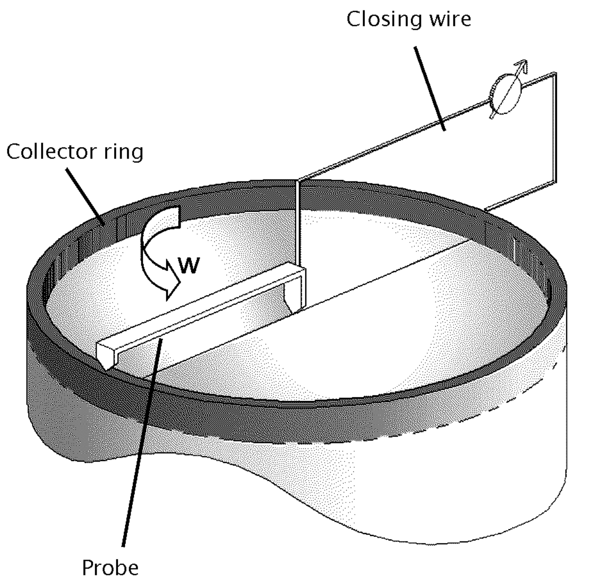


Figure 1: The relevant features of the Homopolar generator

in the minus sign, rests our difference with G-V calculation. Remembering [3] that $A = Ai_2 = (Br/2)\, i_2$, we get $A.dl = -\omega r A dt = -dt\left(\omega B r^2/2\right)$, from which results

$$\varepsilon = -d/dt \int_{probe+collector} -dt\left(\omega B r^2/2\right) =$$

$$= \omega B r^2/2, \quad r < R.$$

For $r > R$, a region in which Lorentz force vanishes, $A = BR^2/2r$ and we get $A.dl = -\left(\omega B R^2/2\right) dt$. Consequently, $\varepsilon = \omega B R^2/2 = constant$ for $r > R$. There is continuity for the *emf* at $r = R$.

At the end, we also wish to point out some rather strange statements recently wrote by G-V:

[1] e-mail: ecogestion@ciudad.com.ar

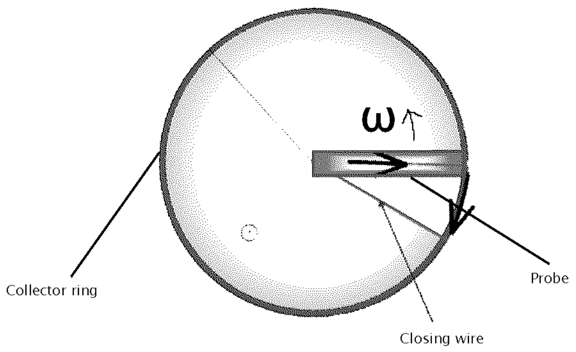


Figure 2: Homopolar generator (top view). The arrows show the integration path available for emf calculation

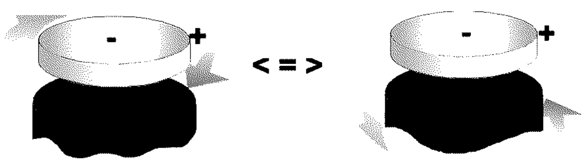


Figure 3: A wire clockwise rotation is equivalent to a magnet Counterclockwise rotation

1. Reference [4]: "Also Jehle's model of the electron must be thoroughly reconsidered. **The standard, classical electromagnetism is confirmed**". Indeed, is Weber's electrodynamics what is confirmed [8], [9], rather than classical theory. According to Panofsky[10] and Shadowitz[11] rotation of an uniform magnet ($\partial B/\partial t = 0$ on nearby conductors) is unable to generate an induced electrical field. G-V experiments rescue the true relativistic nature of electromagnetic induction (Fig. 2), a fact never acknowledged in classical electromagnetism.

2. Reference [5]: "It is worthwhile to stress that the homopolar machine is a famous example where Faraday's flux rule fails". We will show that Faraday's induction law can be sensibly applied to homopolar induction when the topology is correctly understood. Employing Stokes' theorem and remembering that $B = curl A$, Neumann's law becomes $\varepsilon = -d/dt \iint_S B.dS$. We only need to recognize that in dt seconds the probe sweeps the area $dS = r.(r\omega dt)/2$ and B-flux increases in the quantity $BdS = (\omega Br^2/2)\, dt$ across the probe-plus-collector ring plus closing wire closed circuit. The induced current obeys Lenz's rule, since induced current generates a magnetic field that oposes B-flux increase.

References

[1] J. Guala-Valverde and P. Mazzoni, *"The Unipolar Generator, a Genuine Relational Engine"*. Apeiron, **8** 41,52 (2001).

[2] J. Guala-Valverde and P. Mazzoni, *"The Unipolar Motor: A True relativist Engine"*. Spacetime & Substance Journal, **8**, 141 (2001).http://spacetime.narod.ru.

[3] J. Guala-Valverde, *Feynman Lectures, A-field and relativity in rotations"*, Spacetime & Substance Journal, **3**, 94-96 (2002). http://spacetime.narod.ru.

[4] J. Guala-Valverde, *"Spining magnets and Relativity. Jehle vs. Bartlett"*. Physica Scripta, The Royal Swedish Academy of Sciences, **66**, 252 (2002).

[5] J. Guala-Valverde, P. Mazzoni & R. Achilles, *"The Homopolar Motor"*, American Journal of Physics, **70**, 1052 (2002).

[6] A.K.T. Assis, *Weber's Electrodynamics*, Chapter 4.Kluwer, Dordrecht (1994).

[7] J. Palacios, *Electricidad y magnetismo*, Chapter 18. Espasa calpe, Madrid (1959).

[8] A.K.T. Assis, *Weber's Electrodynamics*, Kluwer, Dordrecht (1994).

[9] A.K.T. Assis & D.S. Thober, *Unipolar Induction and Weber's Electrodynamics"*, Frontiers of Fundamental Physics, Plenum, New York (1994).

[10] W.K.H. Panofsky & M. Phillips, *Classical Electricity and Magnetism*, Chapter 22. Addisson Wesley, New York (1955).

[11] A. Shadowitz, *Special Relativity*, Chapter 7. Dover, New York (1968).

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 186–187

WHY HOMOPOLAR DEVICES CANNOT BE ADDITIVE?

Jorge Guala-Valverde[1]

Fundación Julio Palacios and Consejo Provincial de Educación, Salta 326, 8300-Neuquén, Argentina

Received NOvember 10, 2002

Following our task on electrodynamic homopolar induction [1-8], we report new experimental evidence which gives strong support to the **relational** *(i.e. absolutely relativistic) Weber's electrodynamics [9], disproving the customary absolutistic models. The non additivity of homopolar machines only can be understood within a relational framework.*

PACS: 41.20.-q.

1. Short range B-field perturbations

A modified Faraday disk setup allowed us to discover the physics of homopolar phenomena and to locate the seat of the induced ponderomotive and electromotive forces [1-8], a fact denied by Einstein himself in his famous 1905 paper [10]. In our modified version of the Faraday setup, a region of a cylindrical uniform permanent magnet was removed in order to achieve a *B-field* short range inversion, the **singularity** from here on.

The key of the success of the reported experiments lies in the topological features of the magnets singularity. The **short range** field reversion allows the inversion of both ponderomotive and electromotive effects on a *probe* located on the singularity itself, leaving the actions on a *closing wire* **insensitive** to such *B-field* reversion.

In all the quoted experiments we employed a circuit composed by two mecanically decoupled wires: the probe and the closing wire. Electrical continuity was secured making use of mercuy channels.

Now we explore the mechanical behaviour of carrying current rigid loops when interacting with the modified magnet.

Figure 1 (bottom) shows two independent conducting closed loops, axially located on the magnet and symmetrically anchored to the bench probe.The magnet, embedded in a wood cylinder, is firmly anchored to a vertical shaft terminated in sharp points at both ends, the lower one laying on a hard polished surface and the upper one centered by a conical bearing, enabling its almost frictionless rotation.

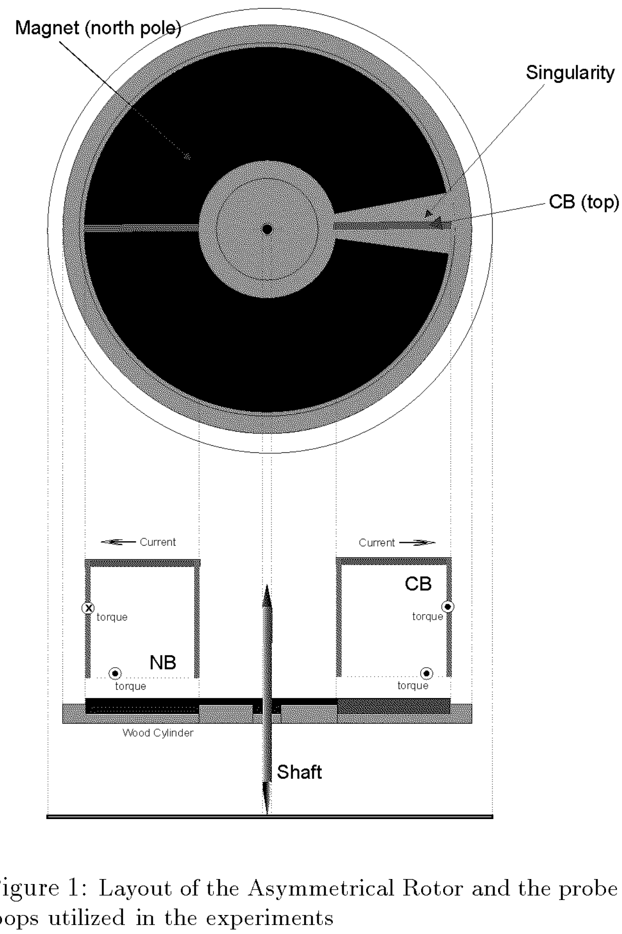


Figure 1: Layout of the Asymmetrical Rotor and the probe loops utilized in the experiments

1.1. Experiment 1.

Direct current (DC) was injected in the left loop as shown in figure1, reaching some 50A. None rotation was

[1]e-mail: fundacionjuliopalacios@usa.net

observed in the magnet, as expected from elementary torque considerations [1-7]. Laplace' s force $Idl \times B$ acting on the wires produces two equal and opposite rotational torques acting respectively on the horizontal *nearby branch* (*NB*) and on the *closing circuit complementary branch* (*CB*). Being *NB* mechanically coupled to *CB*, a null resultant torque acts on the entire loop. According to Newtons Third Law, also the magnet itself experiences a null torque.

Experiment 2

When DC reaches some 1.5A in the right loop, an ostensible (but spatially limited to the singularity) counterclokwise rotation of the magnet is observed.

Due to its short-range nature, the *B-field* perturbation around the singularity is unable to reach *CB*. Briefly speaking, *CB* in the right loop "sees" essentially the same field pattern which *CB* "sees" in the left loop. The entire loop is acted on by a clockwise torque and the magnet reacts with an equal and opposite torque (Newton' s third law), responsible for the observed counterclockwise rotation.

Now both loops were decoupled from the probe bench and then firmly attached to the magnet. None rotation was ostensible when DC reached up to 50 A in both the left loop and the right one.

The three above experiments referred to a motor configuration can be repeated for a generator configuration, with similar outcome. We only report here a crucial experiment in which a 100 turns coil, decoupled from the bench probe, and free to rotate about the shaft, was moved with alternative motion within the singularity. A 250 mV AC signal was measured with the aid of a high impedance metter. The same experiments, with a 300 turns coil was performed in identical conditions, giving a 750 mV AC output.

Later on we attached the 100 turns probe in the magnets singularity. The magnet was here dynamically balanced for accounting of the missing mass. When the set was spun reaching some 1000 rpm, DC voltage never surpassed the 2 mV. The same experiment was repeated, in identical conditions, with the 300 turns coil. Again, DC voltage never surpassed 2 mV.

2. The non additivity of homopolar machines

The outcome of the four above reported experiments becomes trivial working within a relational framework. What matters for the developement of both ponderomotive and electromotive effects, in the relationalists eyes, is the **motion of the loop with respect to the magnet**.

Within an absolutistic framework, what matters in a generator configuration is the **absolute motion** of the conducting probe, being magnets rotation irrelevant as far as $\partial B/\partial t = 0$ at each fixed point in the space. When the loop is located in the magnet's singularity, each branch *NB* and *CB* would behave, in the absolutistic eyes, as two independent generators connected in series, able to deliver the whole emf ε. Consequently, the expected outcome for him would be $N\varepsilon\, mV$ for a N turns coil rotating attached to the magnet in the singularity.

3. Concluding remarks

The non existence of additive homopolar engines is another independent experimental proof that disproves the absolutistic viewpoints on the issue [11, 12,13].

Acknowledgment: To Prof. Adalberto Ipohorsky-Lenkiewicz for helpful comments.

References

[1] J. Guala-Valverde & P. Mazzoni, *Am. J.Phys*, **63**, 228 (1995)

[2] J. Guala-Valverde, P. Mazzoni & Ro. Blas, *Am. J. Phys*, **65**, 147 (1997)

[3] J. Guala-Valverde & P. Mazzoni, *Apeiron*, **8**, 41 (2001).

[4] J. Guala-Valverde & P. Mazzoni, *Spacetime & Sustance,* **3**, 141 (2001).

[5] J. Guala-Valverde, P. Mazzoni & R. Achillles, *Am. J. Phys.*,**70**, 1052 (2002).

[6] J. Guala-Valverde, *Spacetime & Substance*,**3**, 2, 94 (2002).

[7] J. Guala-Valverde, P. Mazzoni & R. Achilles, *New Energy Technologies,* **4**, 7, 37 (2002)

[8] J. Guala-Valverde, *Spacetime & Substance,* **3**, 3, 140 (2002).

[9] J. Guala-Valverde, *Physica Scripta*,**66**, 252 (2002).

[10] A.K.T. Assis and J. Guala-Valverde, *Proceedings of Lanzarote Workshop*. In Press.

[11] A.K.T. Assis a & D.S. Thober, *Frontiers of Fundamental Physics,*, Plenum Press, NY , 409 (1994).

[12] A. Einstein, *Annalen der Physik,* **17**, (1905).

[13] K.K.H. Panofsky, & M. Phillips, *Classical Electricity and Magnetism,* Add. Wes. NY (1955).

[14] A. Shadowitz, *Special Relativity*, Dover, NY (1968).

[15] R. Feynman, *The Feynman Lectures on Physics, II*, Add. Wesley (1964).

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 188–189

DISCUSSION

SYRACUSE UNIVERSITY

DEPARTMENT OF PHYSICS / COLLEGE OF ARTS & SCIENCES

Monday, October 21,2002

Prof. Jorge Guala-Valverde Norpatagonica-R&D Department S. Fe 449 Neuquen, Argentina Q8300BG1

Dear Professor Guala-Valverde,

I must apologize for not having had time to read your paper "The homopolar motor: A true relativistic engine" (Amer. Journal. Physics 70, October 2002) earlier (in preprint form). It is a very fine paper. Please accept my congratulations for producing an excellent piece of work, and please convey this also to your collaborators. Your experiments should remove the last shadow of doubt even of the most skeptical minds, that the electromagnetic phenomena are of a relativistic nature.

Best wishes for future success,

Sincerely yours,

Fritz Rohrlich

Professor of Physics Emeritus

201 Physics Bullding / Syracuse. **New York USA** 13244-1130 **315-443-3901 / Fax:** 31**5-443-9103** / http://physics.syr.edu

STANFORD UNIVERSITY

Stanford Linear Accelerator Center
Mail Address
Bin 76
SLAC, P.O. Box, 4349
Stanford, CA 94309
(415) 926-3988
FAX (415) 926-2395

January 30, 1995

Dr. J. Guala Valverde
Laboratorio Julio Palacios
Sgo, Del Estero 370
8300 Neuquen
ARGENTINA

Dear Dr. Valverde:

Thank you very much for your letter of January 16, 1995, including both the galley proof of your article "The principle of relativity as applied to motional electromagnetic induction" and the unpublished note on "The Motional Electromagnetic Induction Revisited".

You ask for "qualifications" which I interpret to mean comments on the two papers. I have no problem with the analysis in the galley proof of the article to be published in the American Journal of Physics with the exception of its title, I am pleased that you agree that in the circular configuration shown rotation of the magnet in itself will not produce an EMF field. A relative motion between the cylinder and the meter is required to generate an EMF, I am also pleased to read about the experimental verification of these statements; however I see no relationship of these observations using electromagnetic devices which demonstrate a preferred *rotational frames*, but this is not a contradiction with special relativity (which relates to non-accelerated frames) or to general relativity (which describes special properties to frames rotating with respect to a frame in which the preponderance of masses of the universe are at rest).

In view of the above, I find it difficult to understand the concluding remarks in your unpublished manuscript. I see no evidence in your arguments for introducing a preferred ether frame for linear motion.

With many thanks for sending me your material.

Sincerely,

Wolfgang K.H. Panofsky
Director Emeritus

Spacetime & Substance, Vol. 3 (2002), No. 4 (14), pp. 190–192

"GRAVITATION, COSMOLOGY AND RELATIVISTIC ASTROPHYSICS"

2nd Gravitation Conference in Kharkiv

(June 23–27, 2003, Kharkiv, Ukraine)

ORGANIZERS

Ukrainian Gravitational Association
Russian Gravitational Society
Kharkiv National University (Kharkiv, Ukraine)
Institute of Theoretical Physics NASU (Kyïv, Ukraine)
Research and Technological Institute of Transcription, Translation and Replication, JSC (Kharkiv, Ukraine)

SPONSORS

Research and Technological Institute of Transcription, Translation and Replication, JSC (Kharkiv, Ukraine)
Model of the Universe, JSC (Kharkiv, Ukraine)
Kharkiv Regional Foundation for Supporting Enterpreneurship, JSC (Kharkiv, Ukraine)

ADVISORY COMMITTEE

G. S. Bisnovatyi-Kogan (Moskow, Russia)
P.I. Fomin (Kyïv, Ukraine)
V.P. Frolov (Edmonton, Canada)
F.W. Hehe (Koeln, Germany)
B. Korsunoglu (Coral Gables, USA)
V.M. Lipunov (Moskow, Russia)
V.N. Melnikov (Moskow, Russia)
I.D. Novikov (Copenhagen, Denmark)
S.V. Peletminsky (Kharkov, Ukraine)
A. Vilenkin (Medford, USA)
Yu.S. Vladimirov (Moskow, Russia)

LOCAL ORGANIZING COMMITTEE

Yu.V. Alexandrov (Kharkov National University)
L.N. Litvinenko (Radioastronomical Institute)
A.A. Minakov (Radioastronomical Institute)
Yu.P. Stepanovsky (Kharkov National University)
L.V. Verozub (Kharkov National University)
I.I. Zalubovski (Kharkov National University)
V.A. Zakhozhay (Kharkov Astronomical Institute, Chairman)
N.A. Zhuck (RTI TTR, Co-ordinator)

CONTACT ADDRESS

Conference "Gravitation-2003", Kharkiv National University, 61077, Kharkiv, Ukraine
Phone: + 38 (0572) 19-55-77; fax: + 38 (0572) 40-92-98, 40-95-94
E-mail:
grav2003@ukr.net
gravit@univer.kharkov.ua
WWW: www.univer.kharkov.ua/grav2003

WORKING SECTIONS

1. GRAVITATION AND INTEGRATED THEORIES
2. COSMOLOGY AND RELATIVISTIC ASTROPHYSICS
3. ALTERNATIVE THEORIES

THE CONFERENCE WILL TAKE PLACE AT

Kharkiv National University

SPOKEN LANGUAGES

English, Russian, Ukrainian

PUBLICATION OF THE CONFERENCE PAPERS

The Conference Proceedings will be published.

The theses of the reports will have been published by the beginning of the Conference (who have obtained positive recalls of reviewers).

REGISTRATION PAYMENT

The registration fee is equal to 50 US dollars. The registration fees for the participants from Commonwealth of Independent States will be paid by the sponsors.

FINANCIAL SUPPORT

A limited number of the Conference participants from Commonwealth of Independent States will receive the financial assistance of travel and accommodation costs from the Organizing Committee. The personal notifications as for it will be forwarded to the participants until May 15, 2003.

REGISTRATION FORM

Title:
First name:
Middle name:
Family name:
Institution:
Mail address (complete):
E-mail:
Report:
Number of Section — 1/2/3
Title — indicate

Oral / poster — indicate
Lecturer - indicate if there are co-authors
Hotel reservation — Yes / No
Single, double, deluxe — indicate Necessity of financial support (for participants from Commonwealth of Independent States):
Payment of travel — Yes/No
Payment of accomodation — Yes/No
Necessity of the invitation letter — Yes/No

THESES OF THE REPORTS

They are given in one of spoken languages at the conference in the amount no more than 1 (one) page of the format A5 with the margin of 15 mm and the font size 10 pt in the format LATEX 2e or LATEX 2.09. The style file is on the Conference web-page.

TERM OF DELIVERY

The Registration Form and theses of the reports should be sent to the Organizing Committee until April 20, 2003 by E-mail.

VISA

Everyone who needs visa should give the information additionally:
Birth date and birth-place:
Sex:
Passport number and its issue date:
Date of arrival:
Date of departure:
Nationality (citizenship):

Spacetime & Substance
International Physical Journal

INFORMATION FOR AUTHORS

The Editorial Council accepts the manuscripts for the publication only in an electronic variant in the format for LATEX 2.09. They should be completely prepared for the publication. The manuscripts are accepted by e-mail or on diskettes (3.5"). The manuscripts can be adopted in other view only for familiarization.

The original manuscripts should be preferably no longer then 6 pages. They should contain no more than 4 figures. Length of the manuscript can be up to 10 pages only in exclusive cases (at arguing problems of primary importance). If the length of the manuscript exceeds 10 pages, it should be divided by the author into two or more papers, each of which should contain all pieces of a separate paper (title, authors, abstracts, text, references etc.). The Editorial Council accepts for the publication the brief reports too.

The payment for the publication of the manuscripts is not done. Each author gets the electronic version of that Journal edition, in which his paper was published free of charge.

An E-mail message acknowledging the receipt of the manuscript will be sent to the corresponding author within two working days after the manuscript receipt. If a message is not received please contact *krasnoh@iop.kiev.ua* to inquire about the manuscripts.

The Style File and Instructions for its use can be found at http://spacetime.narod.ru (sample.zip, 19 kb).

An abstract (within 20 lines) must be submitted. This one should be concise and complete regardless of the paper content. Include purpose, methodology, results, and conclusions. References should not be cited in the abstract. The abstract should be suitable for separate publication in an abstract journal and be adequate for indexing.

If the argument of an exponential is complicated or long, "exp" rather than "e" should be used. Awkward fractional composition can be avoided by the proper introduction of negative degrees. Solidus fractions (l/r) should be used, and enough enclosures should be included to avoid ambiguity in the text. According to the accepted convention, parentheses, brackets, and braces are in the order { [( )] }. Displayed equations should be numbered consecutively throughout the paper; the number (in parentheses) should be to the right of the equation.

Figures (black-and-while) should be of minimal size providing clear understanding. Breadth of the figure should not exceed 84 mm or 174 mm (in exclusive cases). Figures should be made out as separate files in the format of *.pcx (300 dpi/inch) or *.eps (minimum of kb).

Each figure must be cited in numerical order in the text and must have figure legend.

Tables should be typed as authors expect them to look in print. Every table must have a title, and all columns must have headings. Column headings must be arranged so that their relation to the data is clear. Footnotes should be indicated by reference marks [1], [2] etc. or by lowercase letters typed as superiors. Each table must be cited in the text.

The Editorial Council accepts also response on papers, published in the Journal. They should be no more than 1 journal page in length and should not contain figures but only to refer to the already published materials. But they can contain the formulas. The recalls are publishing in section "Discussion."

The list of references may be formed either by first citation in the text, or alphabetically.

Only works cited in the text should be included in the reference list. Personal communications and unpublished data or reports are not included in the reference list; they should be shown parenthetically in the text: (F.S. Jones, unpublished data, 1990).

The title of paper is permissible not to indicate. It is permissible to give only the initial page number of a paper. The format of the reference list is as indicated below.

References

[1] F.W. Stecker, K.J. Frost, *Nature*, **245**, 270 (1973).

[2] V.A. Brumberg, "Relativistic Celestial Mechanics", Nauka, Moskow, 1972 (in Russian).

[3] S.W. Hawking, in: "General Relativity. An Einstein Centenary Sutvey", eds. S.W. Hawking and W. Israel, *Cambr. Univ. Press*, Cambridge, England, 1979.

Read the Journal before sending a manuscript!

Spacetime & Substance Volume 3, No. 4 (14), 2002

CONTENTS