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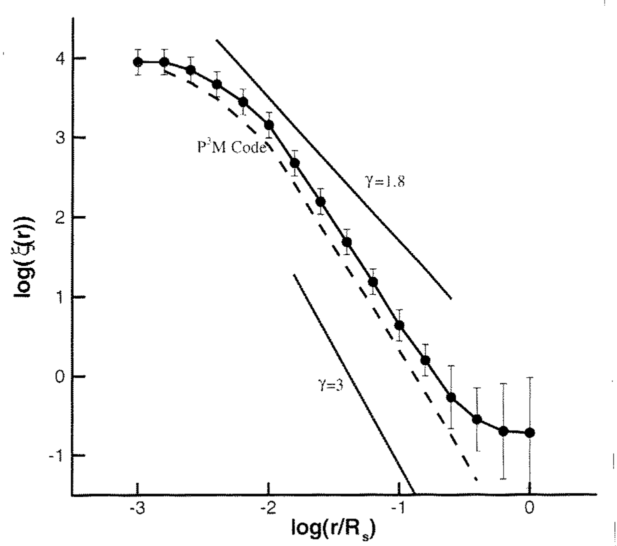
# Spacetime & Substance

**International Physical Journal**

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# Spacetime & Substance

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# QUADRATIC RED SHIFT LAW AND THE NON-ARCHIMEDEAN UNIVERSE

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The quadratic law of the cosmic red-shift proposed recently by Segal et al. and Troitskii et al. is explained in a static non-Archimedean universe by the "compression" of the radial distance metric. The metric is compressed according to the universal mapping function $\nu(x) = tanhx$.

## 1. Introduction

The widely accepted standard cosmology is based on the notion that the space- time metric at an earlier epoch (about 20 billion years ago) was singular. All the matter and the energy of the universe was concentrated in single infinitesimal point. The experimental proof for this physically absurd but widely accepted notion is suppose to have come from the Hubble's law which states that the red shift of distant galaxies is proportional to the distance from the observer. The linear law, first published in 1929 by Hubble was culmination of a decade of earlier research by a number of workers like Slipher, Wirtz and Lundmark on the subject of the cosmic red shift. Later work by Hubble and Humason [1] in 1939 seems to have confirmed the linear law. This resulted in its wide acceptance though there were reservations expressed by Zwicky [2] on account of use of bright galaxy cluster samples. Apart from the simplicity, the major impetus for the acceptance of the linear law came from the fact that it seemed to confirm some earlier theoretical concepts about expanding universe by Friedman, Lemaitre and De-Sitter. Later work by Gamow on remanant cosmic radiation and element abundance and recent work by Penrose and Hawking on gravitational singularities seems to have nailed the issue in favour of the linear law and the expanding universe.

Inspite of these brilliant successes, there were always nagging doubts about the validity of the linear law. In particular Chip Arp and co-workers [3] have been pointing out for some time (apparently without much success) that the red shift from quasars and other large scale structures show an anomolous behaviour which cannot be explained by the linear law. In fact, based on extensive investigations, Segal et al. [4] and Troitskii and his co-workers [5] have recently shown that the quadratic law $z\alpha R^2$ (z is the red shift and $R$ is the distance of the object) is a better fit to the available data on cosmic red shift than the linear $z\alpha R$.

If this law is true, then there must be serious reconsiderations about expanding universe and standard cosmology. There is no way in which the quadratic law can be accomodated in it. To explain this law Segal et al. have resorted to chronometric cosmology [4] while Troitskii has attributed it to gravitational efiects. Very recently [6] we have explained the cosmic red shift in a static universe starting from a very general principle which states that "magnitude of all the physical quantities in the universe is bounded from above". The red shift arises due to radial compression of the metric in a universe of finite extent. In this paper we show how these notions can be used to explain the quadratic law of the red shift.

## 2. Non-Archimedean Algebra

As stated earlier we start from very general principle which states that the magnitude of physical quantities is finite. Now, ordinary algebra and mathematics which is based on the Archimedean axiom that "there is no largest number" is unsuited to deal with such quantities. As we have recently shown in a series of papers [7-12], internally consistent, alternate algebras based on the notion of a "largest number" can be constructed. These algebras which are called "Non-Archimedean" [NA] algebras are most suited to deal with physical quantities whose magnitudes are bounded from above.

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In this section we describe some basic rules of NA which will be used later in the discussion.

Let us begin by declaring $M$ to be the "largest" number. Our first task, then, is to construct an internally consistent number system and an algebra for such number which will respect the axiom that $M$ is the largest number. That is, no operation of this algebra or mathematical operations based on it will ever result in a number greater than $M$. As has been shown [11], such a number field can be obtained from the ordinary unbounded number field by a mapping function $\nu(x)$ and its inverse $\tau(x)$ with following properties [12]:

$$\nu(0) = \tau(0) = 0; \tau(\pm\infty) = M; \tau(\pm M) = \pm\infty. \quad (1)$$

Further, the four basic operation for the new numbers are given by

$$x \overset{\circ}{+} y = \nu[\tau(x) + \tau(y)], x \overset{\circ}{-} y = \nu[\tau(x) - \tau(y)] \quad (2)$$

$$x \circ y = \nu[\tau(x) \times \tau(y)], x \overset{\circ}{\circ} y = \nu[\tau(x)/\tau(y)] \quad (3)$$

Thus once $\nu(x)$ and $\tau(x)$ satisfying Eq. (1) are specified, then the corresponding set of basic operations and the entire mathematical structure based on them is determined. It should be noted that because of isomorphism the NA algebras are as consistent as the usual Archimedean algebra.

What, then is thus universal mapping function $\nu(x)$ according to which all physical quantities are to be manipulated? Clearly mathematical considerations alone cannot uniquely specify $\nu(x)$. This function is to be fixed by the appeal to the experimental observations. The most suitable observation for this is that of the cosmic red shift which we discuss in the next section.

### 2.1. Cosmic Red Shift

To discuss this, we consider a spherically symmetric, at and static universe of radius $R_0$. In this universe the distance between two points $A$ and $B$ is bounded from above, i.e. $R \leq R_0$. Because of this, the radial distance metric is "compressed" as one travels outwards from the point of observation towards horizon [Fig. 1a]. This "compressed" metric is to be obtained by mapping all the points on the radius of an $\infty$ sphere to the radius of a finite sphere $R_0$ using mapping function $\nu(R)$ (which is yet to be determined). Our discussion of the red shift will be based on two principles. (a) cosmological principle - which states that the universe looks identical from all vantage points. Thus if $B$ is the observation point [Fig. 1b] then the universe looks identical with a sphere of radius $R_0$ redrawn around $B$. (b) universe locally is Archimedean i.e. $\nu(R) \simeq R$ for $R \ll R_0$.

Thus NA effect will be apparent only when the magnitudes are close to their upper bounds i.e. $R \leq R_0$. One such object is the radiation from the distant objects. Clearly, one can see that the light emitted by

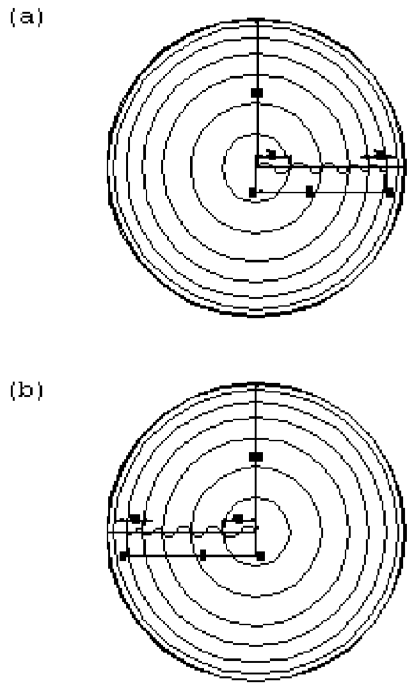

Figure 1: a) Non-Arcimedean universe as seen from point A.The event horizon is the outermost boundry. The light from B is redshifted when received at A. b) The same universe as seen from point B. The light from A is redshifted when received at B.

a star at $B$ will be red shifted when observed at $A$ [Fig. 1a]. Similarly, light emitted by a star at $A$ will be red shifted when observed at $B$ [Fig. 1b]. Thus the phenomenon of red shift is a NA effect in a static universe. The function $\nu(R)$ can be related to the red shift $z$ as follows. Let the light be emitted at time $t = 0$ from $B$ and observed at $t = t_2$ when received at $A$. If the distance between $A$ and $B$ is $R$ then $R/R_0 = \nu[ct_2/R_0]$, where $c$ is the velocity of light. Thus the light is received at $A$ at time $t_2 = \frac{R_0}{c}\tau(\overline{R})$, $\overline{R} = R/R_0$ and the wavelength of the radiation measured at $A$ is $\lambda_2 = ct_2/n$ where $n$ is the number of wavelength between $A$ and $B$. On the other hand, if the universe was infinite, then the light would have been received at time $t_1 = R/c$ and the wavelength would have been $\lambda_1 = ct_1/n$. Thus the red shift $z = (\lambda_2 - \lambda_1)/\lambda_1$ is given by

$$z = \frac{\tau(\overline{R})}{\overline{R}} - 1 \quad (4)$$

Now extensive work by Segal et al. [4] and Troitskii et al. [5] have shown that $z\alpha(R/R_0)^2$. If we choose $\nu(\overline{R}) = R_0 tanh \overline{R}$, $\tau(\overline{R}) = R_0 tanh^{-1}\overline{R}$, then we can easily show that to leading order in $(R/R_0)$

$$z = \frac{1}{3}(R/R_0)^2 \quad (5)$$

Thus the quadratic red shift can be explained in a static, NA universe by postulating the universal mapping function $\nu(x) = tanhx$. Earlier we had shown that corresponding to the linear law $z\alpha\overline{R}$, the universal mapping function is $\nu(x) = Mx/(M+x)$. In fact the idea is to deduce the mapping function $\nu(R)$ from the cosmic red shift data. The situation is somewhat similar to case of geometry where it is experimental observation which are suppose to decide which out of three geometries i.e. hyperbolic, elliptic or at is applicable to the universe. It should be noted that as in Desetter's universe, the boundry $R_0$ in the NA universe is the "event horizon" and the red shift $z \to \infty$ and $R \to R_0$, that is the light will take infinitely long time to go from $R = 0$ to $R = R_0$. To see this, the distance $R$ covered by the light in time $t$ is $R = R_0 tanhct/R_0$, thus $t \to \infty$ as $R \to R_0$. Also in this universe all the points within the metric are equidistant from the event horizon i.e. $R_0 \overset{\circ}{-} R = R_0$ for all $R$.

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# ADELIC QUANTUM COSMOLOGY

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Adelic quantum cosmology [1] is an application of adelic quantum theory to the universe as a whole. Adelic approach takes into account all archimedean and non-archimedean geometries based on real and $p$-adic numbers, respectively. Calculation of the corresponding adelic wave function of the universe exploits Feynman's path integral method. In this contribution we will give a short review of $p$-adic numbers and adeles, as well as motivation and formulation of adelic quantum cosmology. Adelic wave functions for a few minisuperspace models will be presented. There is some discreteness of minisuperspaces, which is a consequence of $p$-adic quantum effects and depends on adelic quantum state of these models.

## 1. Introduction

According to classical mechanics, space and time are continuous, and distances can be in principle measured by any accuracy. However, in quantum mechanics there is the uncertainty relation $\Delta x \Delta k \geq \hbar/2$ that imposes a restriction on simultaneous measuring of position $x$ and momentum $k$. Moreover, quantum mechanics combined with general relativity yields [2] $\Delta x \geq l_{pl} = (G\hbar/c^3)^{1/2} \sim 10^{-35}m$, i.e. the Planck length is the minimum one which can be measured. Thus, nothing can be said about the structure of space-time beyond the Planck scale. In fact, this result is derived using concepts of archimedean geometry and real numbers. Due to this reason it seems to be quite natural at the Planck scale to take into account also non-archimedean geometry based on $p$-adic numbers. Mathematically (Ostrowski theorem), any nontrivial norm on the field of rational numbers $Q$ is equivalent either to the absolute value $|\cdot|_\infty$ or to the $p$-adic norm $|\cdot|_p$ ($p$ is a prime number). Completions of $Q$ with respect to the absolute value and $p$-adic norm give the field of real numbers $R \equiv Q_\infty$ and the fields of $p$-adic numbers $Q_p$, respectively. Any $p$-adic number [3, 4] $x \in Q_p$ can be presented as

$$x = p^\gamma \sum_{i=0}^{\infty} x_i p^i, \;\; x_i = 0, 1, .., p-1, \tag{1}$$

$\gamma \in Z$. If we wish to take into account all possible geometries to study our universe, then a natural mathematical instrument to do that is just adelic theory. An adele [5] $a$ is an infinite sequence

$$a = (a_\infty, a_2, ..., a_p, ...), \tag{2}$$

where $a_\infty \in Q_\infty$ is a real number and $a_p \in Q_p$ is a $p$-adic number, with restriction $|a_p|_p \leq 1$ for all but a finite number of $p$. The set of all adeles $\mathcal{A}$ is a ring under componentwise addition and multiplication. An additive character on $\mathcal{A}$ is

$$\chi(xy) = \prod_v \chi_v(x_v y_v) = \exp(-2\pi i x_\infty y_\infty) \times \prod_p \exp(2\pi i \{x_p y_p\}_p), \quad x, y \in \mathcal{A}, \tag{3}$$

where $\{a_p\}_p$ denotes the fractional part in expansion (1) of $a_p$. In the case of map $f : Q_p \to C$ (also $Q_\infty \to C$) there is well defined translatory invariant Haar measure $dx$ with the property $d_p(ax) = |a|_p dx$, $a \neq 0$. We use here the Gauss integral

$$\int_{|x|_p \leq p^\gamma} \chi_p(ax^2 + bx)dx = \begin{cases} p^\gamma \Omega(p^\gamma |b|_p), & |4a|_p \leq p^{-2\gamma}, \\ \frac{\lambda_p(a)}{|2a|_p^{1/2}} \chi_p\left(-\frac{b^2}{4a}\right) \Omega(p^{-\gamma}|\frac{b}{2a}|_p), & |4a|_p > p^{-2\gamma}, \end{cases} \tag{4}$$

$\gamma \in Z$, where $\lambda_p(a)$ is the arithmetic function for which holds [3] $|\lambda_p(a)|_\infty = 1, \lambda(a)\lambda(-a) = 1, \lambda(ac^2) = \lambda(a), \lambda_p(a)\lambda_p(b) = \lambda_p(a+b)\lambda_p(a^{-1}+b^{-1})$. The characteristic function $\Omega$ in (4), defined by $\Omega(u) = 1$ if $u \leq 1$ and $\Omega(u) = 0$, if $u > 1$, plays a role of a vacuum state in $p$-adic quantum mechanics. Since 1987, there has been a significant investigation in construction of physical models with $p$-adic numbers and adeles (for a review, see [3]).

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## 2. Adelic Quantum Mechanics

Since field of $p$-adic numbers $Q_p$ is totally disconnected, there is no possibility to define $p$-adic "momentum" and "Hamiltonian" operator on appropriate way. This operators in the real case are infinitesimal generators of space and time translations, but in $p$-adic case these infinitesimal translations become meaningless. In $p$-adic quantum mechanics [3] also multiplication $\hat{q}\psi \to x\psi$ ($x$ is a position coordinate, $\psi$ is a wave function) has no meaning because $x \in Q_p$ is a $p$-adic and $\psi \in C$. But finite transformations are meaningful and the corresponding Weyl and evolution operators are $p$-adically well defined.

Dynamics of $p$-adic quantum models is described by a unitary evolution operator $U(t)$ in terms of its kernel $\mathcal{K}$

$$U_p(t)\psi(x'') = \int_{Q_p} \mathcal{K}_t(x'', x')\psi(x')dx'. \tag{5}$$

Ordinary and $p$-adic quantum mechanics can be unified in the form of adelic quantum mechanics which is a triple [6]

$$(L_2(\mathcal{A}), W(z), U(t)) \tag{6}$$

where $L_2(\mathcal{A})$ is the Hilbert space of complex valued functions of adelic variables, $W(z)$ is a unitary representation of the Heisenberg-Weyl group on $L_2(\mathcal{A})$ and $U(t)$ is a unitary representation of the evolution operator on $L_2(\mathcal{A})$.

In adelic approach eigenvalue problem for $U(t)$ reads

$$U(t)\psi_{\alpha\beta}(x) = \chi(E_\alpha t)\psi_{\alpha\beta}(x), \tag{7}$$

where $\psi_{\alpha\beta}$ are adelic eigenfunctions, $E_\alpha = (E_\infty,\ E_2,\ \ldots, E_p, \ldots)$ is the corresponding energy, indicies $\alpha$ and $\beta$ denote energy levels and their degeneration.

Adelic eigenfunction [6] has the form

$$\psi(x) = \psi_\infty(x_\infty) \prod_{p\in M} \psi_p(x_p) \prod_{p\notin M} \Omega(|x_p|_p), \tag{8}$$

where $x \in \mathcal{A}$, $\psi_\infty \in L_2(R)$, $\psi_p \in L_2(Q_p)$ and $M$ is a finite set of primes $p$.

Kernel of the evolution operator (5) is given by the $p$-adic Feynman path integral

$$\mathcal{K}_p(x'', t''; x', t') = \int_{(x',t')}^{(x'',t'')} \chi_p(-S[q])\mathcal{D}q$$
$$= \int \chi_p\left(-\int_{t'}^{t''} L(\dot{q}, q)dt\right)\mathcal{D}q(t) \tag{9}$$

(with the Planck constant $h = 1$). For the systems with quadratic actions (in the case of $n$ uncoupled degrees of freedom) this $p$-adic path integral has the form [7]

$$\mathcal{K}_p(x''_\alpha, t''; x'_\alpha, t') = \lambda_p\left(-\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'_\alpha \partial x''_\alpha}\right)$$
$$\times \left|\frac{\partial^2 \bar{S}}{\partial x'_\alpha \partial x''_\alpha}\right|_p^{1/2} \chi_p(-\bar{S}(x''_\alpha, t''; x'_\alpha, t')), \tag{10}$$

where $\alpha = 1, \ldots, n$. Note that expression has the same form as in standard quantum mechanics.

## 3. Adelic Quantum Cosmology

Adelic quantum cosmology is an application of adelic quantum theory to the universe as a whole. Adelic quantum theory unifies both $p$-adic and standard quantum theory [6]. In the path integral approach to standard quantum cosmology the starting point is Feynman's idea that the amplitude to go from one state with intrinsic metric $h'_{ij}$, and matter configuration $\phi'$ on an initial hypersurface $\Sigma'$, to another state with metric $h''_{ij}$, and matter configuration $\phi''$ on a final hypersurface $\Sigma''$, is given by a functional integral of the form

$$\langle h''_{ij}, \phi'', \Sigma''|h'_{ij}, \phi', \Sigma'\rangle_\infty =$$
$$= \int \mathcal{D}(g_{\mu\nu})_\infty \mathcal{D}(\Phi)_\infty \chi_\infty(-S_\infty[g_{\mu\nu}, \Phi]), \tag{11}$$

over all four-geometries $g_{\mu\nu}$, and matter configurations $\Phi$, which interpolate between the initial and final configurations [8]. The $S_\infty[g_{\mu\nu}, \Phi]$ is the usual Einstein-Hilbert action

$$S[g_{\mu\nu}, \Phi] = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4x\sqrt{-g}(R - 2\Lambda)$$
$$+\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3x\sqrt{h}K$$
$$-\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}[g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + V(\Phi)] \tag{12}$$

for the gravitational field and matter fields $\Phi$. To perform $p$-adic and adelic generalization we first make $p$-adic counterpart of the action (12) using form-invariance under change of real to the $p$-adic number fields [2]. Then we generalize (11) and introduce $p$-adic complex-valued cosmological amplitude

$$\langle h''_{ij}, \phi'', \Sigma''|h'_{ij}, \phi', \Sigma'\rangle_p$$
$$= \int \mathcal{D}(g_{\mu\nu})_p \mathcal{D}(\Phi)_p \chi_p(-S_p[g_{\mu\nu}, \Phi]). \tag{13}$$

The space of all 3-metrics and matter field configurations on a 3-surface is called superspace, which is the configuration space in quantum cosmology. Superspace has infinite dimensions with a finite number of coordinates $(h_{ij}(\vec{x}), \phi(\vec{x}))$ at each point $\vec{x}$ of the 3-surface. In practice, the work with these infinite dimensions is impossible. One useful approximation therefore is to truncate the infinite degrees of freedom to a finite number

$q^\alpha(t)$, $(\alpha = 1, 2, ..., n)$. In this way one obtains a particular minisuperspace model. Usually, one restricts the 4-metric to be of the form $ds^2 = -N^2(t)dt^2 + h_{ij}dx^i dx^j$, where $N(t)$ is the laps function. For such minisuperspaces, functional integrals (11) and (13) are reduced to functional integration over 3-metrics, matter configurations and to one usual integral over the laps function. If one takes boundary condition $q^\alpha(t'') = q^{\alpha\prime\prime}$, $q^\alpha(t') = q^{\alpha\prime}$ then integrals in (11) and (13), in the gauge $\dot{N} = 0$, are standard and $p$-adic minisuperspace propagators, respectively. In this case, for the $v$- adic minisuperspace propagator (unifies standard and $p$-adic), we have

$$\langle q^{\alpha\prime\prime}|q^{\alpha\prime}\rangle_v = \int dN \mathcal{K}_v(q^{\alpha\prime\prime}, N|q^{\alpha\prime}, 0), \tag{14}$$

where

$$\mathcal{K}_v(q^{\alpha\prime\prime}, N|q^{\alpha\prime}, 0) = \int \mathcal{D}q^\alpha \chi_v(-S_v[q^\alpha]) \tag{15}$$

is an ordinary quantum-mechanical propagator between fixed $q^\alpha$ and $N$, and index $v = \infty, 2, 3, \ldots, p, \ldots$ denotes real and $p$-adic cases.

A necessary condition to construct an adelic model is existence of the $p$-adic (vacuum) state $\prod_{\alpha=1}^n \Omega(|q^\alpha|_p)$, which satisfies equation

$$\prod_{\alpha=1}^{n} \int_{|q^{\alpha\prime}|_p \le 1} \mathcal{K}_p(q^{\alpha\prime\prime}, N|q^{\alpha\prime}, 0)dq^{\alpha\prime} = \prod_{\alpha=1}^{n} \Omega(|q^{\alpha\prime\prime}|_p) \tag{16}$$

for all but a finite number of $p$. The corresponding adelic eigenstates have the form (8).

## 4. Some adelic minisuperspace models

To illustrate the above approach we shall consider some one and two dimensional minisuperspace models.

### 4.1. Adelic de Sitter model

The de Sitter model is in quantum cosmology the simplest nontrivial exactly soluble model. This model is given by the Einstein-Hilbert action with cosmological term (12) without matter fields, and by the Robertson-Walker metric

$$ds^2 = \sigma^2(-N^2(t)dt^2 + a^2(t)d\Omega_3^2) \tag{17}$$

where $\sigma^2 = \frac{2G}{3\pi}$ and $a(t)$ is a scale factor. Instead of (17) we prefer metric in the form

$$ds^2 = \sigma^2\left(-\frac{N^2(t)}{q(t)}dt^2 + q(t)d\Omega_3^2\right), \; q(t) > 0, \tag{18}$$

which was considered in the real case [9], and in the $p$-adic and adelic [10] (in the $p$-adic generalization of the non-boundary Hartle-Hawking approach) case, because it leads to the quadratic actions. The corresponding adelic action for this one-dimensional minisuperspace model contains

$$S_v[q] = \frac{1}{2}\int_{t'}^{t''} dtN\left(-\frac{\dot{q}^2}{4N^2} - \lambda q + 1\right), \tag{19}$$

where $\lambda = \frac{\Lambda\sigma^2}{3}$. The classical equation of motion (in the gauge $\dot{N} = 0$) $\ddot{q} = 2\lambda$ with the boundary conditions $q(0) = q'$, $q(T) = q''$ $(T = t'' - t')$ has solution

$$q(t) = \lambda t^2 + (\frac{q'' - q'}{T} - \lambda T)t + q', \tag{20}$$

and the corresponding classical action is

$$\bar{S}(q'', T|q', 0) = \frac{\lambda^2 T^3}{24} - [\lambda(q' + q'') - 2]\frac{T}{4} - \frac{(q'' - q')^2}{8T}. \tag{21}$$

Since the action is quadratic, then the kernel (15) is

$$\mathcal{K}_v(q'', T|q', 0) = \frac{\lambda_v(-8T)}{|4T|_v^{1/2}}\chi_v(-\bar{S}_v). \tag{22}$$

One can show, applying formula (16) and (4), existence of a necessary $p$-adic vacuum state in the form $\Omega$ function, which is

$$\Psi_p(q, T) = \begin{cases} \Omega(|q|_p), & |T|_p \le 1, p \ne 2, \\ \Omega(|q|_2), & |T|_2 \le \frac{1}{2}, p = 2, \end{cases} \tag{23}$$

under condition $\lambda = 4 \cdot 3 \cdot l$, $l \in Z$.

### 4.2. Model with cosmological constant in 3 dimensions

This model in the real case is considered in the paper [11]. Its metric is

$$ds^2 = \sigma^2(-N^2(t)dt^2 + a^2(t)(d\theta^2 + \sin^2\theta d\varphi^2)), \tag{24}$$

with $\sigma = G$. Classical equation of motion $\ddot{a} - N^2 a\lambda = 0$ has the solution

$$a(t) = \frac{1}{2\sinh(N\sqrt{\lambda})}\left((a'' - a'e^{-N\sqrt{\lambda}})e^{N\sqrt{\lambda}t} + (a'e^{N\sqrt{\lambda}} - a'')e^{-N\sqrt{\lambda}t}\right), \tag{25}$$

with the boundary conditions $a(0) = a'$, $a(1) = a''$. For the classical action it gives

$$\bar{S}(a'', N|a', 0) = \frac{1}{2\sqrt{\lambda}}[N\sqrt{\lambda} + \lambda(\frac{2a''a'}{\sinh(N\sqrt{\lambda})} - \frac{a'^2 + a''^2}{\tanh(N\sqrt{\lambda})})]. \tag{26}$$

Quantum-mechanical propagator has the form

$$\mathcal{K}_v(a'', N|a', 0) = \frac{\lambda_v(-2\sinh N)}{|\lambda^{-1/2}\sinh(N\sqrt{\lambda})|_v^{1/2}} \chi_v(-\bar{S}(a'', N|a', 0)). \tag{27}$$

For this model also exists $p$-adic vacuum state

$$\Psi_p(a,N) = \begin{cases} \Omega(|a|_p), & |N|_p \leq 1, p \neq 2, \\ \Omega(|a|_2), & |N|_2 \leq \frac{1}{4}, p = 2, \end{cases} \tag{28}$$

with conditions $|\lambda|_p \leq 1$ and $|\lambda|_2 \leq 2$.

### 4.3. Some two dimensional models

There exists a class of two-dimensional minisuperspace models which after some transformations have the form of two oscillators [12, 13]. These models are: the isotropic Friedmann model with conformally and minimally coupled scalar field and the anisotropic vacuum Kantowski-Sachs model. For all these three models action may be written as

$$S = \frac{1}{2}\int_0^1 dtN\left[-\frac{\dot{x}^2}{N^2} + \frac{\dot{y}^2}{N^2} + x^2 - y^2\right], \tag{29}$$

i.e. this is the action for two oscillators, but one of them has a negative energy. This expression leads to the propagator

$$\mathcal{K}_v(y'', x'', N|y', x', 0) = \frac{1}{|N|_v}$$
$$\times\chi_v\left(\frac{x'^2 + x''^2 - y'^2 - y''^2}{2\tan N} + \frac{y'y'' - x'x''}{\sin N}\right). \tag{30}$$

The linear harmonic oscillator is well analyzed system from real as well as from $p$-adic point of view. One can show that in the $p$-adic region of convergence of analytic functions $\sin N$ and $\tan N$, which is $G_p = \{N \in Q_p : |N|_p \leq |2p|_p\}$, exists vacuum state $\Omega(|x|_p)$ $\Omega(|y|_p)$.

## 5. Conclusion

For some standard minisuperspace models, we constructed the corresponding $p$-adic and adelic minisuperspace models. For all these models there exist adelic wave functions of the form

$$\Psi(q^1, ..., q^n) = \prod_{\alpha=1}^{n} \Psi_\infty(q^\alpha_\infty) \prod_p \prod_{\alpha=1}^{n} \Omega(|q^\alpha_p|_p), \tag{31}$$

where $\Psi_\infty(q^\alpha_\infty)$ are the corresponding wave functions of the universe in standard cosmology. Adopting the usual probability interpretation of the wave function (31) in rational points of $q^\alpha$, we have

$$\left|\Psi(q^1, ..., q^n)\right|^2_\infty = \prod_{\alpha=1}^{n} |\Psi_\infty(q^\alpha)|^2_\infty \prod_p \prod_{\alpha=1}^{n} \Omega(|q^\alpha|_p), \tag{32}$$

because $(\Omega(|q^\alpha|_p))^2 = \Omega(|q^\alpha|_p)$. As a consequence of $\Omega$-function properties we have

$$\left|\Psi(q^1, \ldots, q^n)\right|^2_\infty = \begin{cases} |\Psi_\infty(q^\alpha)|^2_\infty, & q^\alpha \in Z, \\ 0, & q^\alpha \in Q\backslash Z. \end{cases} \tag{33}$$

This result (as those in [1]) leads to the discretization of the minisuperspace coordinates $q^\alpha$, because probability is nonzero only in the integer points of $q^\alpha$. Note that this kind of discreteness depends on adelic quantum state of the universe. When system is in some excited state, the sharpness of the discrete structure dissapears and minisuperspace demonstrates usual continous properties.

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# CONDITIONAL COSMOLOGICAL PRINCIPLE AND FRACTAL COSMOLOGY

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Assuming a fractal distribution of matter in the universe, consequences that follow from the General Theory of Relativity and the Copernican Principle for fractal cosmology are examined. The change in perspective necessary to deal with a fractal universe is highlighted. An ansatz that provides a concrete application of the **Conditional Cosmological Principle** is provided. This fractal cosmology is obtained by arguments closely following those used in standard cosmology. Such a fractal cosmological model can incorporate homogeneous radiation only if its fractal dimension is 2. The resulting model may play a significant role in the debate on whether the universe is a fractal or crosses over to homogeneity at some scale. This model may also be regarded as an idealized fractal model around which more realistic models may be built.

Standard cosmology is based on the assumption of homogeneity and isotropy of the Universe, the so-called Cosmological Principle, on scales greater than $10^8$ light years. During the last decade this assumption has come to be challenged. The number of galaxies $N(r)$ within a sphere of radius $r$, centred on any galaxy, is not proportional to $r^3$ as would be expected of a homogeneous distribution. Instead $N(r)$ is found to be proportional to $r^D$, where $D$ is approximately equal to 2. This is symptomatic of a distribution of fractal dimension $D$.

It has further been argued [1] that available evidence indicates that the fractal distribution of visible matter extends well upto the present observational limits without any evidence of cross-over to homogeneity. This suggests that the entire Universe could be a fractal. At present this question is being hotly debated [2, 3]. Controversy remains, interalia, for want of agreement on proper treatment of raw observational data sets. As a result, the available observational evidence has not been able to pronounce unequivocally whether the Universe becomes homogeneous at large enough scales or whether it continues to remain a fractal indefinitely.

Even though the evidence in favour of a fractal universe that does not homogenize on any scale is not undisputed, it must be remembered that no generally acceptable structure formation scenario has yet emerged within the framework of the standard big-bang cosmologies.

In the absence of a viable alternative, it will be a useful exercise to take the fractal distribution as given a priori and examine the consequences for a cosmological model that follow from the General Theory of Relativity and the Copernican Principle.

If the Universe does not homogenize on any length scale, can one have a cosmological model consistent with this fractal picture, which could also be reasonably concordant with observations.

Apart from controversies in interpretation of observational data, a major obstacle in the acceptance of a fractal cosmography is that no cosmolgical model has been proposed which would support this cosmography, be consistent with the General Theory of Relativity and satisfy the Copernican principle.

Assumptions, like the Cosmological Principle, helped give simplistic solutions to the Einstein's equations. When simplifying assumptions are dropped, chaotic solutions of Einstein's equations can be obtained [6]. Chaotic dynamical systems are intimately linked with fractals. This suggests a need to explore the cosmological implications of chaotic solutions of Einstein's equations. In creating models of the Universe, one invariably encounters fine-tuning problems. In order to overcome these problems, the inflationary scenario was proposed. However, this transfers the fine-tuning problem from cosmology to the underlying particle physics models. Thus there is no successful inflationary model, which can actually achieve the goals for which it was proposed. Models with fine-tuning problems are not regarded as acceptable because they lack predictive capability. In contrast to the approach followed by cosmologists, it should be appreciated that chaotic dynamical systems, by definition, suffer from fine-tuning

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problems, yet they have been useful in modeling physical phenomenon. This indicates that there is a need for seeking chaotic cosmological models exploiting the recent advances and successes in the fields of chaos and fractals. In particular one may expect that the fractal description, so successful in characterizing the statistics of dissipative eddies in turbulent flows, should also be useful for the statistical analysis of gravitational clustering.

Self-similarity is ubiquitous in nature. Perhaps it is the true 'Cosmological Principle'. Although, a fair amount of evidence has been collected in support of inhomogenous distribution of visible matter, there is no evidence to contradict isotropy in a statistical sense from any galaxy. Mandelbrot [5] proposed to replace the standard Cosmological Principle by the Conditional Cosmological Principle. According to this principle the Universe appears to be the same statistically from every galaxy and in every direction. Thus the Conditional Cosmological Principle could allow one to obtain simplified solutions of the Einstein's equations, similar to those for the standard cosmology.

The conditional cosmological principle offers the hope that one can develop cosmological models along the lines of the standard model, while having a simple explanation of the fractal distribution of matter. Although the conditional cosmological principle was proposed about two decades ago, to the best of our knowledge, we are presenting the first ansatz that provides a concrete application of the conditional cosmological principle.

Homogeneity of the Universe in the standard model means that through each event in the Universe, there passes a spacelike "hypersurface of homogeneity". At each event on such a hypersurface the density, pressure and curvature of spacetime must be the same.

In a fractal universe, density is not defined at any point. The concept of density has to be replaced by that of a 'mass measure' defined over sets. The mass measure as obtained by any observer will be the same. By an observer we will mean an observer moving with the cosmological fluid. This precludes observers in a region of void. It should be noted that in a fractal, each of the points belonging to the fractal is on the same footing. But they are not on the same footing for the points not belonging to the fractal. It is known that any sphere centred at a point not belonging to the fractal will be empty with probability 1. Because of this the conditional cosmological principle demands an observer to be situated on a galaxy (point of the fractal) and not in a region of void.

We define a "hypersurface of homogeneous fractality of dimension D" as the hypersurface in which the mass measure over a sphere of radius R centred on the observer is proportional to $R^D$. We say that the Universe is a fractal universe of dimension D, if through each galaxy in the Universe, there passes a spacelike "hypersurface of homogeneous fractality of dimension D".

Isotropy of the Universe means that, at any event, an observer who is "moving with the cosmological fluid" cannot distinguish any space direction from another by local physical measurements.

It is widely believed that isotropy from all points of observation implies homogeneity. Thus an inhomogeneous Universe like a fractal could not be isotropic. It was shown from the observed isotropy of the Universe that the fractal dimension of the Universe could not differ appreciably from 3; ($|D-3| < 0.001$) [7]. To counter this argument, Mandelbrot [4] demonstrated a method of constructing fractals of any given dimension whose lacunarity could be tuned at will to make the distribution as close to isotropy (from any occupied point of the fractal) as desired. Thus in a fractal scenario isotropy from all galaxies may permit inhomogeneity to the extent of admitting homogeneous fractality.

Isotropy of a fractal universe implies that the world lines of the cosmological fluid are orthogonal to each hypersurface of homogeneous fractality. An observer moving with the cosmological fluid can discover by physical measurements the hypersurface of homogeneous fractality relative to which the observer is at rest. His world line would be orthogonal to this hypersurface.

We now choose a hypersurface of homogeneous fractality $S_I$. To all the events on this hypersurface, one may assign coordinate time $t_I$. Galaxies on this hypersurface may be assigned coordinates $x^i$. We now let each galaxy evolve along with the cosmological fluid for proper time $\tau$. We assign coordinate time $t = t_i + \tau$ to the hypersurface formed by the galaxies. We let the spatial coordinates of each galaxy remain unchanged. This would correspond to a "dust approximation" for the fractal distribution. It is easy to see that this hypersurface will be a hypersurface of homogeneous fractality because each galaxy has the same environment and evolves under identical laws.

With coordinates to the galaxies being assigned in the above manner, the interval between two galaxies with coordinates $(t, x^i)$ and $(t + dt, x^i + dx^i)$ will be given by

$$ds^2 = -dt^2 + g_{ij}(t,x)dx^i dx^j \qquad (1)$$

with isotropy demanding the time dependence to seprate as : $g_{ij}(t,x) = a^2(t)\gamma_{ij}(x)$

From isotropy and the Copernican Principle it is ordinarily shown [8] that the 3-metric $\gamma_{ij}(x)$ must yield the same curvature $K$ everywhere. ¿From this it follows that:

$$\gamma_{ij}(x)dx^i dx^j = d\chi^2 + \Sigma^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (2)$$

where

$$\Sigma \equiv \sin\chi \text{ if } k \equiv \frac{K}{|K|} = +1 \text{ (positive spatial curvature)} \qquad (3)$$

$$\Sigma \equiv \chi \quad \text{if } k \equiv K = 0 \text{ (zero spatial curvature)} \tag{4}$$

$$\Sigma \equiv \sinh\chi \text{ if } k \equiv \frac{K}{|K|} = -1 \text{ (negative spatial curvature)} \tag{5}$$

The FRW metric [8] so obtained leads to a constant (same everywhere on the hypersurface of constant time) value of $G^{00}$ given by

$$G^{00}_{\rm FRW} = 3\left\{\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2}\right\} \tag{6}$$

However, this cannot satisfy Einstein's equations as $T^{00}$ is not constant everywhere for a fractal distribution of matter. If matter distribution on a constant time hypersurface satisfies the definition of homogeneous fractality given earlier, we know that

$$\int_{S^3_P(R)} T^{oo} dV = M_P(R) \tag{7}$$

where $S^3_P(R)$ denotes a hypersphere of radius $R$ centered at $P$ on the hypersurface of constant time $t$, and $M_P(R)$ is the mass enclosed in $S^3_P(R)$. If the matter distribution on the hypersurface of constant time $t$ has a fractal dimension $D$, we know that:

$$M_P(R) = \begin{cases} C(t)R^D & \text{if P} \in \text{the fractal} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Einstein's eqns. give

$$\int_{S^3_P(R)} G^{oo}_{\rm fractal} dV = 8\pi M_P(R) \tag{9}$$

As noted earlier, for a fractal distribution of matter, the concept of density is undefined and has to be replaced by the notion of a measure on sets. This implies that curvature (more precisely $G^{00}$) is not defined at any point. However a measure may be defined to satisfy the integrated Einstein's equations over any 3-volume in the constant time hypersurface. In the equation above $G^{00}_{\rm fractal}$ may appear to be a function. It would be more appropriate to regard it as an ansatz for defining a measure on sets containing the point P. In the same way we could express the mass measure by using mass density $\rho$ proportional to $r^{D-3}$. Here $\rho$ would not mean the density at a point, but merely an ansatz to compute the mass measure. In this way we can hope to express the Einstein's equations for a fractal distribution of mass by a relation connecting $G^{00}_{\rm fractal}$ to $\rho$, remembering clearly that these are not functions but ansatz to compute measures. Thus the dependence of $G^{00}_{\rm fractal}$ on $\chi$ and of $\rho$ on $r$ should not be seen as an indication of inhomogeneity but rather as a means of concrete realization of how conditional cosmological principle satisfies the Copernican principle for all occupied points of the fractal.

Now our ansatz is essentially described by:

$$G^{00}_{\rm fractal}(t,\chi,\theta,\varphi) = \begin{cases} f(\chi)G^{00}_{\rm FRW}(t) & \text{if P} \in \text{the fractal} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

On a constant time hypersurface, so long as the origin P is chosen as belonging to the fractal, the integrated $G^{00}$ measure on sets containing P will not depend on the choice of the origin. Thus, despite an apparent dependence of $G^{00}_{\rm fractal}$ on $\chi$, the Conditional Cosmological Principle will be satisfied. Thus,

$$\int_{S^3_P(R)} G^{00}_{\rm fractal}(t,\chi,\theta,\varphi) dV = 8\pi C(t) R^D \quad \text{if P} \in \text{fractal} \tag{11}$$

With $\chi$ chosen so that $R = \Sigma(\chi)a(t)$, eqns. (10) and (11) yield:

$$4\pi G^{00}_{\rm FRW}(t)a^3(t)\int_0^\chi f(\chi)\Sigma^2(\chi)\Sigma'(\chi)d\chi = 8\pi C(t)a^D(t)\Sigma^D(\chi) \tag{12}$$

Hence:

$$G^{00}_{\rm FRW}(t) = 2\nu C(t)a^{D-3}(t) \tag{13}$$

and

$$f(\chi)\Sigma^2(\chi)\Sigma'(\chi) = \frac{D}{\nu}\Sigma^{D-1}(\chi)\Sigma'(\chi) \tag{14}$$

or

$$f(\chi) = \frac{D}{\nu}\Sigma^{D-3}(\chi) \tag{15}$$

where $\nu$ is a constant. We fix this constant by demanding $f(\chi) = 1$ for $D = 3$. This gives $\nu = 3$. Thus,

$$G^{00}_{\rm FRW}(t) = 6C(t)a^{D-3}(t) \tag{16}$$

We denote $C(t)$ by $C_a$, because the scale factor is a function of time. It is easy to establish $a^3C_a = a_o^3C_{a_o}$. This follows from $M_{a_o}(R) = C_{a_o}R^D$; $M_{\beta a_o}(R) = C_{\beta a_o}R^D$. One must have

$$M_{\beta a}(R) = \frac{1}{\beta^3}M_a(R) \tag{17}$$

because increasing the scale factor by a factor $\beta$ would lead to an increase in volume by a factor $\beta^3$, so that mass in a hypersphere of radius $R$ will decrease by a factor $\beta^3$. Thus:

$$C_{\beta a_o}R^D = \frac{1}{\beta^3}C_{a_o}R^D \tag{18}$$

or, with $\beta = a/a_o$;

$$a^3C_a = a_o^3C_{a_o} \tag{19}$$

Therefore we simply get:

$$3\left\{\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2}\right\} = 6\frac{a_o^3}{a^3}C_{a_o}a^{D-3} \tag{20}$$

Giving:

$$\dot{a}^2 = -k + 2a_o^3 C_{a_o} a^{D-4} \equiv -k + \mu a^{D-4} \tag{21}$$

where $\mu = 2a_o^3 C_{a_o}$. If we chose the current epoch as defining $t = 0$; $a(0) \equiv a_0$ and let $D = 2$, then for $k = -1$, we find $\sqrt{\mu + a^2} = t + \sqrt{\mu + a_0^2}$. The solution is singular in the past: $a = 0$ at $t = \sqrt{\mu} - \sqrt{\mu + a_o^2}$. Let us shift the time origin so that $a(0) = 0$ and write the solutions to eqn(21) in general. We describe the solutions for $D = 2$:

$$a(t) = \begin{cases} \mu^{1/4} t^{1/2} & \text{for } k = 0 \\ t^{1/2}\left(2\mu^{1/2} - t\right)^{1/2} & \text{for } k = +1 \\ t^{1/2}\left(2\mu^{1/2} + t\right)^{1/2} & \text{for } k = -1 \end{cases} \tag{22}$$

Eqns (15) and (22) completely describe this fractal cosmological model. In order to make the model more realistic, we may want the to include homogeneous radiation. When this is done in eqn(12), we can obtain solutions only for two cases: (i) D = 3, the usual homogeneous matter distribution and(ii) D = 2, fractal matter distribution [9]. Eqns (15) and (16) generalize to:

$$f(\chi) = \frac{1}{3C_{a_0}a_0^3 + 4\pi\rho_{\gamma_0}a_0^4}\left(\frac{2C_{a_0}a_0^3}{\Sigma(\chi)} + 4\pi\rho_{\gamma_0}a_0^4\right) \tag{23}$$

and

$$G^{00}_{\text{FRW}}(t) = \frac{6C_{a_0}a_0^3 + 8\pi\rho_{\gamma_0}a_0^4}{a^4(t)} \tag{24}$$

respectively. The solutions are again described by eqn(22) with

$$\mu \equiv 2C_{a_0}a_0^3 + \frac{8\pi}{3}\rho_{\gamma_0}a_0^4 \equiv \mu_m + \mu_r \tag{25}$$

There have been treatments of a fractal distribution of matter in the universe as a perturbation on a radiation dominated cosmology [10]. In our approach, we have incorporated the fractal distribution of matter from the beginning. We have shown that for the dynamics of the scale factor, the fractal distribution of matter contributes to the Einstein's equations in the same manner as radiation – both contributing terms that scale as $a^{-4}$. Unlike standard cosmology, the relative importance of matter and radiation remain the same for all epochs as far as the dynamics of the scale factor is concerned.

To summarize, instead of trying to explain the fractal clustering of the universe from hypothetical assumptions about the early universe, we have assumed as given, a fractal distribution of matter. In the present context of ubiquity of chaos and fractals in nature [5] and observational evidence not ruling out fractality, there is (at least) as much (rather much more of a) justification for making *a priori* assumption of homogeneous fractality, as there was for making the assumption of homogeneity at the time that the standard model was proposed.

Just as the standard model follows naturally from the Cosmological Principle when General Theory of Relativity is applied, the fractal model described in this article follows naturally from the conditional cosmological principle, once the necessary change in perspective to deal with fractal distributions is made.

This model may play a significant role in the debate on whether the universe is a fractal or crosses over to homogeneity at some scale. It may be in order to recall the words of Jim Peebles [7], "The geometrical picture of a fractal Universe is elegant, but since it has not been translated into a physical model we can not discuss some of the precision cosmological tests". We hope that the model described in this article would contribute to fill the gap and be regarded as an idealized model around which more realistic models of the universe may be built.

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# STATIC STAR MODEL AND MATHIEU FUNCTIONS

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The Einstein equations with the metric in radiation coordinates of Bondi and energy-momentum tensor of the Pascal perfect fluid are reduced to three differential equations system, one of which can be written as the nonlinear oscillation equation through a new variable. We shall require concurrence of the obtained equation with the equation Mathieu in the finite interval and the star model is described by the Mathieu functions. It leads to a nonlinear equation of the concordance. Inside the star mass density there is a generalization of the parabolic mass density law, which is valid only for an equation of linear oscillator. In a linear approximation estimates are obtained. The star model describes a compact astrophysical object.

## 1. Introduction

The exact solutions of the Einstein equations both for exterior and interior fields is a difficult task even in a static case today. Thus the nonlinearity of the gravitation equations make difficult a solution of this problem. Therefore it is possible to try to exchange an initial problem by a known task, which has a solution.

For an interior static problem description of a spherical symmetric star within the framework of General Relativity theory we shall write the Einstein equations,

$$G_{\alpha\beta} = -æT_{\alpha\beta}, \tag{1}$$

where $G_{\alpha\beta}$ is the Einstein tensor, $T_{\alpha\beta}$ is the energy-momentum tensor, speed of light and Newton's gravitational constant are equal to unit, $æ = 8\pi$ is Einstein's gravitational constant.

In this paper we shall use metric in the Bondi radiation coordinates

$$ds^2 = F(r)dt^2 + 2L(r)dtdr - r^2(d\theta^2 + sin\theta^2 d\varphi^2), \tag{2}$$

where $r$ is a radial variable, $\theta$ and $\varphi$ are angular variables.

Energy-momentum tensor we shall take in an approximation of the Pascal perfect fluid

$$T_{\alpha\beta} = (\mu + p)u_\alpha u_\beta - pg_{\alpha\beta}, \tag{3}$$

where $\mu$ is a mass density , $p$ is pressure, $u^\alpha = dx^\alpha/ds$ is a 4-velocity.

In further all functions depend only on a radial variable.

By replacement $\varepsilon = F/L^2$ the Einstein equations (1) are reduced to a system of the differential equations

$$æ\mu(x) = -\varepsilon'/x + (1-\varepsilon)/x^2; \tag{4}$$

$$æp(x) = F'/(xL^2) - (1-\varepsilon)/x^2; \tag{5}$$

$$(1/2L)(F'/L)' + (1-\varepsilon)/x^2 - \varepsilon L'/(xL) = 0, \tag{6}$$

where the prime designates derivative on a variable $x = r/R$, $0 \le x \le 1$, $R$ is exterior radius of the star. $F(x)$ and $L(x)$ are required metric functions.

Last equation (6) by replacement $F = G^2$ is conversed to the linear differential equation with variable coefficients

$$G'' + f(x)C' + g(x)G = 0, \tag{7}$$

where

$$f(x) = (ln\varphi)', \quad \varphi = \sqrt{\varepsilon}/x,$$
$$g(x) = f(x)/x + 1/(x^4\varphi).$$

Further, passing on to new variable $\zeta$,

$$d\zeta = xdx/\sqrt{\varepsilon}, \tag{8}$$

we obtain an equation of nonlinear oscillator

$$G''_{\zeta\zeta} + \Omega^2(\zeta(x))G = 0, \tag{9}$$

where

$$\Omega^2 = \varphi^2 g(x) = -((1-\varepsilon)/y)'_y; \qquad y = x^2.$$

Connection of functions $\varepsilon$ and $\mu$, obtained by an integration from equation (4),

$$\varepsilon = 1 - (\chi/x)\int \mu(x)x^2dx, \tag{10}$$

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determines that it is necessary to enter function

$$\Phi(x) = 1 - \varepsilon(x), \tag{11}$$

playing role of Newton's gravitational potential inside the star ($\chi = æR^2$). Then the function $\Omega^2$ may be written as

$$\Omega^2 = -(\Phi/y)'_y. \tag{12}$$

For a case $\Omega^2 = 0$ we have the Schwarzschild interior solution with constant value of mass density [1], and at $\Omega^2 = const$ is obtained a known exact solution of gravitational equations with parabolic mass density law [2], which corresponds to linear equation of oscillations.

## 2. Star simulation and Mathieu equation

In more general case ($\Omega^2 \neq const$) the solution of the problem is solved by a reduction of the original nonlinear equation of oscillations (9) to the Mathieu equation on interval $0 \leq x \leq 1$,

$$G''_{\zeta\zeta} + (A + Bsin^2(\zeta))G = 0 \tag{13}$$

with

$$\zeta(x) = \int \frac{xdx}{\sqrt{\varepsilon(x)}}.$$

As shown in [3], it's achieved, for example, by selecting the function

$$\Phi(x) = b^2 sin^2(ax) + cx^6 + ex^{10} + fx^{12}, \tag{14}$$

where $a, b, c, e, f, A, B$ are controlling parameters for reaching the coefficient concurrence of both equations on the indicated interval, and physical behaviour of a mass density and pressure of the star. In this case mass density is regular inside of the star and looks as

$$\chi\mu(x) = \frac{1}{x^2}\Big(b^2 sin^2(ax) - cx^6 - ex^{10} - fx^{12} +$$

$$x(2ab^2 sin(ax)cos(ax) - 6cx^5 - 10ex^9 - 12fx^{11})\Big). \tag{15}$$

The function $\chi\mu(x)$ at center of the star is equal to finite magnitude, decreases to a surface, and, depending on values of parameters may be equal on the star boundary to zero or to step function.

In particular case, for $c = e = f = 0$ by selection of $a$ and $b$ parameters is achieved only concurrence of coefficients values in both equations at the endpoints of interval, since the corresponding functions inside of interval behave as $\sim x^2$ and $\sim x^4$ . Therefore at nonzero parameters $c, e, f$ by setting equal to coefficients at degrees $x$ in expansions of functions $\Omega^2(\zeta) - A$ and $Bsin^2(\zeta)$ in series it is possible to achieve coming together (with a major degree of accuracy) graphs of these functions on the indicated interval of a variation $x$. Thus the parameters $c, e, f, A$ and $B$ are written only through parameters $a$ and $b$ as follows:

$$c = \frac{1}{45}2b^2a^6;\ A = \frac{1}{3}b^2a^4;\ B = \frac{4}{105}b^2a^8;$$

$$e = \frac{2}{14175}b^2a^{10} + \frac{1}{840}b^4a^{10};$$

$$f = -\frac{1}{6300}b^2a^8 + \frac{1}{1680}b^6a^{12}$$

$$-\frac{2}{467775}b^2a^{12} - \frac{1}{4725}b^4a^{12}.$$

By variation of parameters $a$ and $b$ it is possible to influence on the behaviour of a mass density both inside a star, and on its boundary. For example, for values $a = 0.5070$ and $b = 0.4661$ a disagreement for coefficients of the original equation (9) and Mathieu equation (13) less 0.1% is obtained. The junction conditions under $x = 1$ with the Schwarzschild exterior solution guarantees equality to zero of pressure on the star surface.

The estimates show that the used here approximation describes model of a static neutron star with a central mass density $\mu(0) = 9 \cdot 10^{13}$ g/cm$^3$ and radius $R = 10$ km.

## 3. Self-sequence equation and Mathieu functions

However, the more general approach leading to the self-sequence equation is possible. Let's require exact concurrence of the nonlinear oscillator equation on interval $0 \leq x \leq 1$ with the Mathieu equation, taken now as,

$$G''_{\zeta\zeta} + (A_1 - B_1 \cdot cos(2\zeta))G = 0. \tag{16}$$

It's of course, requires restrictions on physical interpretation of a problem. Then we obtain $\Omega^2 = A_1 - B_1 \cdot cos(2\zeta)$ or, using the connection $\Omega^2$ with function $\Phi$, after an integration we get

$$\Phi(y) = Cy - A_1y^2 + B_1y\int_0^y cos(2\zeta)dy =$$

$$Cy - A_1y^2 + B_1y \cdot f(y), \tag{17}$$

with constants $A_1$, $B_1$, C, which may be found from the junction condition with an exterior Schwarzschild solution on a star surface. For parabolic mass density distribution parameter $B_1$ is equal to zero ($B_1 = 0$), then Mathieu equation turns to the harmonic oscillator equation and

$$\varepsilon_0(y) = 1 - Cy + A_1y^2; \tag{18}$$

$$G \equiv G_0(\zeta) = \beta_0 \cdot cos(\zeta + \alpha_0), \tag{19}$$

where it is possible to consider functions $\varepsilon_0(y)$ and $G_0(\zeta)$ now as the initial approximation; $\beta_0$ and $\alpha_0$ are constants of integration. If also $A_1 = 0$, the obtained function $\varepsilon(y)$

corresponds to known case of homogeneous mass distribution (the interior Schwarzschild solution) [1].

Being returned to a general case and substituting instead of function $\Phi(y)$ it expression above mentioned (17), we come to a problem of a solution of the self-sequence equation:

$$2\zeta(y) = \int_0^y \frac{dy}{\sqrt{\varepsilon_0 - B_1 y f(y)}} =$$

$$\int_0^y \frac{dy}{\sqrt{\varepsilon_0(y) - B_1 y \int_0^y \cos(2\zeta(y))dy}} \qquad (20)$$

or it may be written as

$$\zeta = \int_0^y \frac{dy/2}{\sqrt{\varepsilon_0 - B_1 y \int_0^y dy \cos\left(Int\right)}}. \qquad (21)$$

where

$$Int = \int_0^y \frac{dy}{\sqrt{\varepsilon_0 - B_1 y \int_0^y dy \cos(...)}}$$

As a result the correspponding mass density is generalization of parabolic law of behaviour inside of a star

$$\chi\mu(y) = 3C - 5A_1 y + 3B_1 \cdot f(y) + 2B_1 y \cdot f'_y. \qquad (22)$$

Being returned to (16) and written solution of the Mathieu equation as a superposition of Mathieu functions, we have

$$G(\zeta) = C_1 \cdot ce_1(\zeta, B_1) + C_2 \cdot se_1(\zeta, B_1), \qquad (23)$$

where functions $ce_1(\zeta, B_1)$ and $se_1(\zeta, B_1)$ are generalization of a cosine and sine accordingly.

For a case $B_1/A_1 \ll 1$ we obtain in a linear approximation on parameter $B_1$:

$$G(\zeta) \approx G_0(\zeta) - (B_1/16)(G_0(\zeta) \cdot cos(2\zeta) +$$
$$G_0(\zeta + \pi/2) \cdot sin(2\zeta)), \qquad (24)$$

where

$$G_0(\zeta + \pi/2) = dG_0(\zeta)/d\zeta = -\sqrt{\beta_0^2 - G_0^2(\zeta)}. \qquad (25)$$

In view of the self-sequence equation (21) in considered approximation is

$$2\zeta \approx I(y) + (B_1/2) \int_0^y \frac{y \cdot dy}{\varepsilon_0^{3/2}(y)} \int_0^y cos(I(y)dy \qquad (26)$$

with

$$I(y) = \int_0^y \frac{dy}{\varepsilon_0(y)}, \qquad (27)$$

or taking into account equation (18) we have

$$I(y) = (1/\sqrt{A_1}) Arsh(Y/A_1)|_{y-C/2A_1}^{-C/2A_1};$$
$$C = æR^2\mu(0), \qquad (28)$$

where $\mu(0)$ is a central mass density of star.

## 4. Conclusion

At very small values $B_1$ the star model behaves close to the interior star model with parabolic mass density distribution [2] as easy see from relations (22),(24). As well known, one is a Schwarschild-like star model, i.e. our model differs from compact astrophysical object such as a neutron star a little.

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# SUPERHEAVY PARTICLES EITHER FOR UHECR OR FOR MUON ANOMALY

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We show that, according to the scheme of spontaneous breakings starting from a GUT with symmetry $E_6$, it is possible that either a superheavy particle without ordinary interactions is source of ultra high energy cosmic rays (UHECR) or a not so heavy lepton mixes with muon explaining the recently observed discrepancy of the anomalous magnetic moment of the latter.

## 1. Introduction

To the already much discussed problem of the roughly isotropic cosmic rays with energy above $10^{20} eV$ (UHECR) [1], one may add the recently observed discrepancy [2] in the muon anomalous magnetic moment (MAM) as requiring theories beyond the Standard Model (SM) of elementary particles.

There are many explanations of ultra high energy cosmic rays based on decaying superheavy objects with lifetime larger than the universe age [3], as well as possible contributions of new particles to add to the theoretical evaluation of MAM and fill the 2.6 - $\sigma$ gap up to the experimental value [4]. But since the simplest solution of the latter problem is to include a heavy lepton which mixes with muon, one may inquire whether the same scheme offers a superheavy particle which might be origin of UHECR. In the frame of Grand Unification Theories (GUT) the most convenient symmetry is $E_6$, which may come down from the more fundamental superstring theory, because it contains for each generation a heavy charged lepton and one particle without ordinary interactions and therefore with possible great stability.

The feasibility of the modil depends on details of the Higgs fields which produce the breaking of symmetry from $E_6$ down to QCD and electromagnetism giving mass subsequently to superheavy particles and to ordinary ones.

We will show that there are essentially two interesting alternative chains. In the first the particle without ordinary interactions is heavier than the exotic lepton and can decay in it through non-standard gauge bosons, having therefore a short lifetime with no possibility of explaining the UHECR. But since the exotic doublet of leptons mixes strongly with the ordinary one, the muon acquires a relevant flavour-changing coupling with Higgs which may give a contribution to the MAM of the order of the present discrepancy with the experimental value. The second possible chain gives an exotic lepton with a mass larger than that of the particle without ordinary couplings and with a weak mixing with light particles, so that it contributes negligibly to MAM. But now the particle without ordinary couplings decays through virtual exotic fermions whose extremely low mixing with light ones may produce a lifetime as large as the universe age allowing it as source of UHECR.

## 2. $E_6$ and its breaking

The GUT model based on the symmetry of the exceptional group $E_6$ has 78 gauge bosons of which 45 are those predicted by $SO(10)$ and the rest will be called $X$. The left-handed fermions are normally placed in the fundamental 27 - dimensional representation, being the ordinary ones including $\nu^c$ in the representation 16 of $SO(10)$, an exotic lepton doublet $\binom{N}{E}$ together with the singlets $N^c$ and $E^c$ and those of charge $-\frac{1}{3}$ quarks $D$ and $D^c$ in a representation 10, and finally a fermion $L$ without interactions with the 45 $SO(10)$

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bosons in the trivial representation $1$. As it is usual, we work with the charge conjugated left components instead of the right-handed ones.

$E_6$ symmetry must break at a high scale to that of the SM, and the latter to the present one of QCD and electromagnetism at low electroweak (EW) scale. We will assume a detailed high scale chain passing through the maximum subgroups involving the intermediate GUT symmetries $SO(10)$ and $SU(5)$ that is a relevant scheme for producing cosmic strings, which are also of cosmological interest, because of the breaking of the accompanying abelian groups [5]. Obviously to unify couplings of SM according to $SU(5)$ also contributions of supersimmetry (SUSY) are needed [6]. Therefore the succession of symmetries that we consider is

$$E_6 \to SO(10) \times \overline{U}(1) \to SO(10) \to$$
$$SU(5) \times \widetilde{U}(1) \to SU(5) \to$$
$$SU(3)_C \times SU(2)_L \times U(1) \to SU(3)_C \times U(1)_{em} \quad (1)$$

The fermions in the fundamental 27-plet are distributed [7] according to the representations of $SO(10) \times \overline{U}(1)$

$$27 = 16^{1/4} + 10^{-1/2} + 1^1 \quad (2)$$

and then to those of $SU(5) \times \widetilde{U}(1)$ through

$$16 = \bar{5}^{3/2} + 10^{-1/2} + 1^{-5/2}$$
$$10 = 5^1 + \bar{5}^{-1}$$
$$1 = 1^0 \quad (3)$$

The gauge bosons are in the self-adjoint representation 78 which decomposes in $SO(10) \times \overline{U}(1)$ as

$$78 = 45^0 + 1^0 + 16^{-3/4} + \overline{16}^{3/4} \quad (4)$$

with the subsequent $SU(5) \times \widetilde{U}(1)$ components

$$45 = 24^0 + 10^2 + \overline{10}^{-2} + 1^0$$
$$\overline{16} = 1^{5/2} + \overline{10}^{1/2} + 5^{-3/2} \quad (5)$$

The Higgs fields responsible for the breakings shown in Eq. (l) may be in the representations 27 and 78, but it is necessary at least one more to give masses to all the fermions through Yukawa terms according to

$$27 \times 27 = \overline{27} + 351 + 351' \quad (6)$$

which, for the purposes to be discussed in the next Sections, is the 351 with $SO(10) \times \overline{U}(1)$ components

$$351 = 144^{1/4} + 126^{-1/2} + 54^1 + 16^{-5/4} +$$
$$+10^{-1/2} + 1^{-2} \quad (7)$$

that in terms of $SU(5) \times \widetilde{U}(1)$ are

$$144 = 45^{-3/2} + 40^{1/2} + 24^{5/2} + \overline{15}^{1/2} +$$
$$+\overline{10}^{1/2} + 5^{-3/2} + \bar{5}^{-7/2}$$
$$126 = 50^{-1} + 45^1 + \overline{15}^3 + 10^{-3} + \bar{5}^{-1} + 1^{-5}$$
$$54 = 24^0 + 15^2 + \overline{15}^{-2} \quad (8)$$

## 3. Alternative useful for muon anomalous magnetic moment

For the six breakings of Eq.(l) we use eight expectation values of Higgs fields, which for economy will be taken in the representations 78 and 351, to give masses to all fermions and mixing of ordinary with exotic ones.

78 has no influence on fermions as is seen in Eq. (6), but is needed to break $E_6$ because it contains $1^0$ which is invariant under $SO(10) \times \overline{U}(1)$ and to break $SO(10)$ through $45^0$ which contains $1^0$ of $SU(5) \times \widetilde{U}(1)$.

We use all the $SO(10) \times \overline{U}(1)$ representations of 351. $1^{-2}$ is necessary to break $SO(10) \times \overline{U}(1)$ giving mass to $L$ which will be therefore lighter than $X$ gauge bosons, with the possible exception of the $\bar{Z}$ associated to $\overline{U}(1)$, but heavier than the other 10 exotic fermions. $126^{-1/2}$ is used to break $SU(5) \times \widetilde{U}(1)$ giving mass to $\nu^c$, noting that one has to take for the Higgs the complex conjugate of its $SU(5) \times U(1)$ representations according to Eq. (6).

To complete the breakings at GUT scale of $SU(5)$ we use $54^1$ and $144^{1/4}$ which give mass to exotic fermions and mix them with ordinary ones, respectively. This is because they both contain, according to Eq. (7), a 24 of $SU(5)$ which has a component invariant under $SU(3)_C \times SU(2)_L \times U(1)$ that gives [8] the Yukawa couplings

$$\phi(54, 24)\left(D^c D - \frac{3}{2}E^c E - \frac{3}{2}N^c N\right)$$
$$\phi(144, 24)\left(d^c D - \frac{3}{2}E^c e - \frac{3}{2}N^c \nu\right) \quad (9)$$

Due to the fact that mass and mixing terms are analogous, which would not happen in case of including $351'$, at GUT scale the massless states apart from $u$ will be

$$d_0 = d, d_0^c = d^c cos\theta + D^c sin\theta$$
$$e_0^c = e^c, e_0 = ecos\theta + Esin\theta$$
$$v_0 = vcos\theta + Nsin\theta \quad (10)$$

with orthogonal heavy mass states $\hat{E}$ etc.

The peculiar feature of equal mixing, even with large $\theta$, of $e$ and $v$ and no mixing of $d$ and $u$ leads

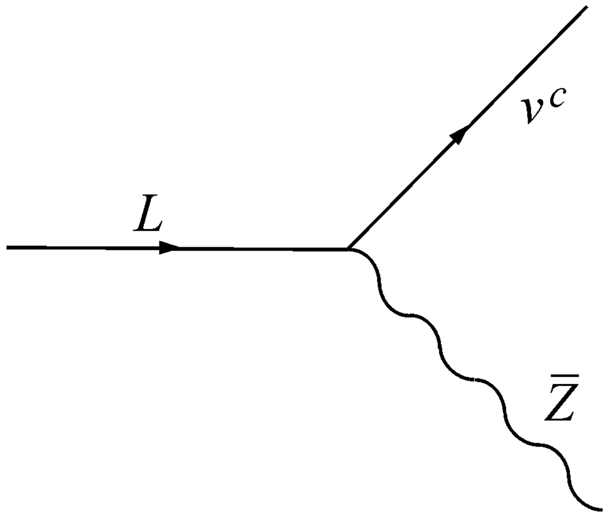


Figure 1: Feynman diagram for the $L$ particle decay in one step

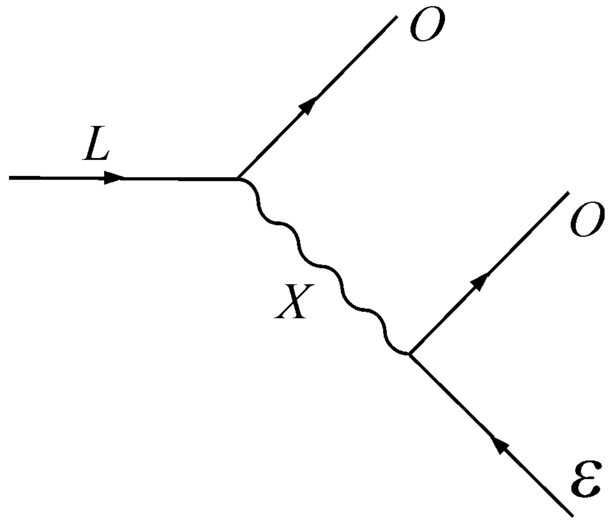


Figure 2: Feynman diagram for the $L$ particle decay in two steps

to unchanged charged weak interactions for ordinary fermions, and additionally Eq. (9) produces no change [9] in the neutral current ones.

At this stage we may already anticipate that this scheme will not allow $L$ to be source of UHECR.

In fact, if the mass of $\bar{Z}$ is of the same order but lower than that of $L$ the latter will decay according to Fig. 1 provided there is mixing of $L$ and $v^c$ because of Eqs. (2, 4). With the approximation $M_{v^c} \ll M_L$, the lifetime $\tau_L$ would be

$$\tau_L^{-1} \simeq \frac{k}{4} M_L \frac{1}{\lambda}\left(1 - 3\lambda^2 + 2\lambda^2\right), \; \lambda = \frac{M_{\bar{Z}}^2}{M_L^2}, \tag{11}$$

where $k$ will depend on the mixing. In case that this is large $k/4 \; 10^{-2}$ because it contains a coupling $\alpha_{GUT}$ and with $M_L \; 10^{16} GeV$ the result would be $\tau_L \; 10^{-38} sec$!

But even without mixing of $L$ and $v^c$ or if $M_{\bar{Z}} > M_L$, the decay of $L$ would be fast enough through Fig.2 which would not require mixing of $L$ and $\varepsilon$ with ordinary fermions $O$ if gauge boson $X$ belongs to 16 according to Eqs. (2, 4).

But mixing $O$ and $\varepsilon$ is necessary for the subsequent decay of the latter. With $\lambda = M_X^2/M_L^2, G = \Gamma_X^2/M_L^2$ and in the approximation $M_L \gg M_\varepsilon, m_O$ the lifetime is

$$\tau_L^{-1} \simeq \frac{\alpha_{GUT}^2}{24\pi} \frac{M_L^5}{M_X^4} \cdot$$

$$\cdot \lambda^2 \int_0^{\frac{1}{2}} dx x^2 \frac{3 - 4x}{(1 - 2x - \lambda)^2 + \lambda G}, \tag{12}$$

which, taking $M_X \; 10^{17} GeV$, gives $\tau_L \; 10^{-32} sec$.

Going to the breaking of SM, this may be done by a Higgs in $10^{-1/2}$ giving mass to ordinary fermions and mixing $L$ with $N$ according to

$$H\,(10, \bar{5})\,(d^c d + e^c e + N^c L) +$$

$$+ H\,(10, 5)\,(u^c u + v^c v + LN) \tag{13}$$

The mixing $L - N$ allows another possible channel of decay of the former but less important than those of Fig. 1, in case of large $k$, and Fig. 2 because it necessarily occurs at EW scale.

Moreover the $16^{-5/4}$ through its $SU\,(5) \times \widetilde{U}\,(1)$ component $1^{-5/2}$ gives a mixing of $L$ with $v^c$ which is small if it occurs at this EW scale, but it might be large if it was produced at $SU\,(5) \times \widetilde{U}\,(1)$ breaking where such expectation value was possible. However $16^{-5/4}$ is the only component of 351 which is not necessary for this alternative.

The eventual contribution of new particles [10] to the MAM has renewed its interest due to the $2.6 - \sigma$ discrepancy between experimental and SM calculation

$$\Delta a_\mu = \Delta \left(\frac{g - 2}{2}\right)_\mu \simeq 4 \times 10^{-9}. \tag{14}$$

Recent explanations come from SUSY [11] and perhaps extra dimensions [12] but also from the exotic fermion [13] of $E_6$.

In fact with the present scheme of breakings based on 351, the Yukawa couplings for $e$ and $E$ of Eqs. (9, 13) through the mixing of Eq. (9) will give a "flavour" changing coupling of the physical four-component $e_0$ and $\hat{E}$ with the Higgs field $h$ added to the vacuum expectation value

$$\mathcal{L}_{FC} = k \bar{e}_0 \left(\alpha - \beta\gamma_5\right) \hat{E} h + h.c., \tag{15}$$

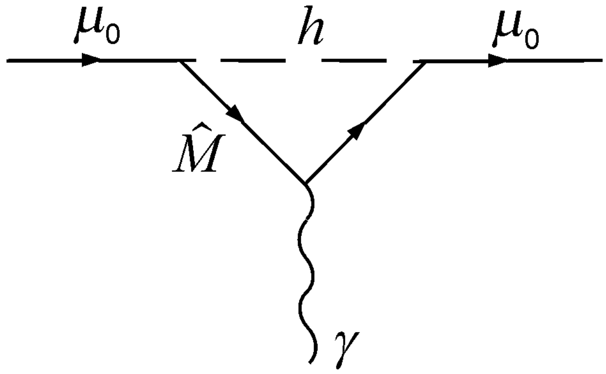


Figure 3: Feynman diagram for the correction of $\mu$ magnetic moment

where $k$ depends on the mixing angle and on the parameters necessary to give the value of the lepton masses. An analogous term for the second generation of fermions will produce a contribution [14] to MAM according to Fig. 3

$$\Delta a_\mu \simeq \frac{1}{8\pi^2}\frac{m_\mu}{M_M}k^2 F\left(\frac{M_h}{M_M}\right), \tag{16}$$

where, for $M_h \ll M_M, F(0) = \frac{1}{2}$. Therefore, being $k \lesssim 1$, to fill the present discrepancy Eq. (13) it is necessary that the mass of the exotic lepton $M_M \lesssim 10^5 GeV$. Since in the present scheme the mixing is large because it occurs at GUT scale, it is not unreasonable to have the above maximum values of $k$ and $M_M$.

It is interesting that in a different approach in which light leptons are com-pled to heavy leptons and pious [15] a contribution to $\Delta a_\mu$ analogous to Eq. (16) may be consistent to UHECR due to the increase of $v$-nucleon cross-section [16].

We may remark that the law of Eq. (16) evades the treatment of Ref. [4] because our coupling Eq. (15) requires mixing of two Higgs. On the contrary, the effective couplings coming from SUSY or extra dimensions would give $\Delta a_\mu \sim \frac{1}{16\pi^2}\left(\frac{m_\mu}{\Lambda}\right)^2$ which, to satisfy the experimental Eq. (13), would require a scale for new physics $\Lambda < 1TeV$. Therefore, corrections of MAM like Eq. (16) are not pressed by a too close new physics scale, though for the alternative explanations different masses of SUSY partners or sum of Kaluza-Klein modes in extra dimension make their situation easier.

## 4. Alternative useful for ultra-high energy cosmic rays

We choose now to break the symmetries of Eq. (1) using eight expectation values of Higgs in all the $SO(10) \times \overline{U}(1)$ components of 27 plus the minimum needed ones of 78 and 351.

Regarding 78, we use as before its components $1^0$ to break $E_6$ and $45^0$, with $1^0$ of $SU(5) \times \widetilde{U}(1)$, to break $SO(10)$. In addition we use again $45^0$, but with its $24^0$ of $SU(5) \times \widetilde{U}(1)$, to break $SU(5)$ keeping the SM symmetry and without influence on fermions.

Also two components of 351 are necessary: $1^{-2}$ to give mass to $L$ and $126^{-1/2}$ for $v^c$ as before.

Now considering 27, $1^1$ is necessary to give mass to the 10 exotic fermions according to

$$\phi(1,1)\left(D^C D + E^C E + N^C N\right) \tag{17}$$

Since presumably the expectation value of $1^1$ will appear at the same scale of $1^{-2}$ corresponding to the breaking of $SO(10) \times \overline{U}(1)$, it will depend on the constants of the Yukawa couplings if $L$ is heavier or lighter than the exotic fermions. It would be also possible that one of these expectation values develops at a lower scale.

To break SM we need $10^{-1/2}$ which will give mass to ordinary fermions and mixing of $L$ and $N$ as in Eq. (12).

Finally $16^{1/4}$ mixes ordinary and exotic fermions, which is necessary because otherwise the latter would be stable, according to

$$\begin{aligned} &H(16,1)\left(d^c D + E^c e + N^c v\right) + \\ &+ H(16,\bar{5})\left(D^c d + e^c E + v^c N\right) \end{aligned} \tag{18}$$

The expectation value of $SU(5)\,\bar{5}$ may appear breaking SM at the EW scale.

On its hand the $1^{-5/2}$ component of the first term of Eq. (18) might appear earlier, at the breaking of $SU(5) \times \widetilde{U}(1)$. If this occurs together with Eq. (17) it would produce a mixing of the same type of Eq. (9) but with smaller angle because of the higher scale of the mass term. Therefore a discussion analogous to that of Sect. 3 will lead to a contribution to MAM smaller than the experimental discrepancy Eq. (13) due to smaller mixing and larger $M_M$ in Eq. (16).

Alternatively, if $H(16,1)$ appears at the same EW scale of $H(16,\bar{5})$ the mixing of exotic and ordinary fermions will be even smaller giving way to negligible corrections of charged and neutral weak interactions and of MAM.

But now the situation regarding UHECR will be different because $L$ cannot decay to $v^c$ since there is no mixing between them, and if $M_L < M_\varepsilon$, the decay of $L$ will be given by $L \to OO\overline{O}V$ with $V$ vector boson of $SO(10)\,45$ which includes the twelve of SM as is seen in Fig.4. The coupling $\varepsilon OV$ is possible, because of $U(1)$ charge conservation of Eqs. (2, 4), only if there is mixing of $\varepsilon$ and $O$.

If e.g. the scale of 27 $(1^1)$ is of the order of breaking of $SO(10) \times \overline{U}(1)$ but that of 351 $(1^{-2})$ is smaller, it is possible to have $M_L$ $10^{12} GeV$ corresponding to

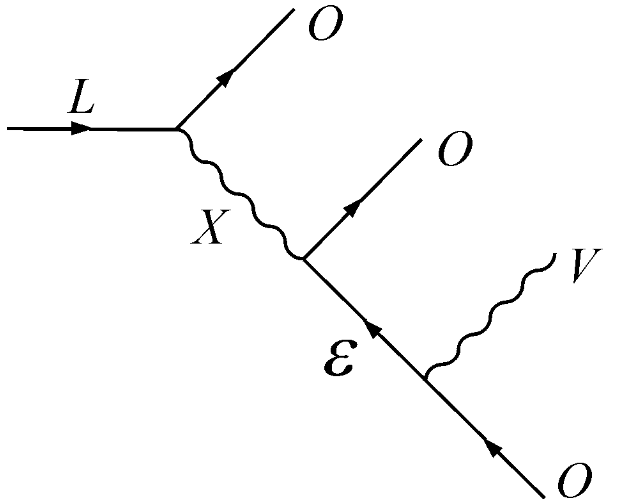


Figure 4: Feynman diagram for the $L$ particle decay in three steps

UHECR whereas $M_{\varepsilon}$ $10^{16} GeV$ and $M_X \sim 10^{17}$ $GeV$ due to breaking of $E_6$.

Therefore the decay of Fig. 4 could be replaced by an effective coupling $\sim \frac{1}{M_X^2 M_{\varepsilon}} \overline{O} \gamma^{\mu} \gamma^{v} O V_v$. Since $M_V \ll M_L$, an estimation of $L$ lifetime is

$$\tau_L^{-1} \sim \alpha_{GUT}^2 \alpha_M 10^{-7} \frac{M_L^8}{M_X^4 M_{\varepsilon}^2 M_V}. \tag{19}$$

Introducing the above masses and $M_V$ $10^2 GeV$ Eq. (19) would give

$$\tau_L^{-1} \sim \alpha_M \frac{10^7}{sec}, \tag{20}$$

of the order of universe age $t_0$ $10^{18} sec$ for $\alpha_M$ $10^{-25}$, not unreasonable for the square of the extremely small mixing between EW and GUT scales. Then $L$ might be origin of UHECR.

## 5. Conclusions

We have seen that one possible alternative for Higgs responsible for the breaking of $E_6$ symmetry passing through $SO(10)$ and $SU(5)$ gives a strong mixing of ordinary and exotic leptons which might allow a not too high mass of the latter and consequently a correction of the $\mu$ magnetic moment of the order of the discrepancy between experimental measurement and Standard Model calculation. The qualitative difference with other explanations based on supersymmetry or extra dimensions is that in our case the scale of new physics is well above the $TeV$ region.

Another alternative of Higgs for the same chain of symmetry breakings starting from $E_6$ produces very heavy fermions in the $SO(10)$ decuplet with extremely small mixing with ordinary ones, so that the $L$ particle in $SO(10)$ singlet with mass $10^{12} GeV$ may have a lifetime of the order of universe age and explain the ultra high energy cosmic rays. It is clear that to avoid present overclosure of universe $L$ particles must have been produced non-thermally to be now a small fraction of dark matter.

Our proposal does not require non-renormalizable interaction for the decay of $L$ at variance from other hypothetical superheavy particles like cryptons [17], protected by half-integer electric charge and a hidden gauge invariance, or those coming from string models in a sector with broken GUT like unitons and singletons, protected by a discrete symmetry [18].

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# POINTLESSNESS AND DANGEROUSNESS OF THE QUANTUM MECHANICS

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The formalism of quantum mechanics produces spectacular results, but its rules, its parameters are empirical, either deduced from classical physics, or from experimental results rather than from the postulates. Thus, quantum mechanics is purely phenomenological; for instance, the computation of the eigenvalues of the energy is generally a simple interpolation in the discrete space of the quantum numbers. The attempts to show that quantum electrodynamics is more precise than classical electrodynamics are based on wrong computations. The lack of paradoxes in the classical theory, the appearance of classical, true interpretations of the wave-particle duality justify the criticism of Ehrenfest and Einstein.
The obscurity of the quantum concepts leads to wrong conclusions that handicap the development of physics. Just as building a laser was considered absurd before the first maser worked, the concept of photon leads to deny a type of coherent Raman scattering necessary to understand some redshifts of spectra in astrophysics, and able to destroy the two fundamental proofs to the expansion of the universe.

> "Llewlyn Thomas, a noted Columbia theorist, told me flatly that the maser could not, owing the basic physics principles, provide a pure frequency with the performance that I predicted. So certain was he that he more or less refused to listen to my explanations. After it worked, he just stopped talking to me altogether. On visiting Niels Bohr, the pioneer of quantum mechanics, in Denmark, he exclaimed: "But that is not possible".
> At a cocktail party in New Jersey, the Hungarian mathematician John von Neumann declared "That can't be right". These objections were founded on principle - the uncertainty principle, the central tenet of quantum mechanics. To physicists steeped in the uncertainty principle, the maser's performance made no sense at all". (C.H. Townes [1]).

## 1. Introduction

At the beginning, relativity and quantum mechanics were strongly criticised; but, while relativity is now widely accepted, so many physicists share now the scepticism of Ehrenfest and Einstein about quantum mechanics that the supporters of this theory look for an absolute proof of its necessity. On the contrary, this paper tries to show that the postulates of quantum mechanics must be rejected.

Quantum mechanics was an important tool in the development of physics in the twentieth century, but using the *formalism* of quantum mechanics does not require the *postulates.*

The following section shows that the symmetry properties often considered as a part of the formalism of quantum mechanics may be justified classically, while the computation of the eigenvalues of the energy uses so disparate methods that it appears phenomenological, and may be bound to either theory. It is only examples because they are too many applications of quantum mechanics.

The next section reinforces the previous one in the field of optics: the defenders of quantum mechanics performed experiments to prove an inadequacy of the classical theory but they used naive hypothesis, so that their demonstrations fail. The flimsiness of quantum mechanics leads to introduce absurd concepts, such as the photon, as a particle, leading to an absurd wording of the EPR experiment. At low lighting, the linearisation of optical effects in function of the amplitude of the electric field appears as a nearly trivial but very useful property.

Next section is a tentative explanation of the wave

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particle duality by (3+0)D solitons whose existence is demonstrated.

It appears finally that the photon leads the astrophysicists to reject the coherent Raman scattering (just as the maser was rejected), giving them the two main (and probably fallacious) proofs of the expansion of the universe. Consequently, they need to imagine fantastic explanations about observations on quasars and other massive objects.

## 2. Classical justification of the formalism of quantum mechanics

A criticism of quantum mechanics is difficult because the frontiers of this theory are not well defined. The symmetries are often introduced by "active transformations" in which the particles are moved by the physicist; the active transformations are, as often claimed, a concept of quantum mechanics. In classical physics, the theoretician observes, does not change the system he observes, using "passive transformations" which act on mathematical tools, such as reference frames, only.

### 2.1. Molecular symmetries

The molecular symmetries are an easy to explain example of symmetries [2] generally studied by "active transformations." Show that classical physics introduces naturally the rules postulated by the theory of active transformations.

The classical problem is setting, at a given instant, the variables which allow to describe a molecule made of punctual atoms, taking into account the hypothesis of the existence of a remarkable configuration (called here equilibrium configuration), generally a configuration for which the potential function is minimal.

Suppose first that the molecule is made of different atoms $A, B, C, \ldots$ of masses $m_A, m_B, m_C, \ldots$. The equilibrium configuration is a geometrical, solid figure of points $a, b, c, \ldots$, defined within a displacement. The real molecule is distorted; the equilibrium configuration must be bound to the molecule, for instance to study small movements, considering that the atoms $A, B, C, \ldots$ are displaced from the corresponding points $a, b, c, \ldots$. For instance, this binding is done using the conditions set by Eckart: First a centre of mass $O$ of the equilibrium configuration is defined by

$$m_a\overrightarrow{Oa} + m_b\overrightarrow{Ob} + m_c\overrightarrow{Oc} + .. = \overrightarrow{0}, \tag{1}$$

The first condition sets that $O$ is the centre of mass of the molecule:

$$m_a\overrightarrow{aA} + m_b\overrightarrow{bB} + m_c\overrightarrow{cC} + .. = \overrightarrow{0}, \tag{2}$$

The second condition is:

$$\overrightarrow{Oa}\wedge(m_a\overrightarrow{aA})+\overrightarrow{Ob}\wedge(m_b\overrightarrow{bB})+\overrightarrow{Oc}\wedge(m_c\overrightarrow{cC})+.. = \overrightarrow{0}. \tag{3}$$

A mobile reference frame $O'xyz$ may be bound to an equilibrium configuration independent of the molecule, obtaining a "reference configuration" which is a mobile, geometrical solid; the reference configuration may be defined by a table of coordinates which will be the components of $\overrightarrow{O'a}, \overrightarrow{O'b}, \overrightarrow{O'c}, \ldots$ in $O'xyz$. The frame is bound to the molecule superposing by a displacement the reference configuration with the equilibrium configuration bound to the molecule by the Eckart conditions. The relative coordinates are defined.

If, in the set $A, B, C, \ldots$ two or more points have the same mass (with an assumed approximation), they will be designed by the same letter, so that the displacement vectors such as $\overrightarrow{aA}$ are not uniquely defined; at a given time, the ambiguity may generally be solved setting, for instance that the sum of the modulus of the displacement vectors is minimal. If, during the movement, the definition of the displacement vectors is not changed, the molecule is said "semi-rigid." To define the coordinates, indices distinguish the $a, b, c \ldots$ points (thus the displacement vectors), when necessary in the table which defines the reference configuration.

If the equilibrium configuration has symmetries, superposing the reference and bound configurations has not a single solution; shifting from a solution to an other by the "symmetry operations," is formally : i) a transformation of the coordinates; ii) a convenient permutation of the indices.

This classical explanation is not trivial, but it does not require postulates. As the transformations do not move the atoms, it is not necessary, when there are interactions with external fields, to move the whole universe or to correct, strangely, the symmetries.

### 2.2. Computation of the energy levels

When quantum mechanics started to develop, the quantum hamiltonians of simple systems were obtained by the correspondence principle, it led to partial derivative equations which were solved using standard computations. Later, it appeared that the resolution of the equations was easier using the raising and lowering operators; finally, it remained only Lie algebra (see an example of this evolution in [3]). Thus, one may think that the quantum theory is a method to introduce these algebra; however, the Lie algebra are now chosen arbitrarily, with the single aim to get good fits of the experimental results. The physical starting point is lost, in molecular, atomic spectroscopy, and elementary particle theories as well. It does not remain much of the old computations: a linear dependence of the energy of the one dimension harmonic oscillator; the remarkable function $j(j+1)$ for the energy of a rotator or a three dimensional harmonic oscillator comes from the isotropy of the space, the O(3) group and its algebra. Thus, the results which bring more than an interpolation over the discrete space of quantum numbers, come from the symmetry of the rotation (or, better, the Poincaré) group which must be taken into account in any problem of physics; the spin comes from the homomorphism of the SO(3) and SU(2) groups. More astonishing, the physicists use other interpolation methods where they work better, for instance the Padé approximants in molecular spectroscopy of diatomic and "spherical top" molecules.

Eigenvalues of the energy appear in most problems of classical mechanics because the potential energy has many relative minimums. Unhappily, their computations are often difficult, involving nonlinearities. We may use the phenomenological formalism of quantum mechanics as an interpolation method convenient to find these classical eigenvalues: the formalism of quantum mechanics gives approximate results of classical (or relativistic) mechanics.

# 3. Classical and quantum electrodynamics in the vacuum

## 3.1. The fundamental unconsciousness of quantum electrodynamics

First, what is an optical mode for the linear Maxwell's equations? Set physically correct boundary conditions in particular that the fields have zero values for infinite space-time variables; then a mode is *any* solution of Maxwell's equations. Setting $w(\nu)\mathrm{d}\nu$ the energy of the mode between the frequencies $\nu$ and $\nu + \mathrm{d}\nu$, the mode may be normalised by

$$\int_0^\infty \frac{w(\nu)\mathrm{d}\nu}{\nu} = h \tag{4}$$

where $h$ is the Planck's constant. Two modes are orthogonal if the energy of the system of the two modes is the sum of the energies of the two modes; a complete set of orthogonal modes (sentence often oversimplified into "set of modes") allows developing any mode on this infinite set of modes.

The fundamental postulate of quantum electrodynamics is the identification of a monochromatic optical mode with a harmonic oscillator. But, if the used set of modes is changed, the quantified energy of the modes is split .... A solution of this problem is "the reduction of the wave packet", a postulate which allows the physicist to identify any mode with any other. This postulate is for me the strongest paradox, no, the strongest absurdity of quantum mechanics: how is it possible to work on objects whose definitions depend on the mood of the physicist?

## 3.2. Some elementary, classical electrodynamics

During a transition, a small, isolated mono- or polyatomic molecule emits a nearly monochromatic wave; an oscillating dipole (or quadrupole ...) which radiates in nearly all directions generally models the radiating molecule. Suppose the frequency low enough to consider the source as small compared to the wavelength.

The electromagnetic energy in a sphere centred on the molecule, and whose radius is small compared to the wavelength allows evaluating the energy radiated by the molecule. If there is no external field, this energy is positive, the molecule loses energy; but an external field may cancel partly the molecular field, so that the molecule may absorb energy or radiate no energy; if it absorbs, it scatters too.

A consequence is that the energy $h\nu$ lost by a molecule cannot be absorbed by a single other molecule, the absorption requires an infinity of molecules, an infinite time: thus the universe is full of residues of electromagnetic fields, the "stochastic" or "zero point" field. It is fundamental to remark that this classical field is an ordinary electromagnetic field, that it must not be neglected, or studied independently of the other fields. The existence of the stochastic field explains, for instance, that the electron of the hydrogen atom generally does not lose energy although it radiates a field (except to reach the Lambshifted frequency).

The evaluation of the stochastic field requires the second Planck's law, not the first which, neglecting the stochastic field, sets that the energy in a mode is:

$$e = \frac{h\nu}{\exp(h\nu/kT) - 1}. \tag{5}$$

Supposing a high enough temperature, the exponential is developed:

$$e \approx \frac{h\nu}{h\nu/kT + (h\nu/kT)^2/2 +} \approx kT - h\nu/2 \tag{6}$$

From thermodynamics, it must be $kT$; thus $h\nu/2$ is added to the energy of the mode to get the second, good Planck's law. This energy is a stochastic electromagnetic energy in the mode [4, 5]; its building shows that the corresponding field is an ordinary electromagnetic field; it provides the field which is necessary to compensate, in the average, the energy lost by radiation; if a system, such as a photoelectric cell requires a low energy to be excited, the long and particularly powerful fluctuations of the stochastic field are able to excite it, it is the noise observed at the lowest temperatures. Marshall and Santos [6] showed a local equivalence between the classical electrodynamics including the stochastic field that they call "stochastic electrodynamics," and quantum electrodynamics. Paradoxically, this equivalence is often used now to set that stochastic electrodynamics is an approximate fruit of quantum electrodynamics, while classical electrodynamics is older.

In the excitation or de-excitation of an electron, the electron provides the quantization of the electromagnetic field, just as the bottles quantify the wine. A common error is setting that this quantization is absolute while it applies only to systems that start from, and end at stationary states. The laws of refraction work at the lowest light levels while a lot of atoms are involved by the refraction by a prism. The temporary absorption of energy by each molecule of the prism during a light pulse is evidently much lower than $h\nu$. At the beginning of the pulse, when the field increases, the atoms get a slight excitation, remaining near their stationary state; if the prism is transparent, its atoms return their excess of energy to the tail of the pulse.

As the atoms are able to amplify the modes of the decreasing field, they are surely able to amplify other modes, but slightly (Rayleigh scattering) without the help of the coherence. Thus, the local stochastic field is increased during a pulse of light, the atoms and the fields get nearly an equilibrium, reversibly unless a big fluctuation of the amplified stochastic field excites an atom up to a transition.

In his thesis, Monnot [7] set a model of two energy levels atom, supposing that the amplitude of its radiating dipole is a quadratic function of the energy of the atom, equal to zero in the two stationary states. He puts a set of such atoms in a reflecting box; the energy of almost all atoms remains next to the eigenenergies, and the stochastic field keeps a nearly constant value; trying to increase or decrease the stochastic field is inefficient, provoking transitions of the convenient number of atoms. In conclusion "photon" must mean "quantity of energy $h\nu$," not particle; W. E. Lamb [8] is not far from this point of view, but he does not make the last step, the rejection of the postulates of quantum electrodynamics.

### 3.3. Experiment of Einstein, Podolsky and Rosen in optics

The emission of a photon is followed by an excitation of an atom by a photon only in the average. A source amplifies the stochastic field, so that the emission of a photon increases the probability that the fluctuations of the stochastic field grow up to values that pump atoms to higher states.

The paradoxes come from the quantization of the electromagnetic field; making a choice between the two locally equivalent theories, paradoxical quantum electrodynamics appears as an approximation of classical electrodynamics.

### 3.4. Optical effects at low light levels

Einstein considers two types of emission of light: the spontaneous emission, evaluated by the $A$ parameter and the stimulated emission evaluated by $B$. The experiments show that these parameters are not independent, the spontaneous emission being an emission stimulated by the stochastic field.

Suppose that a certain optical effect is a non-linear function $f(E)$ of the amplitude of the electric field of an incident beam of light; $E$ is produced by an amplification in a source of a stochastic field $E_0$ and may be written $E = E_0\beta$ where $\beta$ is the amplification coefficient of the source. It is $E$ which is written in $f(E)$ because the stochastic component of the field is an ordinary field which cannot be split from the remainder of the field.

If the light level is low, $\beta$ is nearly 1, so that

$$f(E) = f(E_0\beta) = f(E_0(1+\beta-1)) \approx$$
$$\approx f(E_0) + (\beta-1)f'(E_0). \tag{7}$$

As the stochastic field is, in the average, constant, the effect is a linear function of the electric field, either $E = E_0\beta$, including the stochastic fraction, or $E_0(\beta-1)$ excluding it, as usual [9]. In particular, a photocell detects the available energy it receives, that is the difference between the received energy, including the stochastic field, and a restored stochastic field; at a low level, the signal is proportional to

$$E^2 - E_0^2 = (E_0\beta)^2 - E_0^2 \approx 2E_0^2(\beta-1). \tag{8}$$

It is proportional to the amplitude, not to the intensity.

Seeming to ignore this elementary classical property, many authors gave wrong classical interpretations of experiments to show that quantum electrodynamics is "the good electrodynamics"; they used photon counting to get sub-Poissonian statistics [10, 11], or second order interferences:

### 3.5. Second order interferences

All proposed experiments (see, for instance [12, 13, 14, 15, 16]) are fundamentally equivalent, and they show only that their author's classical interpretations are wrong because the stochastic field is neglected.

Neglecting the stochastic field, the elementary explanation of these experiments [17] gives a contrast 1/2 for the fringes, while the experimental value tends to one with a decreasing intensity of the light. Consider the simplest of these equivalent experiments [13]: two small photocells observe the interference of two small, incoherent, weak, monochromatic sources. These interferences are not visible because the sources are incoherent, their relative phase $\phi$ changes quickly, so that the fringes move too quickly for the eyes. The signal is the coincidences of the "detected photons."

Distinguish the two photocells by an index $j$ equal to 1 or 2, and the differences of the optical paths on the cell $j$ by $\delta_j$; a cell detects proportionnally to the amplitude $\cos(\pi\delta_j/\lambda+\phi/2)$, so that the probability of simultaneous detections is proportional to

$$\cos(\frac{\pi\delta_1}{\lambda}+\frac{\phi}{2})\cos(\frac{\pi\delta_2}{\lambda}+\frac{\phi}{2}). \qquad (9)$$

Taking the mean value during an experiment, that is integrating $\phi$ on $2\pi$, we get a zero value for $\delta_1-\delta_2=\lambda/2$, so that the visibility reaches the right value 1.

For higher light intensities, the signal becomes proportional to the intensity, so that the visibility decreases to 1/2.

## 4. Tentative classical theory of the Wave particle duality

Quantum mechanics claim that it solves the problem of the wave particle duality; having the choice to consider an object as a particle or as a wave is not a true solution. The maser seemed absurd to people who considered the photon as a particle, but the next section will show that, in despite of the popularity of the lasers, the same arguments persist.

While the photon is not a particle, the electron, proton, neutron ... are particles: they have a centre of mass, they may be static. The solitons are waves in non-linear media, which do not dissipate their energies by radiation; they are classified by a (p+q)D symbol, where p is the dimension of the wave, and q the dimension of its propagation. Unhappily our present mathematics allow a rigorous study of a few (1+1)D solitons only. The (3+0)D solitons are fields which have the properties of particles: most of their energy is in a limited region of space, and this region may be static. Unhappily, these solitons do not seem to have been studied up to now. Show the existence of (3+0)D solitons.

### 4.1. Known properties of optical solitons

The nonlinearities which are generally studied in optics are produced by Kerr or photorefractive effects: the permittivity is an increasing function of the electric field, in many computations a quadratic function of this field up to a saturation provided by a sextic term.

The (1+1)D optical solitons have been extensively studied, they are probably used for the transmission of data in optical fibres.

The (3+1)D solitons, called optical bullets, propagate without dispersion, they are similar to particles which could not be stopped.

The optical (2+1)D solitons are light filaments obtained when a powerful laser is focalised in a non-linear medium [18, 19, 20, 21, 22].

The electromagnetic field decreases generally uniformly from the axis to the outside of a slightly converging laser beam, so that the index of refraction is larger near the centre of the wave surfaces; thus, the speed of light is lower near the centre, the curvature of the wave surfaces increases. If the beam is very neat, and the energy not too large, the beam converges to a single filament which is stable if it has exactly a "critical flux of energy," or radiates quickly an excessive energy. If the laser beam is powerful, local fluctuations provoke local convergences; many filaments are produced.

In a filament, the electromagnetic field may be artificially split into two parts: a cylindrical kernel in which the field is high, thus the nonlinearity large, and an evanescent wave whose amplitude decreases quickly with the distance to the kernel, so that it is quickly merged into external fields; the flux of energy in a stable filament free of interactions has the critical value, external fields may slightly change this flux. The properties of these solitons may be deduced from their theory or from their experimental observation.

We are interested here by the solitons in a perfectly transparent medium, but the absorption which occurs in real media brings useful informations: while the filaments lose much energy by molecular excitations, they are nearly parallel and so long [23] that they surely absorb energy from the surrounding field to keep nearly the critical flux of energy; more precisely, there is an equilibrium between the external field and the filament: if the flux of energy in the filament is slightly under the critical value, it absorbs the surrounding field, and vice versa.

An other important result is deduced from the observation: the filaments do not merge, they often make regular figures, they repel each other; but, as they absorb their surrounding field, an interpretation of this repulsion is that the filaments are attracted to the regions where the field is the larger. The theory [24] and the experiments [25, 26] show that the filaments may be curved without a loss of stability by a non-uniform external field, either macroscopic, or created by an other filament.

### 4.2. Theoretical existence of (3+0)D optical solitons

We consider here perfect, isotropic, non-absorbing media having non-linear properties which enable the propagation of infinite filaments [27]. For a filament centred around $Oz$ with a given pulsation $\omega$, the electric and magnetic fields $\overrightarrow{E'}$ and $\overrightarrow{H'}$ are invariant by translations parallel to $Oz$, the lengths of which are integer products of a period $\Lambda$. Set that the evanescent field is negligible over a distance $\rho$ from $Oz$

Consider another problem in which the medium has the previous properties and, in addition, a small, perturbing nonlinearity depending on the amplitude of the magnetic field. This perturbation does not destroy the stability of the filament.

Consider a circle $C$ of radius $R$ larger than $\rho$, whose circumference is an integer multiple of $\Lambda$. Set $\Omega$ a point of $C$, $\zeta_\Omega$ the curvilinear abscissa from $\Omega$ of a variable point $M$ of $C$, $M\xi$ an axis oriented to the centre of $C$ and $M\eta$ the axis making, with a tangent $M\zeta_M$ to $C$ a reference frame. Suppose that, for any point $M$, in a disk of radius $\rho$ and axis $M\zeta_M$, a daemon makes fields of amplitudes $E(\xi,\eta,\zeta_\Omega) = E'(x,y,z)$ et $H(\xi,\eta,\zeta_\Omega) = H'(x,y,z)$, oriented in $M\xi\eta\zeta_M$ just as $\overrightarrow{E'}$ and $\overrightarrow{H'}$ in $Oxyz$.

As the pulsation $\omega$ is a constant, the perturbation may be considered as a function of the amplitude of the curl of the electric field, rather than a function of the amplitude of the magnetic field.

Set $\Pi_M$ the plane orthogonal to $C$ at abscissa $\zeta_\Omega$ and $\Pi_N$ a similar plane at a point $N$ of slightly larger abscissa $\zeta_\Omega + \delta\zeta_\Omega$. Set $\alpha$ the angle of rotation of the tangent to $C$ from $M$ to $N$, that is the angle between $\Pi_M$ and $\Pi_N$.

In a second order approximation, the component along $M\eta$ of the curl of $\overrightarrow{E}$ supposed polarised along $M\xi$, is $\delta E_\xi(\xi,\eta,\zeta_\Omega,t)/(\delta\zeta_M) = \delta E_\xi(\xi,\eta,\zeta_\Omega,t)/(\delta\zeta_\Omega(1+\xi\alpha)) \approx (1-\xi\alpha)\delta E_\xi(\xi,\eta,\zeta_\Omega,t)/\delta\zeta_\Omega$. As $\delta\zeta_M$ is a decreasing function of $\alpha\xi$, the amplitude of the curl and the index of refraction increase with $\alpha\xi$.

Huyghens' construction applied to the wave in plane $\Pi_M$ leads to a distorted and, in the average turned wave surface. Happily Huyghens' construction is too imprecise in a filament whose stability shows that the wave surface is not distorted; however, as the variation of the the index of refraction is odd in $\xi$, the wave surface is turned by an angle $\beta$.

The function $f(\alpha) = \beta(\alpha)/\alpha$ may be adjusted, using the variation of the index of refraction as a function of $\overrightarrow{H}$, so that, for a certain value $\alpha_0$ of $\alpha$, $f(\alpha_0) = 1$, $\mathrm{d}f(\alpha_0)/\mathrm{d}\alpha < 0$. The daemon is not anymore useful, an autocoherent and stable solution is found. The filament is transformed into a torus that traps the electromagnetic field; as the length of the filament and the flux of energy are fixed by the optical parameters, the energy of the soliton is quantified.

The existence of (3+0)D solitons is demonstrated, but a true study seems to require very powerful computers, with, maybe, the previous torus as starting point. If the properties of the filaments remain true, two toruses repulse each other; the regions where the field at the same frequency is large attract the torus.

Some crystals, tourmaline for instance, have remarkable electric and magnetic properties, but they absorb the light so much that it seems impossible to use them to try optical (3+0)D solitons. Are the balls of fire produced by the lightnings optical solitons? They seem made of ionised gas that could have the required properties.

### 4.3. Are particles optical solitons?

Purely electromagnetic interactions, in the $\gamma$ range can produce electron-positron pairs [28]; this shows that the vacuum becomes nonlinear. Remark that quantum mechanics introduce such nonlinearities through virtual particles. Testing whether matter is made of electromagnetic solitons is a big job! Suppose it is true.

If the kernel of a soliton goes through a Young hole, its evanescent wave propagates through both holes. Over the screen, we have a superposition of incoherent fields and of the interferences coming from the evanescent field; the incoherent fields do not interact much with the soliton; the soliton moves to the regions where the interferences are bright. Thus, the torus may be de Broglie's $u$ field while the evanescent field is the $\psi$ field [29].

A lot of different (3+0)D solitons may be defined in a single medium, using, for instance, the following methods:

- Changing the frequency (consequently $R$) and the polarisation of the wave;
- Commuting the roles plaid by the electric and magnetic fields;
- Using as index of refraction a function of the amplitudes of the fields which has many maximums;
- Introducing a torsion of the curved filament.

## 5. Quantum mechanics: a source of errors in astrophysics

Quantum mechanics pretends find a solution to the wave-particle duality, but it is difficult to understand its rules: Why do a single atom absorbs a photon, while all atoms of a prism are necessary to explain the refraction of a single photon? Saying that the maser cannot work, the best physicists are not conscious to make arbitrarily the choice "particle." For this problem, the other choice works, but the problem of the locality appears ....

Unhappily, the astrophysicists made a wrong choice: Forbidding by a rejection of the coherent effects the alternatives to the Doppler (or expansion) effect they are obliged to imagine extraordinary interpretations of observations.

### 5.1. Coherent Raman scatterings

"Coherent Raman scattering" is usually used for the scattering of laser pulses, with frequency shifts. In the region reached first by the laser beam, the emission is spontaneous, then the scattered light is amplified. The process is nonlinear so that few Raman line appear.

The experiments show coherence between the incident light and the ability of matter to amplify a Raman frequency. To maintain the coherence of phase in despite of the dispersion between the incident and scattered lights, their directions of propagation must generally be different, a cone of scattered light is obtained.

This possibility fails if the beam is wide, because the wave surfaces are identical for the incident and scattered lights: the amplification is limited to a "coherence length." At the limit, for Rayleigh scattering, the coherence length is infinite and the refraction produced by an interference of the incident beam with the ($\pi/2$ phase shifted) coherently scattered beam, is a large effect.

The "Impulsive Stimulated Raman Scattering" (ISRS), known since 1968 [30] is used mostly in chemical physics [31, 32, 33]; it uses ultrashort laser pulses. "Ultrashort" has two meanings: it may be relative to the shortest available pulses (usually femtosecond pulses) or to the physics of their interaction with matter: following G. L. Lamb [34] "shorter than all relevant time constants."

The name "Impulsive Stimulated Raman Scattering" is not very convenient because "Raman effect" corresponds to a two photon effect in which the initial and final levels differ. ISRS is a four photons effect which does not pump the molecules. However this parametric effect may be considered as a combination of two, nearly simultaneous Raman effects.

With Lamb's definition of the ultrashort pulses, the time between the collisions must be longer than the length of the pulses, so that the coherence of the scattering is not perturbed by the collisions even if the pulses are weak. The period of the beats between the incident and scattered light is larger than the length of the pulses, so that the beams interfere into a single, monochromatic, but frequency-shifted beam; this type of interferences is usually observed with two lasers, or in a Michelson interferometer, when a mirror moves. Thus, a frequency shift is obtained without any blur of the spectral line or of the images. However, as the power is large, the scattered amplitude is generally proportional to the square of the incident amplitude (stimulated scattering), so that the relative frequency shift is proportional to the intensity: the lines of a polychromatic spectrum have different relative frequency shifts.

### 5.2. Incoherent Light Coherent Raman Scattering (ILCRS)

Natural, incoherent light is made of pulses, the length of which is of the order of 10 nanoseconds.

The first condition to consider it as made of ultrashort pulses is that the collisional time be longer enough than 10 ns; it is an ordinary vacuum, so that the observation of an effect requires a long path.

The second condition is that the period of the beats of the incident and scattered lights are longer enough than 10 ns: The Raman transition must be in the microwaves range, that is the active molecules must have hyperfine structures.

As the power of natural sources is low, the amplitude scattered by ILCRS is proportional to the incident amplitude, the relative frequency shift depends on the frequency by dispersion effects only. The intensity of a parametric effect made of the combination of two effects, is limited by the weaker effect; here the low energy Raman transition, so that, in the visible, the relative frequency shift is almost constant.

ILCRS does not blur either the spectra or the images; it introduces a very nearly constant relative frequency shift: from the spectra and the sharpness of the images, *it is very difficult to distinguish ILCRS redshifts from Doppler redshifts.*

### 5.3. Application to astrophysics

It seems difficult to observe ILCRS in the labs, but astrophysics provides low pressures and long paths.

Which molecules may be this sort of catalyst, able to transfer energy from the redshifted high frequencies to the slightly heated thermal radiation at 2.7K?

All mono or polyatomic molecules are able to acquire Stark or Zeeman hyperfine structures; often, heavy molecules have such structures, but their density is low. The nuclear spin coupling transition of $H_2$ at .21 m is weak, but the molecules which have an odd number of electrons have strong transitions in hyperfine structures. NO, OH, $NH_2$... have been observed in the galaxies.

The ultraviolet radiations can ionise $H_2$ into $H_2^+$, a very stable molecule where the collisions which destroy it are rare. It is easy to understand that the observation of this molecule is difficult: it has many relatively weak lines which are spread by the redshift produced by the molecules themselves; increasing the pressure, $H_2^+$ is destroyed by collisions before a decrease of the ILCRS redshift.

The two main proofs of the expansion of the universe, the cosmological redshift, and the existence of the 2,7K radiation should be tested against their production by an ILCRS effect. A number of $H_2^+$ molecules, of the order of 20 in a cubic metre would produce the whole cosmological redshift [35, 36].

The small dispersion of ILCRS could explain the dispersions observed in the spectra of quasars [37] without the hypothesis of a variation of the constant of fine structure [38].

Near some bright QSOs, powerful thermal radiation observed in the infrared seems produced by heated dust, but the pressure of radiation should push the dust out. Is it an ILCRS effect?

A lot of theories tried to explain the multiplicity of the Lyman lines in the spectra of many quasars; ILCRS proposes a purely static, simple explanation supposing a variable magnetic field in a halo: where the field is low, the lines are written into the spectrum; elsewhere, ILCRS is activated by the Zeeman effect, the spectral lines are so spread by the shift that they are invisible [39].

## 6. Conclusion

Wanting to understand anything, we are tented to use marvellous explanations; the science needs centuries to destroys the marvellous by theories which are coherent and verified by the experiments.

The usefulness of the quantum theory is not a proof of the rightness of its postulates that the paradoxes show fundamentally incoherent; attributing the formalism of quantum theory to the classical theory seems possible, with the strong advantages of precision and lack of paradoxes.

Unavoidably wrong interpretations of quantum theory led the astrophysicists to neglect the search of an optical theory to interpret a part of the redshifts, thus introducing strange hypothesis, unable however to explain hundreds [40], probably now more than thousand observations.

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# PODKLETNOV'S PHENOMENON — GRAVITY ENHANCEMENT OR CESSATION?

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Owing to Vaidya metric including, the model of Expansive Nondecelerative Universe (ENU) is able to localize gravitational energy. Based on the quantification of gravitational field of the Earth, ENU allows to rationalize and quantify the effects of a superconductor-based high voltage impulse gravity generator constructed by Podkletnov. The differences in energy effect observed for two generators used are explained and improvement of the experiment arrangement is proposed.

Keywords: Podkletnov's phenomenon; impulse gravity generator; Expansive Nondecelerative Universe; gravity localization; solar corona temperature

## 1. Introduction

In the recent papers [1-3], the detection of anomalous forces formed within electron discharge from a superconductive ceramic emitter to a targeting electrode has been described. It has been hypothesed that these forces are of gravitational nature. The whole time of a discharge was $10^{-5}$ to $10^{-4}$ s, the peak value of the current at the discharge was of the order $10^4$ A, the voltage varied in the range of 500 kV to 2 MV. The distance between the electrodes was 15 cm to 2 m. Based on the total charge localized on the emitter ($\sim$ 0.1 C) the discharge energy approached $10^5$ J. The gravity impulse accompanying the discharge propagated as a coherent beam in the same direction as the discharge and penetrated through different media (air, brick wall, steel plate) without any noticeable loss of energy. It acted on mobile objects such as spherical 10 to 50 g weighing pendulums made from different materials (rubber, glass, metal, plastics) hanging on a cotton thread like a repulsive force independent on the pendulum material and proportional to their mass. Measurements of the impulse taken at the distance of 3 m, 6 m and 150 m gave identical results.

The experiment was theoretically rationalized by Modanese [3] who stated that the phenomenon could not be understood in the framework of general relativity. He proposed an explanation combining a quantum gravity approach and anomalous vacuum fluctuations.

In this contribution, an independent rationalization of the physical nature of the Podkletnov's experiments, explanation of different efficiency reached using two generators, and proposals for improvement of the efficiency are offered stemming from the background of the model of Expansive Nondecelerative Universe [4-8].

## 2. Background of the model of Expansive Nondecelerative Universe

The basic principles of the Expansive Nondecelerative Universe model were presented in a series of papers [4-8]. The model differs from more frequently used models of inflationary universe in the following features:
a) Schwarzschild metric is replaced by Vaidya metric,
b) the Universe permanently expands by the velocity of light $c$,
c) simultaneous creation of matter and the equivalent amount of gravitational energy (which is, however, negative and thus the total value of mass-energy is constant and equal zero) occurs.
d) the Einstein cosmological constant and the curvature index are of zero value.

Statement b) can be expressed as follows

$$a = ct_c = \frac{2Gm_U}{c^2}, \qquad (1)$$

where $a$ is the gauge factor, $t_c$ is the cosmological time, $m_U$ is the Universe mass (their present values are $a \cong 1.3 \times 10^{26}$ m, $m_U \cong 8.6 \times 10^{52}$ kg, $t_c \cong 1.4 \times 10^{10}$ yr).

In the ENU the state function is formulated as

$$p = -\frac{\varepsilon}{3}, \qquad (2)$$

i.e. a trace of the energy-momentum tensor equals to zero.

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In Vaidya metric [9, 10] the line element is formulated in the form

$$ds^2 = \frac{\Psi'^2}{f^2_{(m)}}\left(1-\frac{2\Psi}{r}\right)c^2dt^2 - \left(1-\frac{2\Psi}{r}\right)^{-1}dr^2 -$$

$$-r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{3}$$

where

$$\Psi = \frac{Gm}{c^2}, \tag{4}$$

$m$ is the mass of a body, $f_{(m)}$ is an arbitrary function. In order to $f_{(m)}$ be of nonzero value, it must hold

$$f_{(m)} = \Psi\left[\frac{d}{dr}\left(1-\frac{2\Psi}{r}\right)\right] = \frac{2\Psi^2}{r^2}. \tag{5}$$

As rationalized in [11] in the ENU

$$\Psi' = \frac{d\Psi}{cdt} = \frac{\Psi}{a}. \tag{6}$$

Vaidya metric may be applied in all cases for which the gravitational energy is localizable, i.e. if

$$r \leq r_{ef}, \tag{7}$$

where $r_{ef}$ is the effective gravitational range of a body with the gravitational radius $2\Psi$.

$$r_{ef} = (r_g a)^{1/2} = (2\Psi a)^{1/2}. \tag{8}$$

Gravitational influence can be thus realized only when the absolute value of gravitational energy density will exceed the critical energy density.

The energy-momentum complex of Einstein pseudotensor is [12]

$$\theta_i^k = \frac{1}{16\pi}\left[\frac{g_{in}}{\sqrt{-g}}\{-g(g^{kn}g^{lm} - g^{\ln}g^{km})\}_{,m}\right]_{,l}. \tag{9}$$

Application of Vaidya metric in Cartesian Kerr-Schild coordinates was solved by Virbhadra [10] who calculated the components of Einstein pseudotensor in a general form. In our approach, instead of a generally formulated $\Psi'$ its actual expression (6) is offered. The complete pseudotensor components are published elsewhere [11], in case of weak fields, its main component

$$\theta_0^0 = -\frac{c^4}{8\pi G r^2}\Psi' \tag{10}$$

represents the gravitational energy density in the first approximation. Relation (10) may be understood as being identical to Tolman equation

$$\varepsilon_g = -\frac{Rc^4}{8\pi G}, \tag{11}$$

where $R$ is the scalar curvature. In Schwarzschild metric, $R = 0$, *i.e.* gravitational energy is not localizable outside a body since $\varepsilon_g = 0$ there. In Vaidya metric

$$R = \frac{6G}{r^2c^3}\frac{d\dot{m}}{dt} = \frac{6Gm}{t_c r^2 c^3} = \frac{3r_g}{ar^2}, \tag{12}$$

where $R$ is the scalar curvature in the distance $r$ for a body having the mass $m$. It follows from (6), (10) and (11) that the density of the gravitational field energy generated by a body with the mass $m$ at the distance $r$ is

$$\varepsilon_g \cong -\frac{3mc^2}{4\pi a r^2}. \tag{13}$$

Within the limits of ENU model it is thus possible to localize and determine $\varepsilon_g$.

Substituting the Earth mass and its radius into (13), the energy density value of the Earth gravitational field is then

$$\left|\varepsilon_{g(Earth)}\right| \cong 25\mathrm{J.m}^{-3}. \tag{14}$$

Stemming from (11), gravitational output $P_g$ is

$$P_g = -\frac{d}{dt}\int\frac{Rc^4}{8\pi G}dV = -\frac{mc^3}{a} = -\frac{mc^2}{t_c}. \tag{15}$$

The energy density of the gravitational field can be expressed also as

$$\varepsilon_g = \frac{E_g}{\lambda_C^3}, \tag{16}$$

where $E_g$ and $\lambda_C$ are the energy of a gravitational field quantum and its Compton wavelength

$$\lambda_C = \frac{\hbar}{mc} = \frac{\hbar c}{E_g}. \tag{17}$$

For the energy of a gravitational field quantum it then hold

$$E_g = \left(\varepsilon_g \hbar^3 c^3\right)^{1/4}. \tag{18}$$

Due to its wave nature, gravitational field may be described by a wavefunction $\Psi_g$

$$\Psi_g = \exp(i\omega t) \tag{19}$$

and using (18) and (19)

$$\omega = \left(\frac{mc^5}{ar^2\hbar}\right)^{1/4}. \tag{20}$$

Based on Schrödinger-like equation for the gravitational waves

$$E_g\Psi_g = i\hbar\frac{d\Psi_g}{dt} \tag{21}$$

the energy of a quantum of gravitational field induced by a body with the mass $m$ at the distance $r$ is given as

$$E_g = -\left(\frac{m\hbar^3 c^5}{ar^2}\right)^{1/4}. \tag{22}$$

Substituting the values for the Earth into (22) it follows

$$|E_g| \cong 10^{-19} \text{ J}. \tag{23}$$

## 3. Interpretation of the Podkletnov's results stemming from the ENU

Within the Podkletnov's experiments [1-3], electrostatic field with the mean voltage intensity

$$E \cong 2.5x10^6 \text{ V/m} \tag{24}$$

was applied which corresponds to the mean energy density

$$\varepsilon_E \cong 27 \text{ J/m}^3. \tag{25}$$

Comparing this value with that given by (14) it is obvious that

$$\varepsilon_E \cong \left|\varepsilon_{g(Earth)}\right|. \tag{26}$$

Our explanation of the Podkletnov's phenomenon lies in a hypothesis that the Earth gravitational field interferes with the electrostatic field created within a discharge. The gravitational attractive force of the Earth results from the attraction of a body (pendulum) by the whole Earth. Provided that it is the only force exerting on a body, its direction is vertical as a result of the vector sum of attractive forces exerted by all the Earth parts. Should the Earth gravitational field be attenuated in a certain location (direction) the other attractive forces prevail. The observed deflection of pendulum thus results from ,,non-vertical“ Earth attraction. The pendulum deflection is, therefore, not a consequence of a repulsive force created within a discharge but lies in a local attractive force decreasing due to reducing the Earth gravitational field caused by its interference with electrostatic field.

In the mathematical language the above ideas can be expressed as follows.

The energy density of the electrostatic field is defined as

$$\varepsilon_E = \frac{\varepsilon_o V^2}{2r_x^2}, \tag{27}$$

where $V$ is the applied voltage and $r_x$ is the distance between electrodes. Based on (13), (14), (26), and (27) for the conditions on the Earth surface it then follows that

$$V^2 \cong \frac{3m_{(Earth)}c^2 r_x^2}{2\pi\varepsilon_o r_{(Earth)}^2 a}. \tag{28}$$

It is obvious that the optimal distance between the electrodes is

$$r_x \cong 0.2 \text{ m} \tag{29}$$

for the applied voltage of $V = 500$ kV, and

$$r_x \cong 0.85 \text{ m} \tag{30}$$

for $V = 2$ MV.

A theoretical treatment of (28) leads to a conclusion that to discharge only one electron from the emitor at the voltage of $V \cong 1$ V, it should hold

$$r_x \cong 10^{-7} \text{ m}. \tag{31}$$

Such an electron would obtain the energy of 1 electron-volt, i.e.

$$E_{(e)} \cong 1.6 \times 10^{-19} \text{ J}, \tag{32}$$

which is close to the energy of a gravitational quantum at the Earth surface (23). It can be, therefore, supposed that in cases when the requirement (26) is fulfilled, each discharged electron creates one quantum of the Earth gravitational field irrespective to the voltage applied. It directly followed from (18) that the energy of a gravitational quantum depends on the energy density only. At the Podkletnov's experiment, the total charge localized on the emitor at the voltage of 2 MV reached

$$Q \cong 0.1 \text{ C}, \tag{33}$$

which corresponds to a number of electrons

$$n_{(e)} = \frac{Q}{e} \cong 10^{18}. \tag{34}$$

Within a discharge at 2 MV, $10^{18}$ quanta of gravitational field are formed, each of them bearing the energy of about $10^{-19}$ J.

Podkletnov observed [3] that the total energy of a deflection depended on the pendulum mass. This observation is consistent with (4) and (8) stating that the higher the mass, the higher the effective gravitational radius. It must hold for the potential energy of a displaced pendulum

$$\Delta E = m_{(P)} n_{(e)} |E_g| (kg^{-1}), \tag{35}$$

where $m_{(P)}$ is the mass of pendulum. For $m_{(P)} =$ 18.5 g (data in [3] are related to this pendulum mass) relation (35) leads to

$$\Delta E \cong 1.8x10^{-3} \text{ J}, \tag{36}$$

which is in good accordance with the experimental data obtained using both a newer equipment (emitter 2)

$$\Delta E \cong 1.3x10^{-3}\ \mathrm{J} \tag{37}$$

and an older one (emitter 1), where

$$\Delta E \cong 2.3x10^{-3}\ \mathrm{J}. \tag{38}$$

Stemming from (30) and comparing the values in (37) and (38) it is obvious that the actual distance between the electrodes in a newer equipment (0.15 to 0.4 m) was far from the optimal distance 0.85 m. We believe that the rearrangement of a discharge chamber so as to permit to reach a higher distance between the electrodes will lead to a higher alteration in the pendulum potential energy. In addition, to preserve a high level of coherency when applying higher voltages, the space between two electrodes should be localized in an external magnetic field having the intensity proportional to $r_x$.

Relation (35) can be obtained by an independent way too. A flow of the gravitational energy of the Earth, $\sigma_{(Earth)}$ through its surface unit (hereinafter, all data are related to the surface of one square meter) is

$$\sigma_{(Earth)} = \frac{P_{g(Earth)}}{4\pi c r_{(Earth)}}, \tag{39}$$

where $P_{g(Earth)}$ is the gravitational output of the Earth (15). Due to a close proximity of the emitter and the pendulum and owing to coherent nature of the pulses, it can be written

$$\sigma_{(P)} \cong n_{(e)}|E_g| = \frac{|E_g|.Q}{e}, \tag{40}$$

where $\sigma_{(P)}$ is the gravitational energy flow from Podkletnov equipment through a surface unit. In such a case it must hold for the energy of pendulum deflection

$$\frac{\Delta E}{U} = \frac{\sigma_{(P)}}{\sigma_{(Earth)}}, \tag{41}$$

where $U$ is the potential energy of the pendulum with the mass $m_{(P)}$

$$U = \frac{G m_{(Earth)} m_{(P)}}{r_{(Earth)}}. \tag{42}$$

It follows from (39) to (42) that

$$\Delta E \cong \left(\frac{4\pi a G|E_g|}{ec^2}\right) Q m_{(P)}. \tag{43}$$

The expression in parentheses is a constant of the value close to 1 $\mathrm{C}^{-1}\ \mathrm{m}^2\ \mathrm{s}^{-2}$. This is why (43) can be rewritten in a simpler form (and expressed in the above unit) as

$$\Delta E \cong Q m_{(P)}. \tag{44}$$

Based on (34) and (35), as well as on the close numerical values of $|E_g|$ and $e$, relations (44) and (35) become thus almost identical.

For a vertical deflection of the pendulum, $h$ it can be written

$$h \cong \frac{Q}{g}, \tag{45}$$

where $g$ is the Earth gravitational acceleration. Once again, it is obvious from (45) that the vertical deflection of the pendulum depends only on the total charge, i.e. on the voltage between the electrodes.

## 4. Prospectives of application of the Podkletnov's phenomenon

The Podkletnov's phenomenon seems to be in principle of qualitatively new nature. If theoretically and experimentally proved more deeply, it could form an advantageous standpoint to rationalize several open phenomena. In this part its connection to the problems of fire balls stability, the hydrogen atom stability, and solar corona temperature is briefly outlined (the treatment of the problems in more details will be published elsewhere).

***a) Stability of fire-ball-like plasmatic bodies***

Our calculations indicate that in a case of simultaneous shortening the time of discharge and increasing the external magnetic field intensity at the voltage exceeding 500 kV, a stable plasmatic body with the radius of about 10 - 30 cm, similar to a fire ball, might be formed. From the technical point of view such an experiment is realizable since the kinetic energy of an electron at $V \geq 500$ kV is comparable to its rest energy. The success of the experiment is conditioned mainly by the type of superconductive emittor.

***b) The hydrogen atom***

The density of electromagnetic energy in the hydrogen atom is

$$\varepsilon_{(H)} = \frac{e^2}{4\pi\varepsilon_o r^4_{(H)}} \cong 10^{12}\ \mathrm{J/m^3}. \tag{46}$$

Putting the hydrogen atom into the gravitational field with a higher energy density, it would transform to a neutron. It is surely not a coincidence that the surface of a neutron star with the mass approaching $10^{30}$ kg and radius of about 10 km (common parameters of neutron stars) is characterized by the gravitational energy density close to that given by (46) (cf. 13). Evaluating the issue from another angle, based on (13) and (46) it is possible to estimate the parameters of neutron stars.

***c) Solar corona temperature***

The magnetic field density of a pulse magnetic field with the intensity of about $H \cong 7\times10^3$ A/m is identical to the absolute value of the Earth gravitational field density. If a pendulum was positioned in such a magnetic field, each pulse would cause a displacement of the pendulum. A mean intensity of the Sun magnetic field is about $10^2$ A/m, at some processes of magnetic field changes, however, the magnetic field intensity rises up to $10^5$ A/m. It is worth pointing out that at the intensity of about $4 \times 10^4$ A/m the magnetic field density is equal to the energy density of the Sun gravitational field. Interference of both the fields might, at certain circumstances, increase the kinetic energy of particles forming the solar corona by means of Podkletnov-like effect and rise its temperature up to the present value of $10^6$ K. The apparent uniformity of the solar corona temperature might be a consequence of periodicity of the magnetic effects.

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# ON SMALL MASS HYBRID STARS WITH QUARK CORE

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The models of layer neutron stars with strange quark core were constructed, based on wide range of realistic equations of state of superdense matter. The parameters of minimal mass layer stars were obtained to be sensitive enough to the selected models for both the neutron and strange quark matter. In particular, within the region of small masses for some models of neutron matter the appearance of additional local maximum on star mass-central pressure curve was revealed. This fact makes possible the existence of stable superdense stars of small masses ($M/M_\odot \sim 0.08$), having quark core with radius of $\sim 1km$, where only 6% of the whole star mass is concentrated. Their radius can reach the values of order of 1000km, that makes them resembling to white dwarfs.
In the considered model an accretion of matter can result in two consecutive transitions to the neutron star with a quark core with an energy release resembling supernovae explosions.

## 1. Introduction

The possibility of formation of strange quark matter in nuclear plasma results in the fact that the equation of state of superdense matter acquires van-der-vaalsian character [1,2]. Thereto energy per baryon $\varepsilon$, as a function of baryon density n, depending on values of parameters, unsufficiently exactly determined in the theory of strong interactions, can have both negative and positive minimum $\varepsilon_{min}$ . This circumstance, in its turn, results in two alternative opportunities.

If a case with $\varepsilon_{min} < 0$ is realized, a self-connected state of strange quark matter is possible and, as a consequence, self-confined configurations wholly consisting of such matter - so-called " strange stars ", - can exist [3] . If the variant with $\varepsilon_{min} > 0$ is realized, then at density higher than the threshold for the formation of strange quark matter, the first order phase transition with density jump takes place. Thus, according to Gibbs conditions, thermodynamical equilibrium between quark matter and nucleon component is possible, i.e. simultaneous coexistence of two phases takes place. The models corresponding to such equation of state, will have a core consisting of strange quark matter and a crust with structure of usual neutron stars, with the jump of density on the phases separation boundary .

In this connection it is necessary to note the calculations of models with the "mixed phase", containing quark formations of different configurations and supposing continuous change of pressure and density in the region of quark phase forming [4]. The results of these authors have shown, that the formation of the mixed phase of quark and nuclear substances can be more or less preferable, than usual first order phase transition from nucleon state to quark one, depending on values of local surface and Coulomb energies, connected with the formation of quark and nuclear structures of the mixed phase.

Below we examine a case supposing such surface tension, which results in first order phase transition with an opportunity of coexistence of two phases.

With the purpose of study of functional dependence of stellar configurations structural and integral parameters on the form of superdense matter equation of state we have considered a wide set of realistic equations of state ensuring the coexistence of neutron and strange quark matter. In the region of small masses it was revealed, that some of these equations of state result in the occurrence of an additional local maximum on star mass - central pressure dependence, that causes an opportunity of existence of new family of stable equilibrium configurations with very interesting distinctive features.

In the given work we present the results received for one of such equations of state, concentrating attention on small masses region.

## 2. Equation of state and the results of calculations

Constructing the equation of state of neutron star substance, the different equations of state for different intervals of density are usually used providing continuity at transition from one interval to another.

In the present paper the equations of state FMT,

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BPS and BBP (for so-called Aen-structure - matter consists of nuclei, degenerated neutrons and electrons) were used for density values below normal nuclear [5].

At supernuclear densities tabulated in [6] relativistic equation of state of neutron matter was used, taking into account two-particle correlations and calculated on the basis of meson-exchange potential of Bonn [7].

The specified equations of state covering in totality the density interval of $7.86 g/cm^3 < \rho < 4.81 \cdot 10^{14}$ $g/cm^3$, describe a matter of neutron star having nucleon structure.

For the description of quark component the MIT "bag" model was used [8], according to which strange quark matter consists of u, d, s- quarks and electrons, being in equilibrium with respect to weak interactions. In the examined equation of state with density jump the quark phase is characterized by the following phenomenological parameters of bag model: "bag" constant $B = 55 MeV/fm^3$, strange quark mass $m_s = 175 MeV$ and coupling constant $\alpha_c = 0.5$.

Gibbs conditions

$P^{(NM)} = P^{(QM)} = P_O,$

$\mu_b^{(NM)} = \mu_b^{(QM)}$

allow to find the pressure $P_O$, baryon number densities $n_N$ and $n_Q$, and mass densities $\rho_N$ and $\rho_Q$, describing the coexistence of two phases (the indices (NM) and (QM), N and Q specify that quantities belong to nucleon and quark phases correspondingly). Here $P$ is a pressure and $\mu_b$ – baryon chemical potential .

The numerical calculations within the framework of this model have resulted in the following values for the characteristics of first order phase transition: $P_O = 0.76 MeV/fm^3$, $n_N = 0.12 fm^{-3}$, $n_Q = 0.26 fm^{-3}$, $\rho_N c^2 = 113.8\, MeV/fm^3$, $\rho_Q c^2 = 250.5\, MeV/fm^3$.

Integral parameters of spherically-symmetrical static superdense star were determined for the given equation of state: coordinate radius of the star $R$, total gravitational mass $M$, total rest mass $M_O$, total proper mass $M_P$ and relativistic moment of inertia $I$, depending on the central pressure $P_c$.

If in the maximal mass region (configuration f with $M = 1.86 M_\odot$, $R = 10.8 km$) these dependences have usual character, in small masses region, where again there is a loss of stability - the condition $dM/dP_c > 0$ is violated (configuration a with $M = 0.0798 M_\odot, R = 254.7 km$), - the curve has a number of features, which are absent in case of other equations of state. Right away after the configuration a there is sharp fracture (configuration b with $M = 0.080 M_\odot$, $R = 205 km$) on the curve caused by the birth of quarks. The section (ab) corresponds to stable neutron stars without quark core. The configurations with small quark cores are unstable - section (bc) of the curve (configuration c is characterized by $M = 0.079 M_\odot, R = 380 km$), where $dM/dP_c < 0$. This corresponds to the results of [9], where for

$\lambda = \rho_Q/(\rho_N + P_O/c^2) > 3/2$

the configurations with small mass core of a new phase were found to be unstable. In the considered case $\lambda = 2.19$, i.e. meets the mentioned condition.

Usually for $\lambda > 3/2$ the sharp fracture abc appears not in the small masses region, but on the ascending branch of $M(P_c)$ curve, and after the configuration c up to the configuration with maximal mass f the curve has monotone ascending character. In the considered case right away after that fracture, again in small masses region the local maximum is formed - configuration d with $M = 0.082 M_\odot$ and $R = 1251 km$. At this configuration both the radius and the moment of inertia dependences on the central pressure have distinguished maximum.

For all specified critical configurations the packing coefficient is positive and, except for the configuration f, has the same order, as that for white dwarfs (configuration e is characterized by $M = 0.072 M_\odot$, $R = 133.2 km$).

As follows from calculations, for configuration d the values of radial coordinate $R_{nd} = 13.24 km$ and accumulated mass $M_{nd} = 0.07 M_\odot$ correspond to the threshold density for the formation of Aen-plasma.

The stars of the same mass corresponding to branches (cd) and (ef), differ considerably from each other by radius. While the stars of branch (ef) have radii of $\sim 10 km$, the stars of branch (cd) have sufficiently large radii of order of $1000 km$, that is characteristic for white dwarfs.

It is necessary to note, that in case of realization of the considered equation of state matter accretion on the neutron star will result in two consecutive catastrophic transitions to a neutron star with quark core, therefore two consecutive processes of energy release will take place. A star with a quark core belonging to the branch (cd) at first is formed; further accretion will result in configurations with radius about $1000 km$, and, at last, as a result of the second catastrophic reorganization a star of branch (ef), having the radius about $100 km$, is formed.

The research has confirmed a regularity of result and has shown, that the variation of the equation of state within the interval $9 \cdot 10^{13} g/cm^3 < \rho < 1.8 \cdot 10^{14} g/cm^3$ can result in some cases even in strengthening of the found out feature of $M(P_c)$ curve.

## 3. Conclusion

The first order phase transition in superdense nuclear matter from nucleon component to strange quark state with the transition parameter $\lambda > 3/2$ usually results in the appearance of small sharp fracture on the stable branch of star mass- central pressure curve. In the considered above model, where the loss of stability in the region of small masses occurs at higher density than in other models ($\rho_c = 2 \cdot 10^{14} g/cm^3$ ) and is close to the threshold for the birth of quarks ($\rho_c = 4.5 \cdot 10^{14} g/cm^3$ ), a new local maximum arises, that makes possible the existence of superdense stars of small masses with the radius, exceeding one thousand kilometer, and having quark core with radius of $\sim 1km$, in which only 6 % of the whole star mass is concentrated. Such stars by their size are similar to white dwarfs, but the main part of their mass is concentrated in Aen-plasma.

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# ON SUBSTANTIATION OF RELATIVISTIC EQUATION OF ROTARY MOTION IN GR MECHANICS

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The present work is aimed for substantiation of relativistic equation of rotary motion.

At present time practical tasks of space exploration needs calculation of planets and satellites motion on the basis of general relativity. That's why the task of substantiation of relativistic equations of translational and rotary motion of bodies in GR are exclusively actual and practically important. Practicing astronomers and specialists on astrodynamics wait for objective relativistic equations of translational and rotary motion of macroscopic bodies with respect to their inner structure.

Before start with a discussion of the main content of the work, let's notice that in our work [1] it has been developed critical analysis of all basic existed equations of rotary motion, that have been found by different authors (Petrova, Ryabushko, Brumberg etc.). As a result of the analysis it was shown that from all that equations on present day the most correct and exhaustive equation of rotary motion is Brumberg equation [2], found from field equations of Einstein by second Fock method. For two bodies Brumberg equation has the form:

$$\dot{\vec{\omega}} = \frac{1}{2c^2}(\vec{v}\vec{\omega})[\vec{v}\vec{\omega}] + \frac{9\gamma m_0}{2c^2r^5}(\vec{v}\vec{r})[\vec{v}\vec{r}] + \frac{\gamma}{c^2}\{\frac{5m_0}{r^3}\vec{\omega}(\vec{r}\vec{v})+$$
$$+\frac{m_0}{r^3}\vec{v}(\vec{r}\vec{\omega}) - \frac{2m_0}{r^3}\vec{r}(\vec{v}\vec{\omega}) + \frac{3I_0^*}{r^5}\vec{\omega}_0(\vec{r}\vec{v}) + \frac{3I_0^*}{r^5}\vec{v}(\vec{r}\vec{\omega}_0)+$$
$$+\frac{3I_0^*}{r^5}\vec{r}(\vec{v}\vec{\omega}_0) - \frac{15I_0^*}{r^7}\vec{r}(\vec{r}\vec{v})(\vec{r}\vec{\omega}_0) + \frac{I_0^*}{r^3}[\vec{\omega}\vec{\omega}_0]+$$
$$+\frac{3I_0^*}{r^5}(\vec{r}\vec{\omega}_0)[\vec{r}\vec{\omega}]\} + \frac{9\gamma m_0 I'}{I^*r^5c^2}[(\vec{r}\vec{\omega})[\vec{r}\vec{\omega}] + \vec{\omega}(\vec{r}\vec{v})+$$
$$+\vec{r}(\vec{\omega}\vec{v}) + \vec{v}(\vec{\omega}\vec{r}) - \frac{5\vec{r}}{r^2}(\vec{r}\vec{v})(\vec{\omega}\vec{r})], \tag{1}$$

where
$I^* = \frac{2}{5}mR^2, \quad I' = \frac{1}{35}mR^4.$

In the same work [2], making note:

$$\vec{S} = I^*\{(1 + \frac{v^2}{2c^2} + \frac{3U}{c^2})\vec{\omega} + \frac{1}{2c^2}\times$$
$$\times[\vec{v}[\vec{\omega}\vec{v}]] + \frac{3m_0}{mc^2}rot\vec{U_M} - \frac{2}{c^2}rot\vec{U}_0 - \frac{6m_0I'}{I^{*2}c^2}rot\vec{U}\}, \tag{2}$$

Relativistic equation of the rotary motion of Brumberg is written in the form:

$$\frac{d\vec{S}}{dt} = [\vec{\omega}\vec{S}]. \tag{3}$$

In the present work we will continue the investigation of the question on a former substantiation of relativistic equation of rotary motion of Brumberg. For that let's find the next, independent from previous, approach for finding GR equation of rotary motion in GR mechanics.

Come from relativistic equation of translational motion for two spherical, homogeneous bodies $(m \ll m_0)$ [1]

$$\frac{d}{dt}\frac{\partial L}{\partial \vec{v}} - \frac{\partial L}{\partial \vec{r}} = 0, \tag{4}$$

Where LaGrange function

$$L = -mc^2 + \frac{1}{2}mv^2 + T + \frac{1}{c^2}[\frac{1}{8}mv^4 + (\frac{1}{3}\varepsilon + \frac{3}{2}T)v^2-$$
$$-\frac{1}{4}I^*(\vec{\omega}\vec{v})^2] + \frac{\gamma m m_0}{r}[1 + \frac{3v^2}{2c^2} + \frac{1}{c^2}(\frac{\xi_0}{m_0} + \frac{\xi}{m})-$$
$$-\frac{\gamma m_0}{2c^2r}] + \frac{\gamma}{2c^2}([(3m_0\vec{S} + 4m\vec{S}_0)\vec{\nabla}\frac{1}{r}]\vec{v})+$$
$$+\frac{2\gamma}{7c^2}\frac{m_0}{m}([\vec{S}\vec{\nabla}][\vec{S}\vec{\nabla}])\frac{1}{r} + \frac{\gamma}{c^2}([\vec{S}\vec{\nabla}][\vec{S}_0\vec{\nabla}])\frac{1}{r}+$$
$$+\frac{2\gamma m}{7m_0c^2}([\vec{S}_0\vec{\nabla}][\vec{S}_0\vec{\nabla}])\frac{1}{r}. \tag{5}$$

Here $T$ - kinetic energy of rotary motion of a test body, $e$ - energy of mutual attraction of particles, that consist the body (taking with contrast sign)

$$\xi_0 = \frac{2}{3}\varepsilon_0 + \frac{8}{3}T_0,$$

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$$([\vec{S}\vec{\nabla}][\vec{S}_0\vec{\nabla}]) = (\vec{S}\vec{S}_0)\triangle - (\vec{S}\vec{\nabla})(\vec{S}_0\vec{\nabla}),$$

$$(\vec{S}_0\vec{\nabla})(\vec{S}_0\vec{\nabla}\frac{1}{r}) = -\frac{S_0^2}{r^3} + \frac{3(\vec{r}\vec{S}_0)^2}{r^5}, \qquad (6)$$

Translational momentum is found from (5)

$$\vec{P} = \frac{\partial L}{\partial \vec{v}} = m\vec{v} + \frac{m\vec{v}}{c^2}[\frac{v^2}{2} + (\frac{2\varepsilon}{m} + \frac{3T}{m}) + 3U] -$$

$$-\frac{1}{2c^2}(\vec{S}\vec{v})\vec{\omega} + \frac{\gamma}{2c^2}[(3m_0\vec{S} + 4m\vec{S}_0)\vec{\nabla}\frac{1}{r}]. \qquad (7)$$

Let's try to find rotary momentum of a test body, analogously to translational momentum, by equation

$$\vec{S} = \frac{\partial L}{\partial \vec{\omega}} = I^*\{(1 + \frac{3v^2}{2c^2} + \frac{8U}{3c^2})\vec{\omega} - \frac{\vec{v}}{2c^2}(\vec{\omega}\vec{v}) +$$

$$+\frac{3m_0}{mc^2}rot\vec{U}_M - \frac{2}{c^2}rot\vec{U}_0 - \frac{8m_0}{7mc^2}rot\vec{U}\}. \qquad (8)$$

One can rewrite this equation using obvious expression

$$[\vec{v}[\vec{\omega}\vec{v}]] = v^2\vec{\omega} - \vec{v}(\vec{v}\vec{\omega}). \qquad (9)$$

Then (8) has the form

$$\vec{S} = I^*\{(1 + \frac{v^2}{c^2} + \frac{8U}{3c^2})\vec{\omega} - \frac{1}{2c^2}[\vec{v}[\vec{\omega}\vec{v}]] + \frac{3m_0}{mc^2}rot\vec{U}_M -$$

$$-\frac{2}{c^2}rot\vec{U}_0 - \frac{8m_0}{7mc^2}rot\vec{U}\}. \qquad (10)$$

One can compare this equation with rotary momentum (2) that comes from equation for rotary motion of Brumberg (1). For that let's rewrite (2) in the form of

$$\vec{S} = I^*\{(1 + \frac{v^2}{2c^2} + \frac{3U}{c^2})\vec{\omega} - \frac{1}{2c^2}[\vec{v}[\vec{\omega}\vec{v}]] +$$

$$+\frac{3m_0}{mc^2}rot\vec{U}_M - \frac{2}{c^2}rot\vec{U}_0 - \frac{15m_0}{14mc^2}rot\vec{U}\}. \qquad (11)$$

Comparing both rotary momentum for a test body, found by different methods, one can see that they are equal. Some slight differences, connected with the coefficients in several terms, perhaps are due to difference in calculation of integrals that depend on the inner structure; Brumberg considers bodies absolutely solid, we consider bodies as liquid in the process of calculation of relativistic equation of translational motion. This way we come to the conclusion that for determination of expressions for translational and rotary momentum it's enough to know LaGrange function of relativistic equation of translational motion (5). This function was found comparatively reliable (with precision $L \sim mq^2\frac{q^2}{c^2}\frac{R^2}{D^2}$) and coincides in Abdildin and Brumberg works. Slight discrepancies of LaGrange function are due to different presumptions on the inner structure of solid bodies (solid or liquid). Knowing the direct expression for rotary momentum from LaGrange relativistic equation for translational motion, we can find relativistic equation of rotary motion, without its finding from equations of gravitational field of Einstein. Really, rotary momentum (10), as any other free vector should correspond to the expression

$$\frac{d\vec{S}}{dt} = \frac{dS}{dt}\vec{e}_S + [\vec{\Omega}\vec{S}], \qquad (12)$$

According to adiabatic theory of motion of bodies in GR there are the following expressions:

$$S = inv, \qquad \vec{\Omega} = \frac{\partial H}{\partial \vec{S}}. \qquad (13)$$

Setting (13) in (12) one has:

$$\frac{d\vec{S}}{dt} = [\vec{\omega}\vec{S}], \qquad (14)$$

i.e. the sought-for relativistic equation of rotary motion .

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# QUANTUM EFFECTS AND CLUSTER FORMATION

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The causal interpretation of quantum mechanics is applied to the universe as a whole and the problem of cluster formation is studied in this framework. It is shown that the quantum effects *may* be the source of the cluster formation.

## 1. Casuel quantum mechanics and gravity

Since the early years of 20th century, there is a causal theory of quantum phenomena called *de-Broglie–Bohm* theory [1, 2, 3, 4, 5]. It is well proved that the causal theory reproduces all the results of the orthodox quantum theory[4, 5], as well as predicting some new results (such as time of tunneling through a barrier [6]) which in principle lets the experiment to choose between the orthodox and the causal quantum theories. Perhaps the most important point about the causal theory is that it presents a causal deterministic description of the reality. But an important by product of this theory is that enables one to put a step towards constructing a successful quantum gravity theory [7, 8, 9, 10, 11, 12].

The causal quantum theory is based on two postulates [3, 4, 5]:

**First Law:** The physical reality is described by $\{\vec{r}(t); \Psi(\vec{x},t)\}$ where $\vec{r}(t)$ is the position vector of the particle and $\Psi(\vec{x},t)$ is a self–field. Both have a causal evolution. $\Psi(\vec{x},t)$ satisfies an appropriate wave equation (like the Schrödinger equation for a non-relativistic particle) while $\vec{r}(t)$ satisfies Newton's equation of motion in which there is an additional force called *quantum force* resulted from $\Psi(\vec{x},t)$. This force can be derived from some *quantum potential* given by $\mathcal{Q} = -\frac{\hbar^2}{2m}\frac{\nabla^2|\Psi|}{|\Psi|}$ for a non relativistic particle.

**Second Law:** The statistical features of the theory is such that the ensemble density is given by $\rho = |\Psi|^2$.

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So the complete description of the physical reality is given by the following two equations for non–relativistic particles:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi, \tag{1}$$

$$m\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}(V+\mathcal{Q})\Big|_{\vec{x}=\vec{r}(t)}; \quad \mathcal{Q} = -\frac{\hbar^2}{2m}\frac{\nabla^2|\Psi|}{|\Psi|}. \tag{2}$$

The relativistic extension is easy [4, 5]. If one works in the flat Minkowski space–time one should replace the Schrödinger equation with some relativistic wave equation (such as the Klein–Gordon equation for spin zero particles). The quantum potential would be generalized to:

$$\mathcal{M}^2 = m^2(1+\mathcal{Q}) = m^2 + \alpha\frac{\Box|\Psi|}{|\Psi|}; \qquad \alpha = \frac{\hbar^2}{m^2c^2} \tag{3}$$

and the equation of motion is given by:

$$\mathcal{M}\frac{dx^\mu}{d\tau^2} = \left(\eta^{\mu\nu} - \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right)\partial_\nu\mathcal{M}. \tag{4}$$

It is important to note that $\mathcal{M}$ plays the role of the (quantum) mass field of the particle [5, 7, 8, 9, 11, 12].

The extension to the curved space–time is trivial [5, 7, 8, 9, 11, 12]:

$$\mathcal{M}^2 = m^2 + \alpha\frac{\Box|\Psi|}{|\Psi|}; \qquad \alpha = \frac{\hbar^2}{m^2c^2}, \tag{5}$$

$$\mathcal{M}\frac{dx^\mu}{d\tau^2} = \left(g^{\mu\nu} - \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right)\nabla_\nu\mathcal{M}. \tag{6}$$

An important result of all these things is that the quantum effects of matter are in fact geometrical in nature [7]. One can easily transforms the above equation of motion to the ordinary geodesic equation via a conformal transformation of the form [7, 8, 9, 11]:

$$g_{\mu\nu} \to \widetilde{g}_{\mu\nu} = \frac{\mathcal{M}^2}{m^2} g_{\mu\nu}. \tag{7}$$

So that the matter quantum effects are included in the conformal degree of freedom of the space–time metric [7, 11]. A corollary of this result is that one can always work in a gauge (classic gauge) in which no quantum effect be present or in a gauge (quantum gauge) in which the conformal degree of freedom of the space–time metric is identified with the quantum effects of matter [11].

An important question is that if this quantum gravity theory leads to some new results. This theory is investigated for the BigBang and black holes in reference [7, 11]. But here we are interested in investigating whether this theory has anything to do with the cluster formation or clustering of the initial uniform distribution of matter in the universe. The problem of cluster formation is an important problem of cosmology and there are several ways to tackle with it [13]. Here we don't want to discuss those theories, and our claim is not that the present work is a good one. Here we only state that *the cluster formation can also be understood in this way*. It is a further task to decide if this work is in complete agreement with experiment or not.

## 2. Claster formation and the quantum effects

In order to investigate if the cluster formation can be viewed as a quantum effect produced by the quantum potential, we consider the universe as a fluid. The hydrodynamics equation is given by:

$$\mathcal{M}\left(\frac{\partial p}{\partial x^\nu} g^{\mu\nu} + \frac{1}{\sqrt{-g}}\left(\sqrt{-g}(p+\rho)U^\mu U^\nu\right) + \right.$$

$$\left. + \Gamma^\mu_{\nu\lambda}(p+\rho)U^\mu U^\nu\right) = \rho\left(g^{\mu\nu} - U^\mu U^\nu\right)\frac{\partial \mathcal{M}}{\partial x^\nu}, \tag{8}$$

where we have introduced the quantum force in the right hand side just as it is introduced in the equation of motion of a single particle (see equation (6)). It must be noted that the metric itself must be calculated from the corrected Einstein's equations including the back–reaction terms [7, 11]. In fact one must solve the above equation and the metric equation simultaneously to obtain the metric and the density. We shall not do in this way because solving those equations (equations of references [7, 11]) is difficult. We shall do in a similar way. It is an iterative way and is based on the fundamental assumption of this theory, that is equation (7). As the first order of iteration, we consider the space–time metric as given by the classical Einstein's equations (Robertson–Walker metric) and solve the above equation for the density, then using the result obtained, calculate the quantum metric using equation (7). Then the new metric can be used to obtain the density at the second order and so on.

In the comoving frame and with the assumption that the universe is in the dust mode ($p = 0$) with the flat Robertson–Walker metric, we have from the equation (8):

$$\frac{d\rho^{(1)}}{dt} + 3H\rho^{(1)} = 0, \tag{9}$$

$$\frac{\partial \mathcal{Q}^{(1)}}{\partial x^i} = 0, \tag{10}$$

where $^{(1)}$ denotes the first order of iteration and $H = \dot{a}/a$ is the Hubble's parameter. The solution of the above two equations is:

$$\rho^{(1)} = \frac{\mathcal{X}^{(1)2}}{t^2}; \qquad \mathcal{Q}^{(1)} = \text{constant}, \tag{11}$$

where $\mathcal{X}^{(1)}$ should yet be determined. The constancy of the quantum potential leads to:

$$\mathcal{Q}^{(1)} = \frac{\alpha}{2}\frac{\Box\sqrt{\rho^{(1)}}}{\sqrt{\rho^{(1)}}} = -\frac{\alpha}{2a^2}\frac{\nabla^2\mathcal{X}^{(1)}}{\mathcal{X}^{(1)}} \tag{12}$$

so that:

$$\nabla^2\mathcal{X}^{(1)} + \beta\mathcal{X}^{(1)} = 0, \tag{13}$$

where:

$$\beta = \frac{2\mathcal{Q}^{(1)}a^2}{\alpha}. \tag{14}$$

This equation for $\mathcal{X}^{(1)}$ can simply be solved either in the Cartesian coordinates or in the spherical ones. The solution is:

$$\mathcal{X}^{(1)} = \sin\left(\sqrt{\frac{\beta}{3}}x\right)\sin\left(\sqrt{\frac{\beta}{3}}y\right)\sin\left(\sqrt{\frac{\beta}{3}}z\right) \tag{15}$$

or:

$$\mathcal{X}^{(1)} = \sum_{l,m}\left(a_{lm}j_l(\sqrt{\beta}r) + b_{lm}n_l(\sqrt{\beta}r)\right)Y_{lm}(\theta,\phi). \tag{16}$$

This is the first order approximation. At the second order, one must use the equation (7) to change the scale factor $a^2$ to $a^2(1+\mathcal{Q})$, and then from the relation (9) we have:

$$a^2(1+\mathcal{Q}) = t^{2/3}\mathcal{X}^{(1)-4/3}. \tag{17}$$

So that:

$$\mathcal{Q}^{(2)} = -1 + \mathcal{X}^{(1)-4/3} \tag{18}$$

and then using this form of the quantum potential in the relation (13) or (15) leads to the following approximation for the density:

$$\rho^{(2)} = \frac{1}{t^2}\sin^2\left(\sqrt{\frac{\gamma}{3}}x\right)\sin^2\left(\sqrt{\frac{\gamma}{3}}y\right)\sin^2\left(\sqrt{\frac{\gamma}{3}}z\right), \quad (19)$$

where:

$$\gamma = \frac{2a^2}{\alpha}\left(-1 + \mathcal{X}^{(1)-4/3}\right) \quad (20)$$

and $\mathcal{X}^{(1)}$ is given by the relation (15). This procedure can be done order by order.

## 3. Results

In the figures (1), (2), (3), (4) and (5) the density at four times are shown and the clustering can be seen easily. These figures are plotted using the solution in the Cartesian coordinates.

In the figures (6), (7), (8), (9) and (10) the $(l, m) = (00)$ mode of equation (16) is shown at five time steps.

In the figures (11), (12), (13), (14) and (15) the $(11) \oplus (1-1)$ mode is shown at five time steps.

The second order solution (19) is shown in figure (16), where both large scale and small scale structures are shown.

It is important to note that the clustering can be seen in any of these figures. In the last figure, however, one observes that at the large scale the universe is homogeneous and isotropic, while at the small scale these symmetries are broken.

At the end, in order to see whether or results are in agreement with the observed clustering, the correlation function $(\xi(r))$ is obtained from the third order of iteration and is compared with the cases $\xi = (r/r_0)^{-\gamma}$ with $\gamma = 1.8$ and $\gamma = 3$ and with the standard result of a typical $P^3M$ code [13]. As it can be seen in figure (17) our results are in good agreement with the $P^3M$ code and with observation.

As we stated previously, our claim here is not that this theory is a good one for the cluster formation problem. But it is only claimed that in the framework of causal quantum theory, the quantum force *may* be a cause for the cluster formation.

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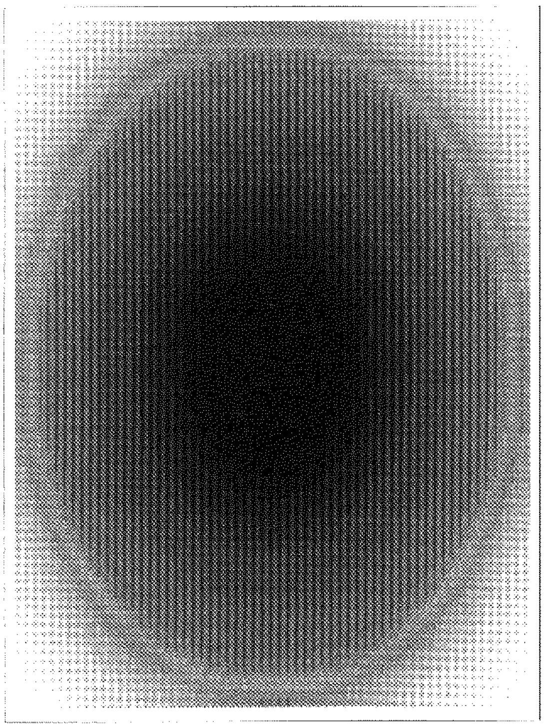

Figure 1: Cartesian mode after expansion by the factor 1.1

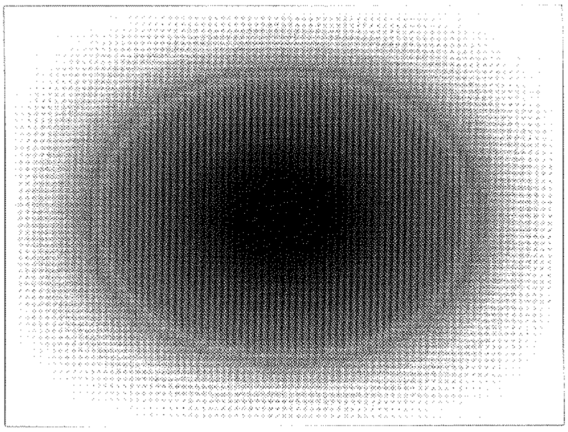

Figure 2: Cartesian mode after expansion by the factor 1.2

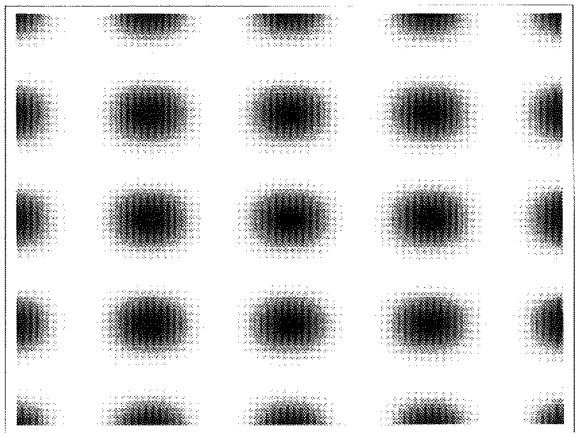

Figure 5: Cartesian mode after expansion by the factor 6

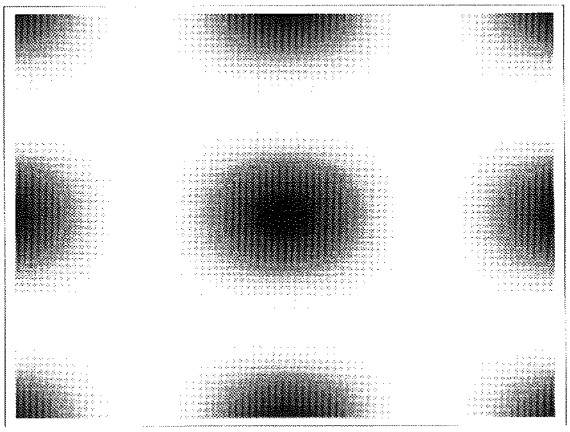

Figure 3: Cartesian mode after expansion by the factor 3

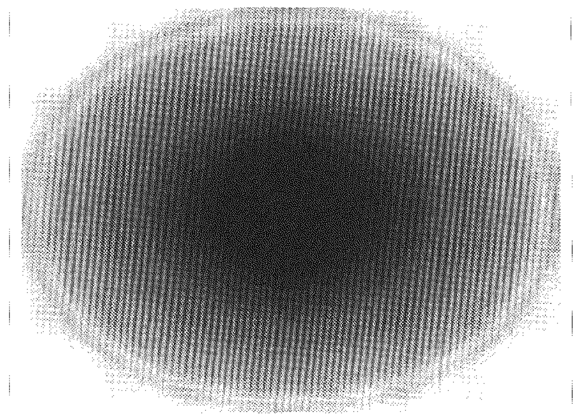

Figure 6: (00) mode after expansion by the factor 1.1

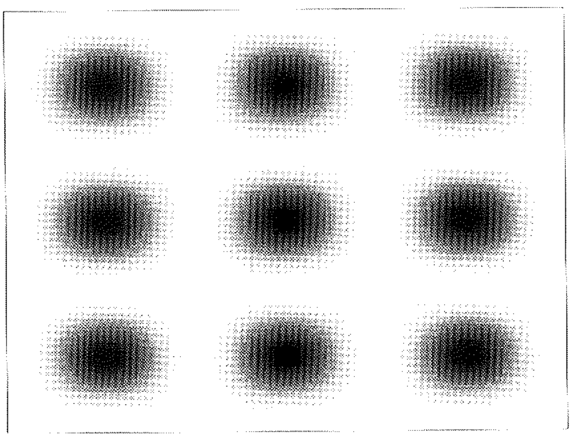

Figure 4: Cartesian mode after expansion by the factor 5

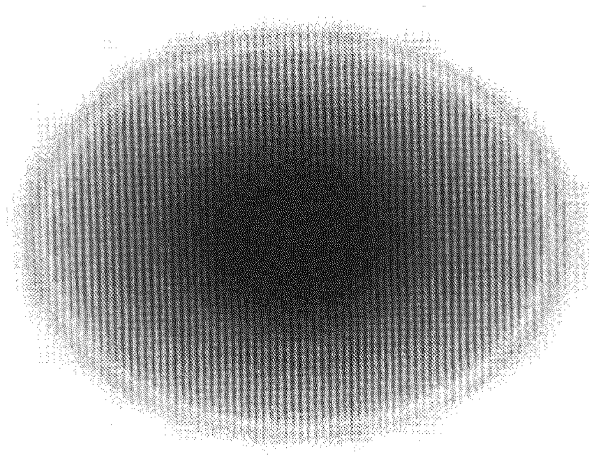

Figure 7: (00) mode after expansion by the factor 1.2

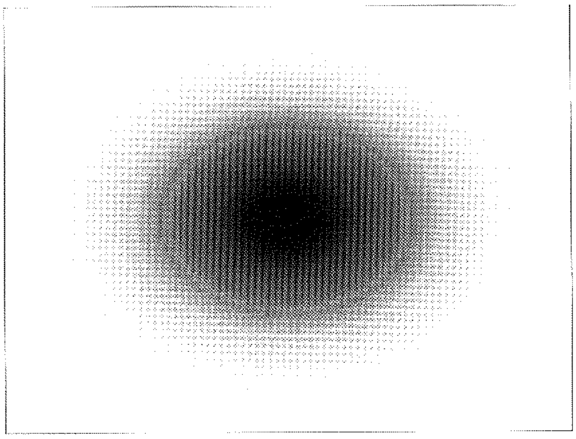

Figure 8: (00) mode after expansion by the factor 3

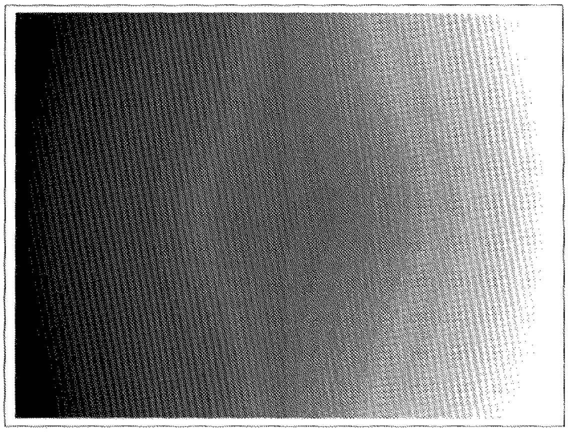

Figure 11: (11)$\oplus$(1−1) mode after expansion by the factor 1.2

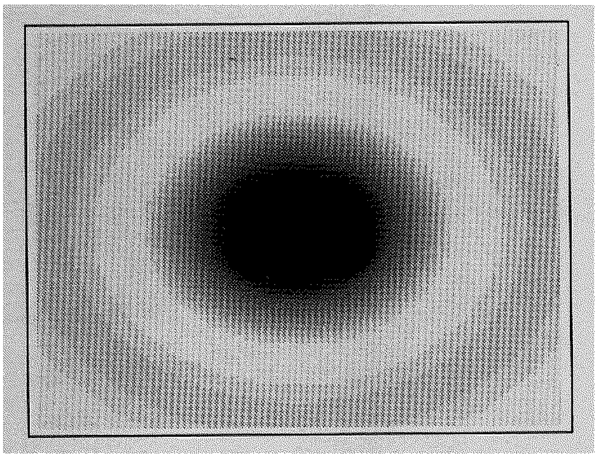

Figure 9: (00) mode after expansion by the factor 5

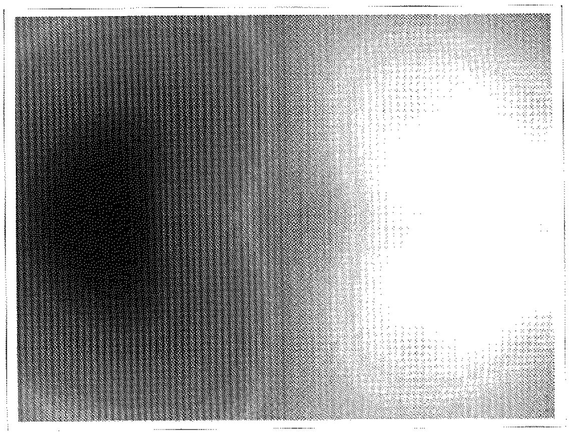

Figure 12: (11)$\oplus$(1−1) mode after expansion by the factor 3

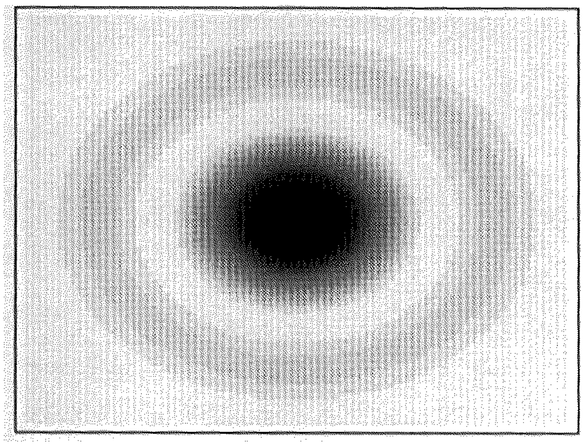

Figure 10: (00) mode after expansion by the factor 6

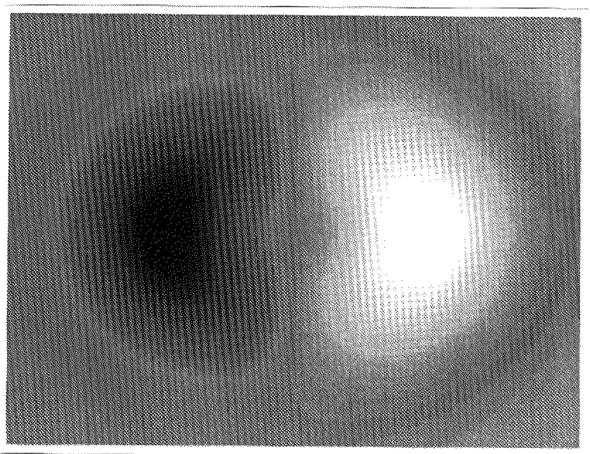

Figure 13: (11)$\oplus$(1−1) mode after expansion by the factor 5

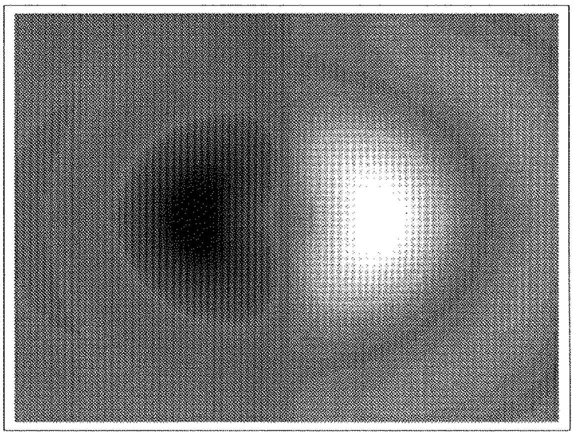

Figure 14: (11)$\oplus$(1−1) mode after expansion by the factor 6

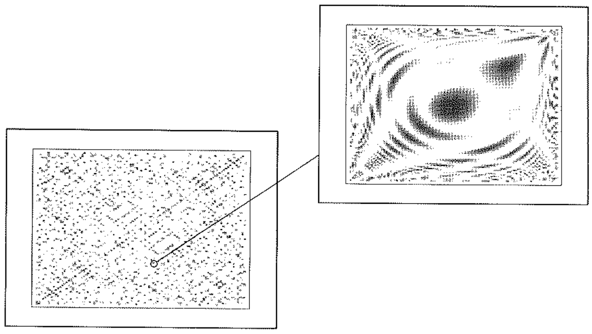

Figure 16: Second order solution after expansion by the factor 8

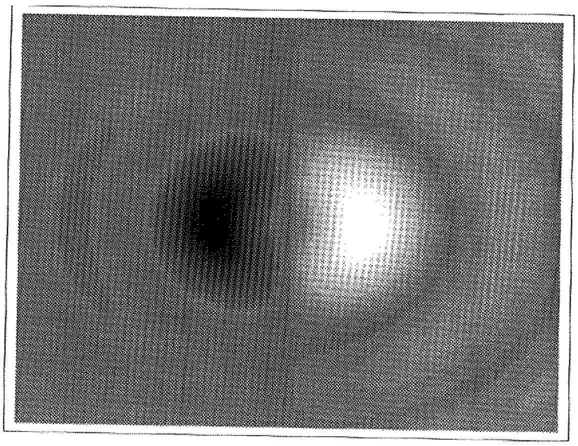

Figure 15: (11)$\oplus$(1−1) mode after expansion by the factor 8

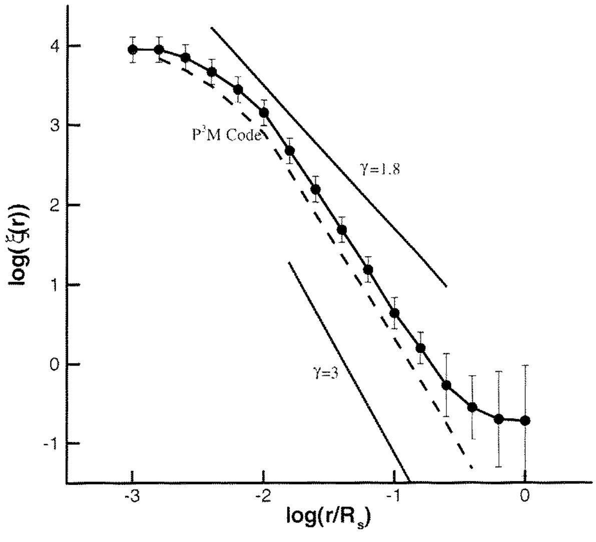


Figure 17: Correlation function after expansion by the factor 8

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# THE UNIPOLAR MOTOR: A TRUE RELATIVIST ENGINE

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*Fundación Julio Palacios and Consejo Provincial de Educación, Salta 326, 8300-Neuquén, Argentina*

*Received October 8, 2001*

Following our task concerning unipolar devices, started as far as 1994 in the American Journal of Physics [1], [2], we describe a new experiment available to locate, without ambiguities, the seat of ponderomotive forces in such engines. Our findings are in full accordance with relativistic physics and disprove absolutistic views on the whole issue.

*"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances."*

**Is. Newton.**

## 1. The asymmetric rotor

The figure 1 shows a ceramic-type, axially magnetized permanent magnet $M$ which exhibits a circular sector **free of magnetic matter**. The above sector, amounting some 1/30 of the whole piece, was cutted out in order to allow $B$ "lines" to reverse its sense. As customed, the symbol $\bullet$( $\times$ ) labels an outgoing (ingoing) $B$ field.

The magnet itself is embodied in a wood cylinder whose rim locates two semicircular channels. All the above is firmly anchored to a conducting axle $X$ ended as sharp points able to rotate "quasi free" of frictional forces when pressing upon a hard, polished, glass surface.

The inner ends of the conducting branches $aX$, $bX$ are soldered to $X$. The outer ends $a, b$ are each inmersed in the proper semicircular channel previously filled with mercury.

We label as $R$ (*rotor*) the whole above apparatus, and as $CC$ the closing circuit wires which allow electrical connection with the power supply ($PS$). The whole apparatus $R$ is free to rotate in the lab around its own symmetry axis (see photographs).

## 2. Experimental

### 2.1.

When DC is injected from the $PS$ at $a_1$, being its return path a sliding wire which connects the $PS$ with $X$, **a neat** $CCW$**rotation** starts when the current reaches some 2 A. The above current ensures us the minimun

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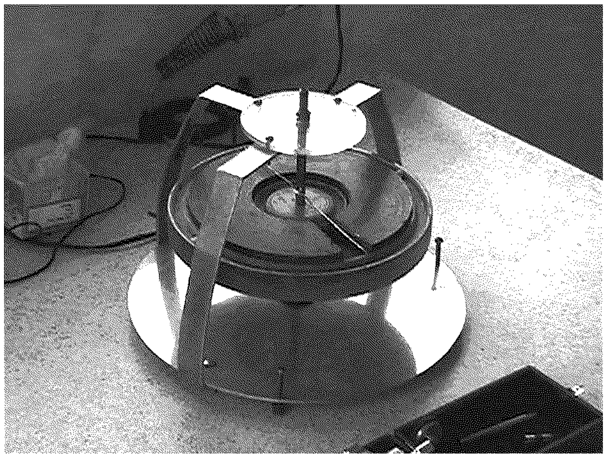

Figure 1: Ceramic-type, axially magnetized permanent magnet $M$ which exhibits a circular sector **free of magnetic matter**

torque able to overcome the whole frictional forces, mainly due to the stationnary wire/mercury contact .

### Analysis

In the following, we label as **A** and **R** when referring to hypothetical absolutist and relativist advocates, respectively. The above due to the following facts: An absolutist don't need to consider *any relative motion* between the wires and the magnet, in order to explain the observed facts. In his view the branches $aX$, $bX$ are the seat of ponderomotive forces. Consequently, he (she) ensures that the above branches, when carrying DC current, are able to "drag" the magnet as a whole. What matters for a relativist is, on the contrary, the motion of the magnet *relative* to the wires .

**A** view: the branch $aX$ plays the *active* role in the generation of *ponderomotive forces*. Thus, the Laplace force, $dF$=$I(dlxB)$, when integrated along the entire branch, is the responsible for the observed $CCW$ torque. Briefly speaking, **A** ensures that when current is flowing, $aX$ "drags" $M$ as a whole in the $CCW$ sense.

Following the **R** view, $M$ plays the major role in the force interaction. Thus, $M$ repels (attracts) $aX$ in the $CCW$ sense and attracts (repels) the $CC$ wire in the $CW$ sense. Consequently, $aX$ repels (attracts) $M$ in the $CW$ sense. Being $aX$ soldered to $M$, the whole interaction $aX$-$M$ cannot generate a neat rotational torque. Only remains the interaction $M$-$CC$ wire in order to explain rotation. Briefly speaking, $M$ is pushed (pulled) by the $CC$ wire in the $CCW$ sense (Newton?s third law), as observed.

### 2.2.

Now DC is injected from the $PS$ at $X$, being its return path at $b_1$ , crossing the $bX$ branch. A neat $CW$ of $R$ takes place when the current reaches some 2 A. Also note that frictional forces are the same as in 2.1.

### Analysis

**A** view: As well as in 2.1, the Laplace force acting on $bX$ must "drag" $M$ in the $CCW$ sense, with wich he (she) becomes unable to explain the observed facts..

**R** view: As well as in 2.1, the interacction $bX$-$M$ is unable to deliver a rotational torque and, since also $M$ moves the $CC$ wire in the $CCW$, then the $CC$ wire moves $M$ clockwise. At this point we can see a nice phenomenon: the branch $bX$ bends in the $CCW$ sense, as expected both by **A** and**R**, wherein $R$ moves in the opossite sense, as expected by **R**.

### 2.3.

At the end, DC is injected at $a_1$ and returned at $b_1$. We failed to detect the slighest rotation when current was raised from 1 up to **100 A**. The branches $aX$, $bX$ showed ostensible bending (and heating) in the $CCW$ sense.

The above striking facts are trivially expected by **R** since when seeking for the $CC$ wires he (she) only finds some portions of the semicircular mercury channels, and the wires connecting with the $PS$, both unable to deliver any rotational torque on $M$. Therefore, rotational motion is clearly forbidden both for the magnet as for the closing circuit wires.

## 3. Topologycal considerations

Figure 2 attempts to sketch the whole action of the only two torques able to produce *relative motion* between $M$ and the $CC$ wire. Being Laplace force perpendicular to the current, forces acting on the mercury circular path, and on the circular wires connecting with th $PS$ cannot contribute to rotation of the $CC$. Consequently,

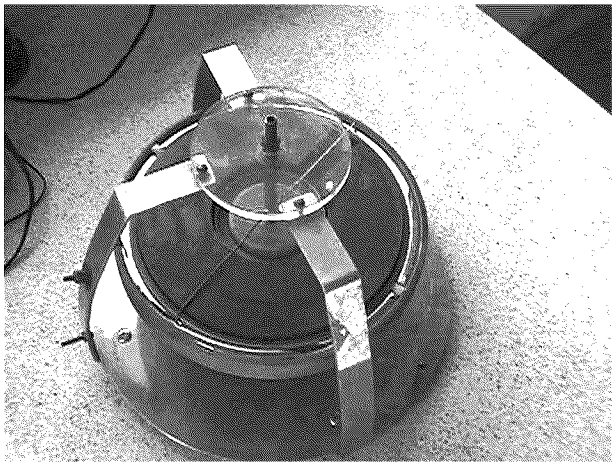

Figure 2: The whole action of the only two torques able to produce *relative motion* between $M$ and the $CC$ wire

the $CC$ cannot substantiate any rotational torque on $M$ (principle of *action-reaction*). With the aid of the fundamental property $divB= 0$ [2] and some elementary topologycal considerations it is not difficult to generalize the above statement for any arbitrarily shaped $CC$ wire, a fact also easyly corroborated by the experiment.

The cut in which $bX$ lies only introducts a **minor local perturbation** in the overall spatial distribution of the $B$ field in the space surrounding $M$, unable to denaturalize the main ponderomotive effects which take place on the $CC$ wire (fig.3). There is not reversion of $B$ field in the main parts of the $CC$ wire. In other words, the $CC$ wire "don?t see" the singularity (cut). As an additional proof of the above statement, we have prepared another magnet in which the cut only amounted some 1/150 of the entire annulus surface being the outcome, as far as the reversion of $B$ concerns, identical to the former, albeit the original field distribution remainded almost umperturbed.

## 4. Final considerations

Our main actual aim is to offer conclusive, easy to perform and not expensive experiments available for a better understanding of basic electromagnetic phenomena at the undergraduate level. These phenomena are often darkened due to a lack of cleverly designed experiments. Briefly speaking, with the aid of soundly conceived experiments we will contribute to avoid posterior endless discussions, and waste of time and money. Don?t forgett the recent growing interest in electromagnetic phenomena [3,4,5,6,7,8,9,10,11,12,13].

The behavior of unipolar devices falls within the far reaching realm of true relativistic physics, as clearly stated by Weber towards the middle of the $19^{th}$ century

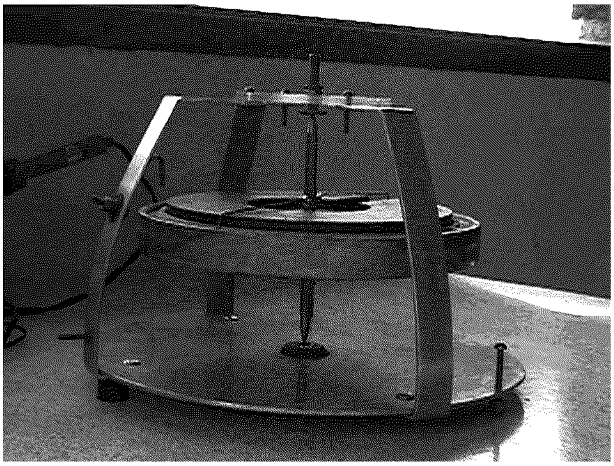

Figure 3: The cut in which $bX$ lies only introducts a **minor local perturbation** in the overall spatial distribution of the $B$ field in the space surrounding $M$, unable to denaturalize the main ponderomotive effects which take place on the $CC$ wire2

[15]. Nevertheless, our work has nothing to do with the observer-based pseudo relativistic theories customarily accepted (see the wrong and untenable statements of Shadowitz on the issue [14] ).

**Acknowledgements**

To Laboratorio Julio Palacios SRL and CAPEX Termoelctrica SA for financial support. To C.N. Gagliardo for helpful technical assistance.

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# DISCUSSION

**From**: V.G. Aleshinsky[1]
**To**: N.A. Zhuck
**Article**: A. Einstein, "The equations of the gravitational field"
**Journal**: Vol. 1 (2000), No. 2, p. 51
**Data**: July 25, 2001

It is possible to explain interaction between charges located in empty space without presence of medium and an ether.

I have found, that it is possible to explain a Coulomb interaction between charges located in empty space without the account of medium and an ether, with the help of ultra microscopic particles - coulombtones, released by charges.

Thus to not upset a series of principles, which we have formulated, (as, for example, principle prohibiting screening by fixed charges one another without origin of dipoles) these particles - coulombtones should be, at least, of two types and have in essence various properties: primary coulombtones and secondary coulombtones. Primary coulombtones are divided into two subtype.

Released by one charge primary coulombtones transit through second charge and prolong to fly further. But they call in the second charge occurrence of secondary coulombtones. Depending on, whether are the interreacting charges one or various signs, the outgoing direction of secondary coulombtones will be variously. Secondary coulombtones take off from a charge in a direction against a motion primary at interaction of charges of one sign and on a motion primary - at interaction of charges of various signs.

Thus primary coulombtones prolong to beat out secondary at all meeting at them on trajectories of charges. Secondary coulombtones on coming across by them the charges do not act on trajectories.

**From**: N.A. Zhuck[2]
**To**: V.G. Aleshinsky
**Article**: DISCUSSION, V.G. Aleshinsky
**Journal**: Vol. 2 (2001), No. 3, this page
**Data**: July 25, 2001

Many authors already has shown, that the gravitational interactions are transmitted by means of medium (ether).

So, the philosophical substantiation of this thesis is given in the paper [1], the cosmological substantiation is given in the works [2-8], and the microscopic analysis is shown in the article [9-12].

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# NEW BOOKS

Robert K. Soberman[1] and Maurice Dubin, "DARK MATTER ILLUMINATED," Infinity Publishing.com, USA, 2001, 166 pp.

This book is about the answer to question about a substance from the sky: what is the dark matter of the Universe?

As dark matter exists and has mass, it must be attracted and accreted into stars. As a direct result the mass of stars increases continuously. This inescapable conclusion is independent of the nature of dark matter. The only possible counters are that the gain in mass from the addition of dark matter is negligible over stellar lifetimes or the dark matter is repelled or diverted. Those assertions, if voiced, are inconsistent with the accepted conclusion that there exists at minimum nine times more "dark" than visible matter and the former must be pervasive. Experimental evidence is cited herein, using our Sun as a stellar example, that the quantity in the solar system is significant and the accre tion while small is measurable and significant on the scale of a stellar lifetime.

Large optical telescopes were the fundamental tools of the astronomer in the first half of the twentieth century. Theory formulated to explain observation was based upon visual observation. In consequence, it is not surprising that an unseen component did not appear in the prevailing model. When Fritz Zwickey [1937] discovered the gravitational influence of unseen masses upon the motions of galaxies the "standard model" was already well established. When Rubin et al. [1980] found the rotation rate of spiral galaxies was being influenced by external unseen masses the prevailing theoretical astrophysical scenario was so entrenched that it was barely modified to include the large scale gravitational influence of this "dark matter."

Today, while the very nature of this component is hotly debated, the prevailing astrophysical model speaks only to the large-scale gravitational force of dark matter [Sadoulet 1999]. While admitting that dark matter composes over ninety percent of the universe, the standard model purports to explain all astrophysics based upon observation of the remainder. What better way to illustrate this then the very name given to a hypothetical body proposed to constitute this mass? That name is Weakly Interacting Massive Particle with the seemingly appropriate acronym WIMP.

Cosmoids (a contraction of cosmic meteoroids) is the name we coined [Dubin & Sobennan 1991] for meteoroids, interstellar and beyond. These small, near invisible, ubiquitous bodies are key to the workings of the Sun and stars. Cold, dark loose aggregates of volatile matter, predominantly hydrogen with a size distribution extending from macromolecules to kilometers, cosmoids are the dark matter of the universe. There is a wealth of experimental data attesting to their existence, beginning decades before their recognition/discovery in the results of the three meteoroid experiments carried on the PIONEER 10 and 11 spacecraft [Dubin & Soberman 1991].

Any model describing the astrophysics of the universe and its components which lacks an understanding of cosmoids, their nature and interactions, requires constant revision and contrived hypothetical physics to explain observations undreamed when the prevailing model was first formulated three quarters of a century ago.

Gravitationally attracted to stars including our sun they are a major influence upon stellar behavior and evolution. The negation of the Russell-Vogt fundamental astrophysical premise is significant. A theoretical patch on the standard model cannot accommodate the collapse of this assumption. If stars grow with time, then their evolution differs significantly from current belief. Stars are born small and add mass as intuition would dictate. The largest most massive stars are not, as currently believed, the youngest, but rather among the oldest in the heavens. This is but one facet of the new astrophysical paradigm demanded by the se revelations.

The following describes this new astrophysical model for the Sun, stars and oar universe based upon the interaction with dark matter (cosmoids). Many enigmas are resolved herein, including tile power heating the Sun's upper atmosphere, driving solar flares and mass ejection. Among numerous other explanations, the driving force for solar and planetary atmospheric super-rotation, the solar neutrino solution, and the link between the Sun's radiation and the Earth's climate.

The model is a product of the space age. Most of the measurements required for its formulation come from instruments and platforms of space age technology hence could not have been articulated earlier. It should be noted that the model hangs together like a chain with each link an integral part that cannot be separated from the whole. Acceptance of a single link requires acceptance of the entirety. Each link is supported by measurement. The observational support spans a diverse group of physical and astrophysical specialties. If the cited data do not suffice, there are tests suggested that might be carried out with existent facilities that can distinguish between popular theory and the model presented in the book.

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# Spacetime & Substance

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